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Shared energy storage strategy with a capacity exchange mechanism

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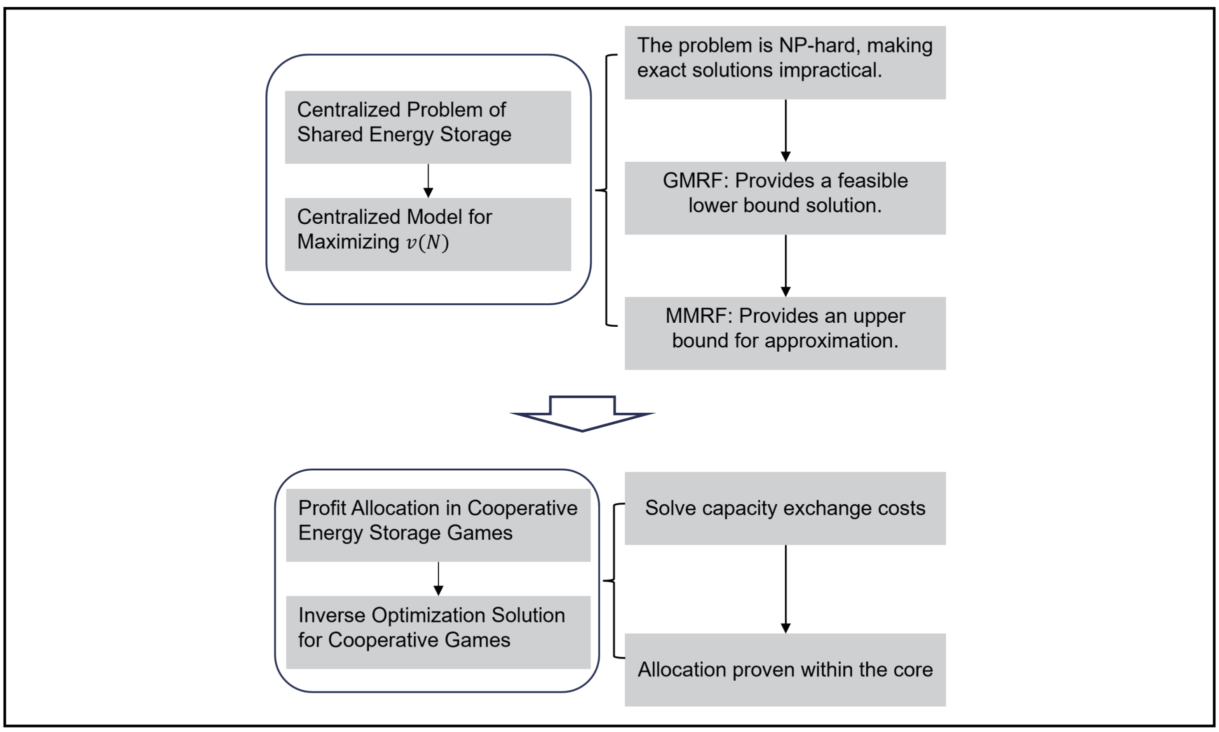
Graphical abstract



Strategy for shared energy storage with capacity exchange, focusing on optimization and cooperative profit allocation.

Public summary

■ To tackle the NP-hard centralized shared energy storage (SES) problem, we introduced an improved Max-Revenue with Flow (MRF) Algorithm. This algorithm reduces computational complexity, demonstrating superior efficiency in case studies compared to existing methods, offering a practical solution for SES optimization.

■ We resolved the conflict between energy efficiency and fair profit distribution by applying inverse optimization techniques to set reasonable capacity exchange costs. This ensures fair profit allocation while maximizing the benefits of the SES alliance, encouraging active participation from all members.

■ Using real data from Qingpu District, Shanghai, we demonstrated SES systems' economic advantages over individual storage solutions. We also proposed a profit allocation mechanism tailored to the district, ensuring budget balance and stability. This mechanism supports the sustainable operation of the SES alliance, offering a practical approach for real-world adoption.

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Abstract: As renewable energy adoption increases globally, the demand for energy storage (ES) has risen accordingly. The sharing economy has emerged as a promising solution to mitigate the high costs associated with ES. However, shared energy storage (SES) faces challenges in balancing efficient energy use and ensuring fair benefit allocation within a SES alliance. This paper addresses these challenges by introducing the Centralized Model for Sharing Storage Decisions (CSSD), which aims to maximize profits for the SES alliance. To solve the CSSD, we propose an enhanced version of the Max-Revenue with a Flow (MRF) Algorithm, called the Greedy Max-Revenue with a Flow (GMRF) Algorithm, which ensures the efficient solution of the centralized problem. Moreover, this paper introduces the concept of capacity exchange cost within the SES alliance. This concept allows ES operators to procure or lease energy storage capacity, generating revenue opportunities. We use linear programming models and inverse optimization to determine the appropriate exchange costs, ensuring that individual profits are maximized while benefiting the entire alliance. The feasibility and effectiveness of these optimization and allocation strategies are validated through simulations conducted in Qingpu district, Shanghai.

Keywords: cooperative game; profit allocation; inverse optimization

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1 Introduction

Renewable energy, while being developed worldwide, presents a set of challenges that need to be addressed. One major challenge lies in the intermittent nature of certain renewable energy sources, resulting in fluctuations in availability due to factors such as weather conditions. This intermittency poses obstacles to ensuring a stable power supply[1]. Furthermore, the high penetration of renewable energy sources has introduced instability into the power grid[2]. For example, the Texas power system serves as an efficient model of a self-sufficient power system by combining stable power supplies such as natural gas with wind energy[3]. However, the collapse of the Texas power system during the February 2021 cold snap shook confidence in renewable energy systems. Additionally, inefficiencies may occur when excess wind energy and excess solar energy are not fully utilized on the demand side. To address these challenges, energy storage (ES) systems have emerged as crucial components of renewable energy systems[4]. Nonetheless, the costs associated with individual ES systems have proven insufficient to meet the growing demand for renewable energy[5].

The sharing economy provides a promising solution to the high costs associated with ES systems by leveraging idle resources to reduce waste and improve efficiency[6]. Shared energy storage (SES) systems, in particular, minimize idle capacity, meet more storage demands, and generate higher profits. Numerous scholars have conducted comparative studies on the returns of individual ES systems versus SES systems, consistently demonstrating that SES systems can yield greater returns[7, 8]. In practical terms, many companies are actively involved in SES projects. For instance, Power Ledger and LO3 Energy have made commitments to utilize blockchain technology as a means to provide SES solutions. These companies have successfully developed a blockchain-based platform that facilitates energy exchange and sharing among individuals and businesses within distributed energy networks. The platform designed by Sunverge promotes the mutual utilization of ES devices by households and businesses, optimizing energy consumption, storage, and distribution.

An SES system consists of three key components: ES operators, storage users, and the SES manager. ES operators own and manage their respective storage systems, providing the infrastructure for energy storage and distribution. Users, including residential, commercial, or industrial entities, rely on the SES system to meet their storage needs, especially during peak demand periods or when there is an excess of renewable energy. The SES manager acts as a coordinator, managing the SES alliance and ensuring its long-term stability and operational efficiency. ES operators unite to form the SES alliance by interconnecting ES systems to share resources. Currently, there are two main ways to connect these systems: indirect connections through the grid and direct connections via lines. The grid-based connection is suitable for urban energy sharing, industrial parks, and charging station networks, as it enables coordination and energy dispatch among different storage systems to optimize energy usage across regions. On the

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other hand, direct connections are ideal for microgrids, corporate alliances, neighborhood energy sharing, and remote infrastructure, enabling energy exchange directly through local networks or systems, thereby reducing reliance on the grid and enhancing dispatch efficiency. These methods are applied flexibly based on specific needs.

The following details explain how the tripartite collaboration operates: ES operators determine their service prices on the basis of user demand. Interaction between users and operators typically occurs in two scenarios. In the first scenario, users proactively submit storage orders, specifying storage duration and energy requirements. This scenario is prevalent in well-planned demand situations, such as factories storing electricity on the basis of production schedules, residential solar users storing excess daytime energy during periods of low electricity prices, or electric vehicle charging stations preparing power in advance on the basis of anticipated demand. In the second scenario, operators use users' historical electricity data to proactively plan storage schedules. This approach is often applied in smart grid systems, large data centers, or households with regular usage patterns to help optimize energy consumption. However, while we describe the interaction between users and operators in terms of service demand and storage usage, the origin of the power supply itself is not explicitly considered in our proposed model. This decision is intentional, as the core focus of our research is on the sharing and coordination of storage resources rather than on the generation or procurement of electricity. Each operator already has their own storage plan, which is tailored to their specific energy supply scenarios. These plans account for factors such as electricity supply, costs, and profit, with operators optimizing for various objectives such as profit maximization or peak shaving. As such, our model builds on the assumption that power supply management is handled individually by participants, and we concentrate on coordinating shared storage resources within these pre-existing plans without revisiting the power supply issue. The SES manager plays a central coordinating role, managing the idle resources of operators and user demands while taking into account factors such as service pricing and energy loss during storage. They allocate storage resources and establish trading rules between operators to maximize the alliance's benefits and ensure the long-term stable operation. Typically, this role is undertaken by third-party entities such as power grid companies. Although the SES manager can technically coordinate different types of storage resources, in practice, aligning storage resources with similar operating frequencies is often more efficient. For example, pairing lithium-ion batteries with pumped hydro storage is not feasible.

However, the SES framework involves coordinating multiple ES operators. It presents distinct challenges in energy allocation, fairness in profit distribution, and system stability. First, unlike other cooperative games, SES cooperative games must consider storage capacity limitations, energy loss during storage and transmission, and the constantly changing demand and storage status. The integration of demand and storage requires adjusting the operational strategies of the SES system to ensure efficient energy storage, transmission, and utilization over different time periods. Second, in cooperative settings, individual ES operators often prioritize their own profits, which can lead to the neglect of potential collective benefits. As Kong et al.[9] emphasized, the key challenge lies in balancing energy efficiency and fair profit allocation. Since the profit allocation in SES systems must consider not only current energy usage but also the long-term effects of storage decisions across different time periods, this challenge becomes even more complex. Therefore, a successful SES system requires not only effective collaborative storage strategies but also a fair profit allocation mechanism that considers both current and future impacts, incentivizing each ES operator to make optimal collaborative decisions[10].

The remaining sections of this paper are organized as follows. In Section 2, we provide a review of relevant research on SES cooperative games. In Section 3, we present the centralized model for the SES alliance. In Section 4, we model the individual perspective for ES operators. In Section 5, we formulate allocation strategies and design cooperative mechanisms. In Section 6, we validate the effectiveness of the proposed solutions through simulations and real-world data. Finally, we conclude the paper in Section 7.

2 Literature review

The problem of SES games can be categorized into two groups[11]: non-cooperative game models and cooperative game models. In non-cooperative game models, the SES manager and consumers are treated as price-makers or price-takers, often modeled through bi-level optimization or Stackelberg games[12–15]. However, such models focus on individual profits and do not consider the overall interests of the cooperative alliance.

In the application of cooperative game theory, several classic studies have provided a crucial theoretical foundation for understanding and solving complex resource allocation problems. Dror and Hartman[16] analyzed how firms could optimize inventory management and reduce costs through joint ordering and explored the stability of cooperation. Unlike inventory management with infinite capacity, the finite nature of ES storage capacity must be considered in SES cooperative games. Hall and Liu[17] investigated cooperative games in capacity allocation, establishing conditions for equilibrium. In these cases, the resources are typically considered non-depleting and constant over time. However, in SES systems, energy storage experiences losses. Mitridati et al.[18] explored community-based market mechanisms, extending core, Shapley value, and nucleolus allocation methods. While this work contributes to energy system allocation, its assumption of static market mechanisms is inadequate for addressing the multi-temporal demands of the SES system. Similarly, Zheng et al.[19] designed an incentive scheme for a producer responsibility organization in waste tire management, offering an optimized solution for static incentives but with limited applicability to cross-period management challenges in SES cooperative games. Therefore, although these classic studies offer valuable theoretical frameworks, the models and methods applicable to SES cooperative games require further development and adjustment to address the unique challenges of finite resources and time dependency.

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Unlike traditional cooperative game studies, the storage and transmission decisions in SES cooperative games are dynamically adjusted on the basis of changing demand, adding extra complexity to the cooperation strategies. Current approaches to address this issue largely rely on predictive results. Existing literature explores cooperative games within the SES context. In cooperative game research, an SES manager is introduced to replace individual ES operators. It consolidates the energy resources and demands of all operators to formulate central charging and discharging plans. This issue is known as *the centralized problem*[20]. In SES cooperative game research, many studies have focused on centralized optimization models. For example, Song et al.[21] developed a planning model to optimize multi-site power generation with the SES, whereas Ma et al.[22] used a dual-layer optimization model to provide optimal strategies for multiple stakeholders. However, these models address centralized optimization without fully considering the competition between users for limited storage capacity. Similarly, Hafiz et al.[23] and Shen et al.[24] introduced uncertainty into SES models to account for renewable energy variability. Despite these advances, these studies still face challenges in ensuring fair and efficient storage capacity allocation among competing users.

Cooperative game theory provides a comprehensive mathematical framework for equitable cost and benefit sharing among collaborators. Importantly, a well-designed allocation mechanism within this theory can ensure alignment between individual and collective benefits. Cooperative game models have already been widely researched across various fields. Shapley[25] introduced the Shapley value, which provides fair allocations regardless of the core's status, but it may not always align with the core, potentially leading to instability. Schmeidler[26] proposed the nucleolus, which minimizes the dissatisfaction of the least satisfied coalitions. If the core is non-empty, the nucleolus will be within it, ensuring stability. However, finding the nucleolus can be complex and does not always meet monotonicity criteria. Nash's bargaining solution addresses surplus division between two parties but is limited to two-party scenarios[27]. Gillies[28] introduced the core, which represents allocations where no coalition can improve their outcome. One of the well-known results in finding solutions in the core, as studied by Owen[29], shows that an allocation within the core can be obtained by solving a dual linear program. However, dual payoffs are not always in the core[10].

Existing literature explores cost or profit allocations within the SES context. Proportional profit allocation is the simplest and most prevalent method for allocating profits in cooperative games[30]. For example, Liu et al.[31] proposed an optimal planning method for SES games, in which benefits are allocated on the basis of each retailer's contribution. Kim et al.[32] maximized regional profits through arbitrage trading, with curtailment allocations of each photovoltaic power producer based on power sensitivity. Many studies on SES games use Shapley values to share cooperative costs or benefits. Yu et al.[33] proposed a community power coordinated dispatching model based on blockchain technology, establishing a community power coordination scheduling model that considers shared energy and demand response. They used the improved Shapley value method to allocate coordination scheduling costs, encouraging the community to join the blockchain. Moreover, profit allocation based on the nucleolus is more stable than the Shapley value method is, but it is often computationally challenging[34]. However, profit allocation within existing SES alliances often faces challenges due to the fixed allocation of storage capacity, leading to inefficiencies from unused capacity. Moreover, improving computational efficiency and ensuring fairness in allocation remain key challenges in SES alliance studies.

While existing studies provide valuable insights into SES optimization and cooperation strategies, they face several limitations, such as fixed allocation of storage capacity, inefficiencies from unused capacity, and challenges in ensuring fairness among participants. To address these limitations, our paper proposes several key improvements. First, we introduce an enhanced network flow algorithm to solve the centralized optimization problem more efficiently and accurately. Second, we propse a novel concept of *capacity exchange cost*, which allows users to redistribute benefits more flexibly. Finally, we employ inverse optimization techniques to ensure that both alliance and individual stakeholders benefit from the decisions made.

In summary, SES cooperative games present unique challenges due to the finite and dynamic nature of energy storage. Existing models have laid important theoretical foundations but often fail to account for these complexities. By introducing new optimization algorithms and capacity allocation methods, our study aims to provide more effective and equitable solutions to the SES problem.

3 A centralized model for sharing storage decisions

In this paper, we explore the SES system depicted in Fig. 1. Each community consists of storage users and a storage device managed by an ES operator. The primary goal of ES operators is to generate profits by offering storage services that enable users to store energy. To maximize their collective gains, multiple ES operators cooperate to establish a shared energy storage system, aiming to fulfill a broader range of user demands. To facilitate the coordination of charging and discharging actions among the community storages, an SES manager is assumed to be responsible. Furthermore, each community's storage is equipped with a net metering device, which monitors the flow of energy. The motivation behind ES operators seeking collaboration stems from their recognition that cooperative operations yield benefits that go beyond individual endeavors. This paper specifically focuses on the development of a cooperative mechanism for the SES system, with the primary goal of facilitating resource sharing among ES users.

To ensure the tractability and computational efficiency of the model, we propose the following set of assumptions. First, the formulation and execution of ES via plans are based on discrete time intervals. This discretization allows for efficient calculations and decision-making processes. Second, the storage price per unit of electricity is assumed to be known, and different ES operators have the flexibility to set distinct unit prices on the basis of varying storage periods and amounts.

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Third, the operating costs associated with the storage system are considered to be fixed throughout the operating cycle and are not explicitly addressed in this paper. Fourth, demand information for storing electricity encompasses both the desired electricity volume and the intended storage period. Moreover, demands with the same storage period can be consolidated to simplify the optimization process. Fifth, any losses in electricity during transmission and storage processes are promptly compensated. These losses are taken into account in the program through the use of appropriate cost coefficients, ensuring an accurate representation of the system dynamics.

Specifically, the problem is modeled within the context of a cooperative game, a domain of game theory that examines the collaborative actions of rational agents. Therefore, the space-time network model described in this paper can be simplified to the network shown in Fig. 2. We assign labels to the ES operators as $\mathcal{N}=\{1,2,\cdots,N\}$, indexed by i. The ES can be indirectly connected through the grid or directly connected in sequence via self-built lines. Within an SES alliance consisting of N communities, we define any group of communities $\mathcal{S}\subseteq\mathcal{N}$ that shares storage as a sub-alliance. The operating time units of energy storage devices can differ. As shown in Fig. 2, power can be transmitted between the corresponding energy storage operation time nodes. Consequently, the combination of the user set $\mathcal{N}$ and time points forms a spatiotemporal network for the SES system, denoted as $G=(V,E)$, where $e=(v,w)$ for $v,w\in V$ and $e\in E$. The demand for storing electrical power by community i is denoted as $D^{(s,d,i)}$, where s and d represent the start and end nodes corresponding to the demand in the spatiotemporal network. The decision of community i to satisfy demand $D^{(s,d,i)}$ is denoted as $q_{(d,s)}^{(s,d,i)}$. Additionally, $q_{(u,v)}^{(s,d,i)}$ represents the flow decision of $q_{(d,s)}^{(s,d,i)}$ on edge (u,v). As stated in Assumption 5, the costs associated with power transmission and losses are quantified by the coefficient $\eta_{(u,v)}$, allowing for the estimation of losses during power transmission and charging/discharging processes, as well as losses during the storage period. Each community determines the unit revenue $R^{(s,d,i)}$ for fulfilling demand $D^{(s,d,i)}$. The characteristic function $v(S)$ measures the economic gain of an SES alliance S. We represent the coalition game with N players as $(\mathcal{N},v)$, where $v:2^{\mathcal{N}}\to\mathbb{R}$ maps the value $v(S)$ to each subgroup $S\subseteq\mathcal{N}$. The notations are shown in Table 1.

In this paper, we characterize the value of an SES alliance $v(\mathcal{N})$ by the profit generated by the alliance. To maximize profit, we present a Centralized Model for Sharing Storage Decisions (CSSD) expressed as follows.

$$v(\mathcal{N})=\max_{q}\sum_{(s,d,i)\in D}q_{(d,s)}^{(s,d,i)}\cdot R^{(s,d,i)}-\sum_{(s,d,i)\in D}Cost^{(s,d,i)} \tag{1}$$

$$\text{s.t.}\quad \sum_{(u,v)\in E\bigcup\{(d,s)\}}q_{(u,v)}^{(s,d,i)}-\sum_{(v,w)\in E\bigcup\{(d,s)\}}q_{(v,w)}^{(s,d,i)}\le 0,\quad \forall v\in V,\quad \forall(s,d,i)\in D; \tag{2}$$

$$q_{(d,s)}^{(s,d,i)}\le D^{(s,d,i)},\quad \forall(s,d,i)\in D; \tag{3}$$

$$\sum_{(s,d,i)\in D}q_{(u,v)}^{(s,d,i)}\le C_{(u,v)},\quad \forall(u,v)\in E; \tag{4}$$

$$Cost^{(s,d,i)}=\sum_{(u,v)\in E}q_{(u,v)}^{(s,d,i)}\cdot\eta_{(u,v)},\quad \forall(s,d,i)\in D; \tag{5}$$

$$q_{(u,v)}^{(s,d,i)}\ge 0,\quad \forall(u,v)\in E,\quad \forall(s,d,i)\in D. \tag{6}$$

The objective function (1) aims to maximize the profit of the SES system. Constraint (2) ensures energy conservation within the collaborative system by balancing supply and demand. Constraint (3) guarantees that the electricity supplied does not exceed the corresponding demand. Constraint (4) imposes capacity constraints for both storage and transmission lines. Constraint (5) calculates the total cost associated with each flow. Finally, constraint (6) imposes a non-negativ-

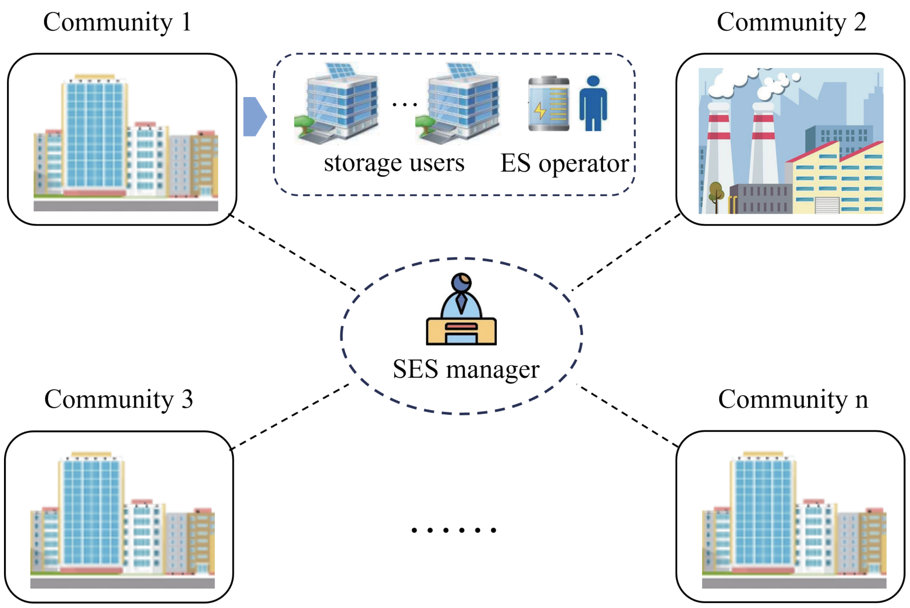

Fig. 1. The shared energy storage system.

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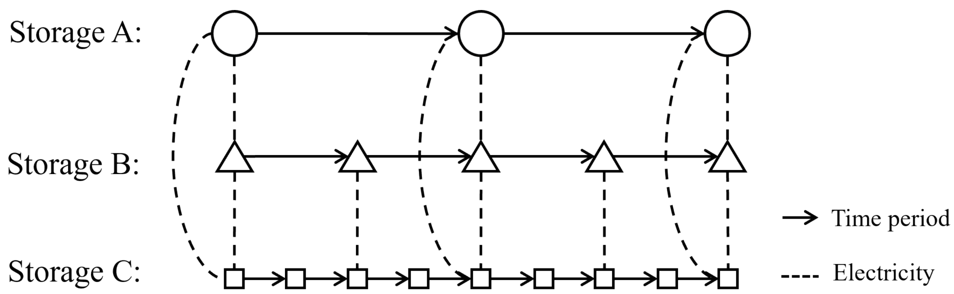

Fig. 2. Spatiotemporal network.

Table 1. Notations.

Notation	Description
$\mathcal{N}=\{1,2,\cdots,N\}$	Set of participants in the shared storage cooperative game
$\mathcal{T}=\{1,2,\cdots,T\}$	Set of time points
$\mathcal{D}^i$	Set of Power storage demands for ES operator i
$\mathcal{D}=\bigcup_{i=1}^{n}\mathcal{D}^i$	Set of demands for all ES operators
$D^{(s,d,i)}$	Power storage demand from point s to point d for ES operator i
(u,v)	Edges in time-space network
$q_{(u,v)}^{(s,d,i)}$	Power transmitted along edge (u,v) to satisfy the demand $D^{(s,d,i)}$
$q_{(d,s)}^{(s,d,i)}$	Total power transmitted to satisfy the demand $D^{(s,d,i)}$
$R^{(s,d,i)}$	Revenue generated for ES operator i when fulfilling the demand of one unit, $D^{(s,d,i)}$
$\eta_{(u,v)}$	Cost of power transmitted along edge (u,v)
$Cost^{(s,d,i)}$	Cost of power transmitted to meet demand $D^{(s,d,i)}$
$C_{(u,v)}$	Capacity of edge (u,v)
$cost_{(u,v)}$	Storage capacity exchange cost

ity requirement.

Theorem 1. The CSSD problem is NP-hard.

The CSSD problem belongs to the NP-hard class. We establish its NP-hardness by reducing it to a well-known NP-hard problem, the 0-1 knapsack problem.

4 Modeling the individual perspective

The CSSD provides optimal decisions denoted as q^* for the SES alliance. However, in practice, individual ES operators often prioritize their own interests, which poses difficulties in achieving adherence to these optimal decisions. This challenge complicates the implementation and maintenance of the optimal decision-making process. To address this issue, this section introduces a model that takes into account the perspective of individual ES operators. The proposed approach involves the introduction of a concept called the capacity exchange cost. This concept enables each community to meet its storage needs by leasing energy from other communities or generating revenue by leasing out its own ES capacity to others. By incorporating capacity exchange costs, the primary objective of this model is to promote cooperation and resource sharing among ES operators, thereby ensuring the stability and long-term sustainability of the alliance.

First, we introduce binary variables $\gamma_{(u,v)}^k$ to denote whether the storage belongs to ES operator k:

$$\gamma_{(u,v)}^k=\begin{cases}1, & (u,v)\text{ is operator }k\text{'s time period;}\\ 0, & \text{otherwise.}\end{cases} \tag{7}$$

Therefore, the profit gained by operator k through ES capacity exchange is as follows:

$$s^k=\sum_{(u,v)\in E}cost_{(u,v)}\cdot\gamma_{(u,v)}^k\cdot\sum_{(s,d,i)\in D\setminus D^k}q_{(u,v)}^{(s,d,i),k}-\sum_{(u,v)\in E}cost_{(u,v)}\cdot(1-\gamma_{(u,v)}^k)\cdot\sum_{(s,d,i)\in D^k}q_{(u,v)}^{(s,d,i),k}.$$

The first part of the equation represents the revenue earned by operator k by leasing out their own storage capacity. The latter part denotes the cost incurred by operator k for renting storage capacity from other participants. When $s^k\geq 0$, s^k is interpreted as the profit obtained by renting out ES capacity. Conversely, when s^k is negative, it can be seen as the cost of renting ES capacity from other participants. To ensure an equitable distribution of profits within the SES alliance, it is crucial to design appropriate adjustment factor $cost_{(u,v)}$. These adjustment factors play a vital role in balancing and optimizing the allocation of profits among alliance members, thus fostering fairness and collaboration. The primary goal for each individual ES operator is to maximize its own profit, and the CSSD^k model can be formulated as follows.

$$\max_q\sum_{(s,d,i)\in D^k}q_{(d,s)}^{(s,d,i),k}\cdot R^{(s,d,i)}-\sum_{(s,d,i)\in D^k}\sum_{(u,v)\in E}q_{(u,v)}^{(s,d,i),k}\cdot\eta_{(u,v)}+\sum_{(u,v)\in E}\left[\gamma_{(u,v)}^k\sum_{(s,d,i)\in D^k}q_{(u,v)}^{(s,d,i),k}-(1-\gamma_{(u,v)}^k)\sum_{(s,d,i)\in D^k}q_{(u,v)}^{(s,d,i),k}\right]cost_{(u,v)} \tag{8}$$

$$\text{s.t.}\quad\sum_{(u,v)\in E\bigcup\{(d,s)\}}q_{(u,v)}^{(s,d,i),k}-\sum_{(v,w)\in E\bigcup\{(d,s)\}}q_{(v,w)}^{(s,d,i),k}\leq 0,\quad\forall v\in V,\ \forall(s,d,i)\in D; \tag{9}$$

$$q_{(d,s)}^{(s,d,i),k}\leq D^{(s,d,i),k},\quad\forall(s,d,i)\in D; \tag{10}$$

$$\sum_{(s,d,i)\in D}q_{(u,v)}^{(s,d,i),k}\leq C_{(u,v)},\quad\forall(u,v)\in E; \tag{11}$$

$$q_{(u,v)}^{(s,d,i),k}\geq 0,\quad\forall(u,v)\in E,\ \forall(s,d,i)\in D. \tag{12}$$

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The objective function (8) aims to maximize the profit of operator k. Constraint (9) ensures the conservation of energy within the collaborative system, maintaining a balance between supply and demand. Constraint (10) guarantees that the electricity supplied does not exceed the corresponding demand. Constraint (11) establishes capacity constraints for both energy storage units and power lines. Finally, constraint (12) imposes a non-negativity requirement.

In traditional optimization, the goal is to find the best decision given an objective and constraints. In contrast, inverse optimization starts with known decisions and works backward to determine the objective or constraints that make those decisions optimal. Essentially, inverse optimization aims to find the parameters of a “forward” model such that the given decisions are either exactly or approximately optimal. This process is called inverse optimization.

We begin with the optimal solution $q^* = [q_1^*, q_2^*, q_3^*, \cdots, q_N^*]$ for the CSSD and aim to solve the exchange cost $cost_{(u,v)}$ that would make q^* optimal for the CSSD^k model. $cost_{(u,v)}$ is defined as inverse feasible with respect to q^* if q^* is an optimal solution to the CSSD^k model at a given $cost_{(u,v)}$. To compute $cost_{(u,v)}$ for which every q_k^* is inverse feasible, we solve the inverse optimization problem as follows[35].

$$\min_{cost} \{0 \mid q^* \in optimal(\text{CSSD}^k(cost))\ \forall k \in \{1, \cdots, N\}, cost \geq \mathbf{0}\}. \tag{13}$$

For player k, let (CSSD^k) denote the dual problem of the CSSD^k. The dual variables associated with constraints (9), (10), and (11) are denoted as (α^k), (δ^k), and (ε^k), respectively. A solution q^* is optimal for the CSSD^k, and the tuple $(\alpha^k, \delta^k, \varepsilon^k)$ is the corresponding optimal solution for the (CSSD^k), only if q^* and $(\alpha^k, \delta^k, \varepsilon^k)$ are feasible for their respective problems and satisfy the complementary slackness conditions.

Therefore, we can establish the inverse optimization problem (ICSSD^k) as (14)–(21). Constraints (15)–(18) represents the constraints of the (CSSD^k) problem. Constraints (19) and (20) indicate the complementary slackness conditions that must be satisfied when the optimal solution to the CSSD^k problem is q^*. Constraint (19) states that when the demand for a community during a specific time period is not entirely met, the corresponding dual variable must be equal to 0. Constraint (20) expresses that when there is available storage capacity during a certain time interval, the corresponding dual variable must be equal to 0. Notably, the dual variables corresponding to the edges representing the power lines are set to 0 because of the assumption of an infinite capacity for the power lines. Finally, constraint (21) introduces the non-negativity requirement.

If $(\alpha^{(s,d,i),k} u, \delta^{(s,d,i),k}, \epsilon^k(u,v), cost_{(u,v)})$ is a feasible solution for the ICSSD^k, it indicates that setting the exchange price to $cost_{(u,v)}$ makes the optimal solution q^* for the CSSD model also optimal for the CSSD^k model. This encourages the operator k to align with the optimal decision of the alliance q^*, promoting cooperation and coordination. To ensure that every operator $k \in N$ adheres to the optimal alliance decision q^*, it is necessary to find a suitable $cost_{(u,v)}$ that satisfies the constraints of the ICSSD^k for each operator k. Taking ICSSD^N as the set of constraints composed of all the ICSSD^k constraints for each operator $k \in N$, we have the following theorem.

$$\min\ 0 \tag{14}$$

$$\text{s.t.}\quad \alpha_u^{(s,d,i),k} - \alpha_v^{(s,d,i),k} + \varepsilon_{(u,v)}^k \geq \gamma_{(u,v)}^k \cdot cost_{(u,v)},$$
$$\forall (s,d,i) \in D/D^k, \quad \forall (u,v) \in E; \tag{15}$$

$$\alpha_u^{(s,d,i),k} - \alpha_v^{(s,d,i),k} + \varepsilon_{(u,v)}^k \geq (\gamma_{(u,v)}^k - 1) \cdot cost_{(u,v)} - \eta_{(u,v)},$$
$$\forall (s,d,i) \in D^k, \quad \forall (u,v) \in E; \tag{16}$$

$$\alpha_d^{(s,d,i),k} - \alpha_s^{(s,d,i),k} + \delta^{(s,d,i),k} \geq 0, \quad \forall (s,d,i) \in D/D^k; \tag{17}$$

$$\alpha_d^{(s,d,i),k} - \alpha_s^{(s,d,i),k} + \delta^{(s,d,i),k} \geq R^{(s,d,i)}, \quad \forall (s,d,i) \in D^k; \tag{18}$$

$$\delta^{(s,d,i),k} \cdot (f_{(d,s)}^{*(s,d,i)} - D^{(s,d,i)}) = 0, \quad \forall (u,v) \in E; \tag{19}$$

$$\varepsilon_{(u,v)}^k \cdot \left(\sum_{(s,d,i) \in D} f_{(u,v)}^{*(s,d,i)} - C_{(u,v)} \right) = 0,$$
$$\forall (u,v) \in E, \quad \forall (s,d,i) \in D, \tag{20}$$

$$cost_{(u,v)} \geq 0 \quad \forall (u,v) \in E. \tag{21}$$

Theorem 2. ICSSD^N is always feasible.

Theorem 2 guarantees the existence of feasible solutions for the ICSSD^N, thereby ensuring the availability of capacity exchange costs $cost_{(u,v)}$, which enables individual ES operators to make decisions aligned with the collaborative optimal solution. This result establishes the foundation for achieving coordination and cooperation among ES operators within the alliance, as it guarantees the feasibility of determining appropriate exchange costs that facilitate the adoption of the optimal solution for the collective benefit of the system.

5 Solutions and allocations

5.1 Algorithm for the centralized model for sharing storage decisions

On the basis of Theorem 1, it can be inferred that the CSSD problem is NP-hard. This complexity arises from the large number of variables present in the linear programming formulation of the CSSD, even for problems of moderate size. The considerable scale of the model comes from the exponential growth in the number of possible cycles. To address this challenge, we propose the Greedy Max-Revenue with a Flow (GMRF) algorithm, which is based on the Max-Revenue with a Flow (MRF) Algorithm. This algorithm aims to provide a feasible solution that closely approximates the optimal solution in a short timeframe. The steps involved in this approach are as Algorithm 1.

The GMRF algorithm, as described in Algorithm 1, first sorts all the demands of operator i in set S in descending order on the basis of their unit profit. It then assigns a sequential priority label to each demand. In cases where resources are limited, the GMRF algorithm prioritizes fulfilling the de-

Algorithm 1. GMRF algorithm.

Require: The original network G. The capacity matrix C. The revenue matrix W. The number of customers, N. $[S_i, E_i, D_i, R_i]$ of each customer.

Ensure: The total revenue TR_{GA} under Greedy Algorithm.

1: $TR_{GA} = 0$.

2: Sort all the tuples of $[S_i, E_i, D_i, R_i]$, ensuring that $R_1 \geq R_2 \cdots \geq R_N$.

3: Build up the original network. Setting S_0, E_0 as the source and tank node of the graph.

4: **for** $i = 1$ to N **do**

5: Add an edge from S_0 to S_i. $C(S_0, S_i) = +\infty$, $W(S_0, S_i) = 0$.

6: Add an edge from E_i to E_0. $C(E_i, E_0) = D_i$, $W(E_i, E_0) = R_i$.

7: Build up the residual network G_r.

8: Conduct the MRF Algorithm on G_r and get the result MR and f.

9: $TR_{GA} = TR_{GA} + MR$.

10: **for** $(u,v) \in G$ **do**

11: $C(u,v) = C(u,v) - f(u,v)$.

12: **end for**

13: **end for**

mands that generate the highest economic benefit, as demonstrated in Step 2 of Algorithm 1. The algorithm avoids evaluating all possible combinations, significantly narrowing the search space and shortening the computation time. Simultaneously, GMRF progressively allocates resources, updating capacity and flows at each step, providing a near-optimal solution within a limited time. Moreover, the GMRF algorithm operates on the fundamental principle of sequentially fulfilling demands while ensuring that flow balance, demand quantities, and capacity constraints are met. This ensures that the flow values obtained through the greedy algorithm align with the specified constraints (2–4), guaranteeing a feasible solution for the CSSD. Therefore, the GMRF algorithm effectively establishes a lower bound for the CSSD.

Furthermore, to address the problem with relaxed constraints and explore potential upper bounds for the optimal solution, we introduce the Mixed Max-Revenue with Flow Algorithm (MMRF), as illustrated in Algorithm 2.

The inclusion of constraint (2) adds complexity to the problem since it necessitates balancing the flow at each network node for each demand, resulting in longer solution times. The MMRF algorithm mitigates this complexity by relaxing constraint (2), as depicted in Steps 3–5 of Algorithm 2. This hybrid algorithm effectively reduces the burden imposed by constraint (2) by modifying it to:

$$\text{s.t.} \sum_{(s,d,i)\in D} \sum_{(u,v)\in E^{(s,d)}} q^{(s,d,i)}_{(u,v)} - \sum_{(s,d,i)\in D} \sum_{(v,w)\in E^{(s,d)}} q^{(s,d,i)}_{(v,w)} \leq 0 \quad \forall v \in V. \tag{22}$$

The MMRF algorithm simplifies the problem by relaxing the flow balance constraints at the network nodes, which accelerates the computation. By loosening strict balance constraints (2), the algorithm expands the solution space and identifies high-revenue solutions, reducing computational difficulty without significantly increasing accuracy. This relaxed constraint approach allows the MMRF to quickly find an upper bound of the optimal solution within a broader solution space.

Algorithm 2. MMRF algorithm.

Require: The original network G. The capacity matrix C. The revenue matrix W. The number of customers, N. $[S_i, E_i, D_i, R_i]$ of each customer.

Ensure: The total revenue TR_{MA} under Mixed Algorithm.

1: $TR_{MA} = 0$.

2: Build up the original network. Setting S_0, E_0 as the source and tank node of the graph, respectively.

3: **for** $i = 1$ to N **do**

4: Add an edge from S_0 to S_i. $C(S_0, S_i) = +\infty, W(S_0, S_i) = 0$.

5: Add an edge from E_i to E_0. $C(E_i, E_0) = +\infty, W(E_i, E_0) = R_i$.

6: **end for**

7: Build up the residual network G_r.

8: Conduct the MRF Algorithm on G_r and get the result MR and f.

9: $TR_{MA} = MR$.

5.2 Allocation mechanism design

In this paper, we leverage cooperative game theory to develop a collaborative mechanism for SES alliances. The application of principles from cooperative game theory allows us to evaluate the effectiveness of the allocation mechanism employed. Within the realm of cooperative games, an allocation x_i is in the core if it satisfies two key properties: ① The budget balance property, which ensures the equitable distribution of all benefits, can be expressed as $\sum_{i\in N} x^i = v(\mathcal{N})$. ② Stability property, which indicates that the gains obtained by each participant in the grand coalition exceed what they can achieve by leaving the grand coalition and is expressed as $\sum_{i\in S} x^i \geq v(S) \quad \forall S \subset \mathcal{N}$. An allocation x_i is considered stable when it is proven to be within the core, satisfying these properties.

Having established our ability to efficiently solve the CSSD and optimize the SES alliance, we now shift our focus to the allocation of profits. It has been proven that ICSSD^N always has feasible solutions, indicating the existence of exchange cost $cost_{(u,v)}$ that aligns individual storage decisions with optimal alliance decisions. As a result, the objective function of CSSD^i represents the allocation for each participant in the alliance, denoted as x^i with:

$$x^i = p^i + s^i, \tag{23}$$

$$p^i = \sum_{(s,d,i)\in D^k} q^{(s,d,i),k}_{(d,s)} \cdot R^{(s,d,i)} - \sum_{(s,d,i)\in D^k} \sum_{(u,v)\in E} q^{(s,d,i),k}_{(u,v)} \cdot \eta_{(u,v)}, \tag{24}$$

$$\begin{aligned} s^i = & \sum_{(u,v)\in E} cost_{(u,v)} \cdot \gamma^k_{(u,v)} \cdot \sum_{(s,d,i)\in D\setminus D^k} q^{(s,d,i),k}_{(u,v)} - \\ & \sum_{(u,v)\in E} cost_{(u,v)} \cdot (1-\gamma^k_{(u,v)}) \cdot \sum_{(s,d,i)\in D^k} q^{(s,d,i),k}_{(u,v)}. \end{aligned} \tag{25}$$

We now examine whether the allocation x^i presented in Eq. (23) belongs to the core of the cooperative game for SES systems. First, the allocation provided by Eq. (23) guarantees the budget balance property. According to Theorem 2, it is ensured that we can find $cost_{(u,v)}$ such that each individual's op-

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timal decision aligns with the optimal decision of the alliance q^*. Substituting q^* into Eq. (24) reveals that $p^i = v(\mathcal{N})$. Furthermore, when we evaluate the profit or cost generated through the exchange of energy storage, we find that $\sum_{i\in\mathcal{N}} s^i = 0$. Consequently, the allocation satisfies the budget balance constraint, since $\sum_{i\in\mathcal{N}} x^i = v(\mathcal{N})$.

The subsequent step involves analyzing the stability property of the allocation x^i. By outlining the calculation process for $v(\mathcal{S})$ and presenting its dual, we can establish that for any $i \in \mathcal{S}$, there exists $\sum_{i\in\mathcal{S}} x^i \geq v(\mathcal{S})$. This demonstrates that the allocation satisfies the stability property, as explicitly stated in Theorem 3.

Theorem 3. The allocation x^i shown in Eq. (23) has the stability property.

6 Case study

In this section, we conduct a comprehensive evaluation of the GMRF algorithm as a solution to the CSSD. To thoroughly assess the effectiveness of the algorithm in various scenarios, we employed multiple sets of simulated random data. Furthermore, we applied the proposed allocation mechanism to real-world data obtained from the Qingpu District in Shanghai, providing valuable insights into its practical application and its impact on promoting fairness and efficiency within cooperative alliances. All computations were performed on a computer equipped with 32 GB RAM and a 4.0 GHz Intel Core i7-6700K processor.

6.1 Algorithm evaluation

The objective of the experiment was to evaluate the performance of the GMRF and MMRF algorithms. To conduct a comprehensive assessment, we varied several parameters, including the spatio-temporal network structure of alliance members, ES capacity, and load demand scale. Our experiment had two primary objectives. First, we compared the computational speed of the GMRF and MMRF algorithms with that of Gurobi. This comparison allowed us to assess the efficiency of our proposed algorithms in solving the CSSD. Second, our objective was to determine whether the GMRF algorithm could provide a comprehensive explanation of the optimal solution.

To evaluate the computational speed improvements of the proposed algorithm, this study conducted random simulations using three spatio-temporal network configurations of varying scales: (a) 3 ES operators with 4 decision time units, (b) 5 ES operators with 6 decision time units, and (c) 5 ES operators with 24 decision time units. Three demand density levels (low, moderate, high), represented by the ratio D_i of total user demand to total energy storage capacity, were tested. The simulation parameters were configured as follows:

(i) Storage costs: Per-unit-time storage cost $\eta_{(u,v)}$ was sampled from a uniform distribution over $(0,1)$, then scaled by 1×10^{-1}.

(ii) Transmission costs: Inter-operator transmission cost $\eta_{(u,v)}$ followed a uniform distribution over $(0,1)$ scaled by 1×10^{-3}.

(iii) User demand: ① Order times were uniformly distributed across the decision cycle; ② Electricity storage demands followed a discrete uniform distribution over integers [1, 180].

Table 2 presents a comparison of the results obtained from the GMRF, MMRF algorithms, and the Gurobi-based solution. The first column shows the network size, and the second column shows the demand-to-storage ratio. The third, fourth and fifth columns present the optimal solutions from Gurobi, the GMRF algorithm, and the MMRF algorithm, respectively. The sixth, seventh, and eighth columns show the CPU times required by Gurobi, GMRF, and MMRF, respectively. The ninth column shows the relative difference between the GMRF and Gurobi solutions.

As shown in columns 6, 7, and 8 of Table 2, both the GMRF and MMRF algorithms significantly improve the processing speed. As shown in columns 3, 4, and 5, the GMRF consistently outperforms the MMRF in terms of solution quality. In particular, MMRF solutions are not always feasible for the original problem, unlike GMRF solutions. Thus, MMRF should be used as a supplementary tool, whereas the GMRF remains the primary choice. When Gurobi fails to deliver the optimal solution, the large difference between the GMRF and MMRF results indicates the accuracy of the GMRF. For larger-scale cases, the GMRF is preferred for its superior performance and reliability.

6.2 Experimental results

Table 2. Comparison of the GMRF, MMRF, and Gurobi results.

(N,T)	Di	G^*	Gr^*	M^*	Gt (s)	Grt (s)	Mt (s)	$GgGap$
(3,4)	0.36	415.34	415.34	417.35	0.03	0.05	0.03	0.0000%
(3,4)	0.73	409.49	409.49	413.42	0.02	0.06	0.03	−0.0072%
(3,4)	1.45	386.60	386.59	397.97	0.02	0.06	0.04	−0.0041%
(5,6)	0.25	1901.22	1901.22	1901.38	0.85	0.05	0.04	0.0000%
(5,6)	0.49	1884.59	1884.41	1884.82	0.89	0.08	0.04	−0.0095%
(5,6)	0.99	1772.13	1771.00	1795.83	0.86	0.07	0.04	−0.0634%
(5,24)	0.25	2116.41	2116.41	2155.16	778.48	0.07	0.06	0.0000%
(5,24)	0.51	2075.32	2074.09	2141.78	1332.86	0.09	0.07	−0.0593%
(5,24)	1.02	1958.84	1952.41	2070.27	1285.32	0.12	0.08	−0.3286%

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As one of China's most urbanized cities, Shanghai has high energy consumption. The abundant sunlight in the city also favors the development of photovoltaic power. The Shanghai government actively promotes clean energy development through a series of favorable policies. With the rapid growth of photovoltaics, the penetration of renewable energy in the Shanghai grid is steadily increasing. However, the volatility and uncertainty of renewable energy can destabilize the power system and affect its quality. In this context, innovative energy storage (ES) systems are crucial for transforming the energy structure and increasing the use of renewable energy. ES devices will be essential for balancing supply and demand, storing surplus energy, and improving the reliability and stability of Shanghai's power system. Although Shanghai is actively promoting the development of ES, breakthroughs in advanced technologies will take time. Efficient management and operational models are essential for improving the cost-effectiveness of ES systems.

This study analyzed the operational patterns of ES systems using 15-minute interval data from 1236 photovoltaic (PV) users in Shanghai's Qingpu District during May 2022, including installed capacity, power generation, and grid interaction records. The district was divided into five geographically defined communities with the following configurations:

Community 1: 421 PV users across Baihe Town, Xianghuaqiao Subdistrict, Chonggu Town, Yingpu Subdistrict, and Xiayang Subdistrict (Total capacity: 3850 kW).

Community 2: 522 PV users in Huaxin & Zhaoxiang Towns (Total capacity: 5240 kW).

Community 3: 299 PV users in Xujing Town (Total capacity: 1790 kW).

Community 4: 554 PV users in Zhujiajiao Town (Total capacity: 5500 kW).

Community 5: 251 PV users in Jinze & Liantang Towns (Total capacity: 1980 kW).

Key simulation parameters were set as follows:

(Ⅰ) Spatio-temporal network: During the simulation process, we divided users into five communities on the basis of their geographical distribution, assuming that energy storage installations are centrally located within each community. In addition, we defined the planning horizon as one day, with decision intervals set every two hours. Therefore, the spatio-temporal network for this study consists of five energy storage operators and twelve decision time points.

(Ⅱ) Transmission cost of electricity: The loss of electricity in transmission between ES operators is calculated by multiplying the stored electricity by a coefficient related to the distance between the ESs of each community, which is related to the distance of the ESs of each community.

(Ⅲ) Storage cost of electricity: The loss of storing electricity during the storage process is calculated on the basis of the stored electricity multiplied by a coefficient that reflects storage performance, which is related to the performance of the storage.

(Ⅳ) Storage demand of users: Combining users' actual generation data and demand data, we simulate the electricity storage demand for each user. The storage demands for users within the same time period for each community have been aggregated, with the results shown in Table 3.

(Ⅴ) Revenue from meeting storage demands: Two sets of control experiments are conducted, one determined by time-of-use tariffs and the other with individual pricing by each ES operator. During peak hours in Shanghai (6:00–22:00), the selling price of electricity is 0.617 yuan per kilowatt hour, whereas during non-peak hours (22:00–6:00), it is 0.307 yuan per kilowatt hour. The configuration of energy storage enables users to charge during low-price periods and discharge during high-price periods to arbitrage the price difference. Each operator's price will depend on the duration and volume of users' storage orders.

(Ⅵ) ES capacity: According to relevant policies in Shanghai, each community is equipped with ES capacity at a ratio of 0.2 to the installed photovoltaic capacity.

On the basis of the community ES demand described in Table 3, this article, which is based on the content of Chapters 4 and 5, formulates a cooperative SES plan for photovoltaic users in Qingpu District. We establish a community ES cooperation optimization model and utilize an improved network flow algorithm (refer to Chapter 4) to optimise the SES collaboration plan. Subsequently, leveraging the inverse optimization process proposed in Chapter 5, it calculates the ES capacity exchange prices for each time period. Each community engages in transactions on the basis of the calculated capacity exchange prices. The exchange prices for Community 5 are shown in Table 4, while the prices for the remaining communities are listed in the Appendix.

From Table 4, it is evident that through collaborative sharing of SES capacity, the communities collectively achieve an increase of over 80% in revenue, optimizing the overall benefits of the SES. After the capacity exchange prices are allocated through inverse optimization, the lowest community revenue increases by 16.74%, whereas the highest increases by 209.71%. All communities experience revenue growth, and the sum of individual community revenues matches the total revenue from the alliance collaboration. Thus, the revenue distribution achieved through the inverse optimization process is stable and budget-balanced.

Despite the increase in revenue through collaboration, Table 5 shows that the utilization rate of ES under time-of-use pricing remains relatively low. This is due to the high simultaneous demand for ES during peak hours. In addition, the be-

Table 3. Proportion of the day-ahead ES plan.

Operator	Storage duration of electricity	Demand for electricity (kw)	Profit (yuan/(kw·h))
1	00:00–06:00	431	0.31
1	04:00–16:00	687	0.31
1	12:00–22:00	10000	0.31
2	04:00–08:00	10000	0.31
2	04:00–14:00	10000	0.31
2	04:00–24:00	4160	0.31
3	00:00–06:00	10000	0.31
3	04:00–12:00	10000	0.31
3	04:00–22:00	7709	0.31
…	…	…	…

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Table 4. Profit and allocation.

Operators	Non-cooperative	Cooperative	After allocating	Improvement
1	237.4680	0	286.7458	20.7513%
2	324.0416	561.8176	568.9322	75.5738%
3	110.6936	572.7200	342.8345	209.7148%
4	338.3600	0	395.0008	16.7398%
5	244.5696	1130.8556	671.8800	174.7193%
Sum	1255.1328	2265.3932	2265.3932	80.4903%

Table 5. Capacity exchange prices.

Operator	Time interval	Shared capacity	Capacity exchange cost
5	00:00–02:00	Not open	0.413458
5	02:00–04:00	Not open	0.397138
5	04:00–06:00	396	0.216792
5	06:00–08:00	396	0.190681
5	08:00–10:00	Not open	0.300138
5	10:00–12:00	Not open	0.365274
5	12:00–14:00	Not open	0.394002
5	14:00–16:00	Not open	0.407986
5	16:00–18:00	Not open	0.412558
5	18:00–20:00	Not open	0.407545
5	22:00–24:00	Not open	0.387138

nefits of storing energy are reflected only in price differences during time-of-use pricing, which limits overall utilization. This paper addresses this issue by introducing a new transaction pricing strategy to perform a comparison experiment. The unit price is inversely proportional to the quantity of stored energy, which means that a higher quantity of stored energy results in a lower unit price. The results of this new pricing strategy are detailed in Tables 6 and 7.

As shown in Table 7, the proposed cooperative optimization model and the inverse optimization method are effective under the revised pricing model. Table 6 shows a significant improvement in both ES utilization and SES sharing rates after the change in the transaction price. Therefore, this paper

Table 6. Capacity exchange prices based on independent pricing.

Operator	Time interval	Shared capacity	Capacity exchange cost
5	00:00–02:00	396	0.371195
5	02:00–04:00	396	0.255490
5	04:00–06:00	396	0.156257
5	06:00–08:00	0	0.188151
5	08:00–10:00	238	0.235264
5	10:00–12:00	238	0.196359
5	12:00–14:00	396	0.183943
5	14:00–16:00	396	0.180349
5	16:00–18:00	0	0.157007
5	18:00–20:00	0	0.273484
5	22:00–24:00	0	0.179079

Table 7. Profit and allocation based on independent pricing.

Operators	Noncooperative	Cooperative	After allocating	Improvement
1	4524.0984	6458.6182	5892.9496	30.2435%
2	1878.8544	0	2582.0099	37.4247%
3	641.8224	0	828.9990	29.1633%
4	4830.3200	8710.6886	5360.8505	10.9833%
5	1604.8608	1372.4526	1876.9503	16.9541%
Sum	13479.9560	16541.7594	16541.7594	22.7137%

offers a rational mechanism for SES systems. However, to further advance SES transactions and development in Shanghai, it is essential to establish appropriate pricing mechanisms for SES transactions.

7 Conclusions

This paper tackles the challenge of balancing energy utilization efficiency and equitable benefits within Energy Storage (ES) systems by proposing a collaborative mechanism that optimizes the capacity of the Shared Energy Storage (SES) alliance. Specifically, we address two main problems:

In the first problem, we establish a centralized optimization model aimed at maximizing the profit of the SES alliance. To solve this problem efficiently, we design an enhanced Max-Revenue with a Flow (MRF) algorithm. This approach significantly reduces the computational complexity of the centralized problem, providing a practical solution for SES optimization. We demonstrate, through case studies, that our MRF algorithm outperforms existing methods in computational efficiency.

In the second problem, we introduce the concept of capacity exchange cost within the SES alliance. This concept allows energy storage operators (ES operators) to procure ES capacity for specific time periods, creating new revenue-generating opportunities. Simultaneously, ES operators with available capacity are incentivized to lease their resources to others for financial gain. We develop linear programming models for each ES operator to calculate appropriate exchange costs, aimed at maximizing individual profits. By employing inverse optimization techniques, we align these individual models to achieve a collectively optimal solution. This ensures that the SES alliance maximizes overall benefits while maintaining active participation and enthusiasm from each member.

The paper contributes to the field in several significant ways:

(Ⅰ) We propose a cooperative game model that optimizes the integration of SES operations and provides an effective cost allocation mechanism. This enables efficient collaboration among SES participants, thereby maximizing the overall benefits of the alliance.

(Ⅱ) To tackle the NP-hard nature of the SES centralized problem, we introduce an enhanced MRF algorithm, which simplifies the solution process while maintaining accuracy and scalability. Our case studies illustrate the superior computational performance of this method compared to traditional optimization tools like Gurobi.

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(Ⅲ) By introducing capacity exchange costs via inverse optimization, we address the key issue of balancing energy utilization efficiency and fair profit allocation. This approach ensures a fair distribution of profits and maximizes the benefits for the entire SES alliance, thus maintaining participants' motivation and engagement.

(Ⅳ) We demonstrate the economic advantages of adopting SES systems over individual ES systems using real-world data from Qingpu District, Shanghai. We also propose a tailored profit allocation mechanism for the district that ensures budget balance and meets validity constraints, offering a sustainable and practical solution for real-world implementation.

(Ⅴ) The feasibility and effectiveness of the optimization methods and distribution strategies are validated through simulations conducted in Qingpu District. The findings highlight the commercial potential of ES systems and underscore the importance of appropriate storage pricing strategies to encourage optimal utilization of ES resources.

There are certain limitations that could be addressed in future research. First, the proposed models assume ideal conditions, such as accurate data on capacity availability and demand, which may not always hold in real-world applications. Future work could focus on incorporating more dynamic and uncertain factors, such as fluctuations in energy demand or the impact of external market conditions.

Finally, the focus of this study is on SES alliances within a specific district in Shanghai. Extending the proposed models to other regions with varying energy needs and infrastructure could help validate the generalizability of the approach. Moreover, investigating the long-term sustainability of the SES system, including environmental and regulatory impacts, will be crucial for its widespread adoption.

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Conflict of interest

The authors declare that they have no conflict of interest.

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