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Trade credit contracting in a risk-averse supply chain under adverse selection and moral hazard

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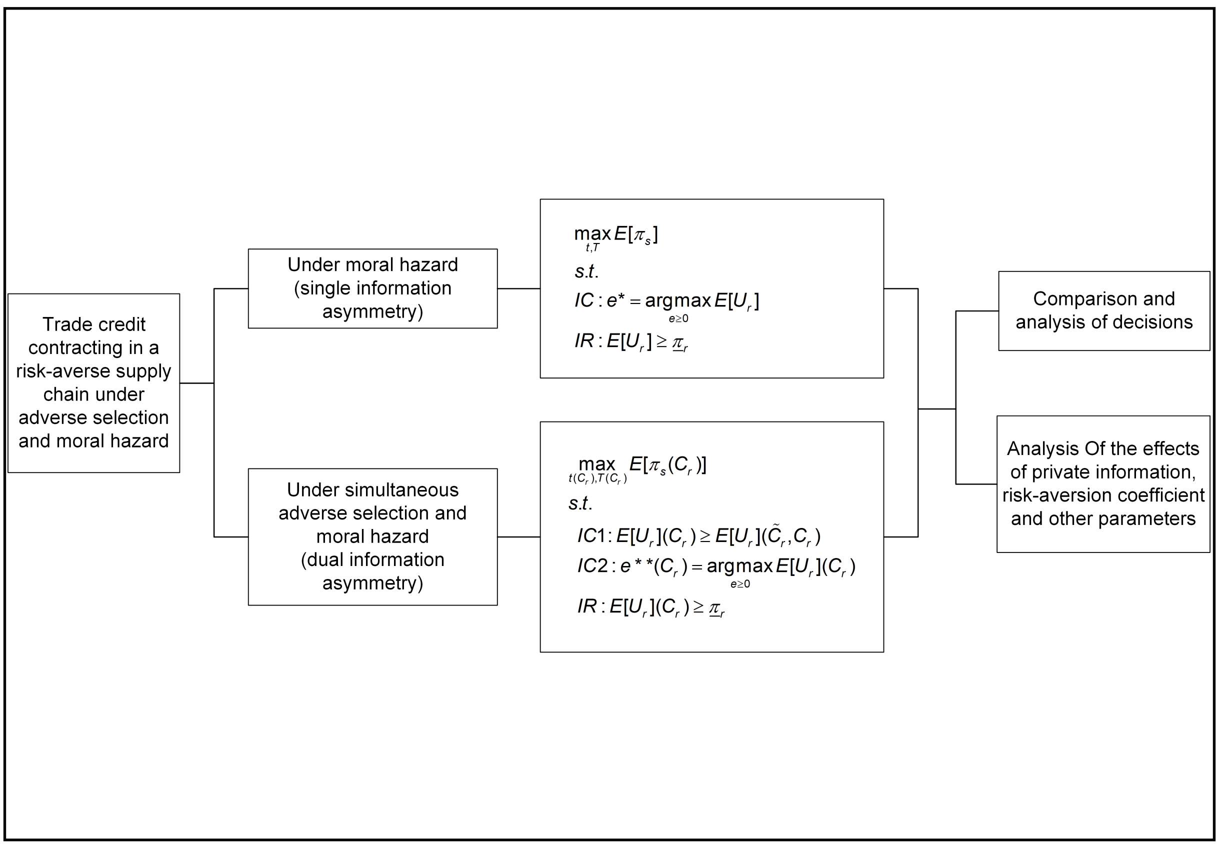
Graphical abstract



The overall framework of our delivery selection model.

Public summary

- The study explores how the manufacturer designs optimal trade credit contracts when adverse selection and moral hazard coexist.
- The manufacturer can identify the retailer's private information and motivate it to exert the optimal effort level by designing acceptable and appropriate trade credit contracts.
- The manufacturer can identify the retailer's private information and motivate it to exert the optimal effort level by designing acceptable and appropriate trade credit contracts.

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Abstract: Trade credit, as an effective tool for integrating and coordinating material, information, and financial flows in supply chain management, is becoming increasingly widespread. We explore how a manufacturer can design optimal trade credit contracts when a risk-averse retailer hides its sales cost information (adverse selection) and selling effort level (moral hazard). We develop incentive models for a risk-averse supply chain when adverse selection and moral hazard coexist, which are then compared with the results under single information asymmetry (moral hazard). Moreover, we analyze the effects of private information and risk-aversion coefficient on contract parameters, selling effort level and the profit or utility of the supply chain. The study shows that when the degree of retailer's risk aversion is within a certain range, reasonable trade credit contracts designed by the manufacturer can effectively induce the retailer to report its real sales cost and encourage it to exert appropriate effort. Furthermore, we find that the optimal trade credit period, optimal transfer payment, and retailer's optimal sales effort level under dual information asymmetry are less than under single information asymmetry. Numerical analysis are conducted to demonstrate the effects of the parameters on decisions and profits.

Keywords: trade credit; risk averse; adverse selection; moral hazard; supply chain

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1 Introduction

Insufficient working capital has become a problem that can seriously affect the survival and development of enterprises across the world. Trade credit is an important source of short-term financing and an effective incentive for coordination; it provides indirect financing services that compensate for deficiencies in the financial flow. Trade credit has thus been widely used by enterprises in different regions, such as China, the United States, and Europe[1–3]. Under trade credit, manufacturers that are larger and well-funded function as both product providers (PPs) and financial service providers (FSPs), providing products and financing services to downstream small and medium-sized retailers that are vulnerable to cash flow difficulties. That is, under a trade credit contract, manufacturers allow retailers to pay for purchases at the end of the trade credit period. Intuitively, trade credit provides a win–win solution to enterprises involved in the supply chain. Trade credit can help supply chains to improve efficiency and effectiveness and gain competitive advantage[4–7].

Most extant articles of trade credit assume that information is symmetric, that is, all information is common knowledge to manufacturers and retailers. However, information asymmetry often arises in trade credit contracting. For example, retailers possess information on their private sales cost because they know their own operation status and information on market demand because they directly face the market, meaning that retailers can obtain preferential credit terms from manufacturers, which is a typical "adverse selection" problem. After manufacturers and retailers sign trade credit contracts, the retailers may become lazy because manufacturers cannot accurately observe retailers' selling efforts, which is a typical "moral hazard" problem. These information asymmetry problems often occur at the same time, which results in inefficient operations and even failures of trade credit. Therefore, under the dual information asymmetry of adverse selection and moral hazard, it is crucial that manufacturers design reasonable contracts that not only motivate retailers to report their real information and promote their sales efforts, but also help manufacturers gain as much profit as possible.

Moreover, the operational environment of trade credit involves an increasing number of uncertainties, such as information asymmetry, market demand uncertainties, price fluctuations, and changes in corporate funds. Supply chains have become more vulnerable to such uncertainties, and decision makers focus more on risk[8–10]. Smaller enterprises that are short of funds often react more strongly to risks and show the characteristics of risk aversion, which may influence the decisions and profits of the supply chain.

Based on the above analysis, the following questions may arise: How can acceptable and appropriate incentive contracts be designed to induce the retailer to report its real sales cost and exert appropriate effort and thus achieve supply chain equilibrium? What impact does the degree of risk aversion have on incentive contracts and supply chain decisions?

How does it affect the benefits of supply chain members and the system?

To answer these questions, we consider a supply chain composed of a risk-neutral manufacturer and a risk-averse retailer. The manufacturer provides trade credit to the retailer. We first develop a principal-agent model by considering the retailer's risk aversion in the case when adverse selection and moral hazard coexist simultaneously. We then further investigate trade credit incentive contracts and analyze the effects of risk aversion on the decisions and profits of the supply chain.

The contributions of our paper are as follows. First, our paper explores how the manufacturer can optimally design trade credit contracts when both the retailer's sales cost and selling effort level are unobservable (in which adverse selection and moral hazard coexist). Second, we develop a principal-agent model that incorporates adverse selection, moral hazard, and risk aversion in a trade credit setting. The retailer's risk aversion and asymmetric information have important effects on trade credit contracts, operations decisions, and the supply chain's profits. To the best of our knowledge, no theoretical exploration of the incentive of trade credit in risk-averse supply chains under dual information asymmetry exists in the operations management literature.

The rest of this paper is organized as follows. The literature review is presented in Section 2. Section 3 focuses on mathematical models under single information asymmetry and dual information asymmetry. Section 4 presents the model analysis. A set of numerical experiments and analyses are conducted to illustrate the models in Section 5. At last, Section 6 provides conclusions and future research.

2 Literature review

Our paper is closely related to three bodies of literature, trade credit as well as risk aversion and information asymmetry in trade credit. We will review the literature as follows.

2.1 Trade credit in operations management

One stream of research has focused on trade credit, which is widely used as a marketing tool to promote sales and a mechanism to coordinate a supply chain. Wang and Liu[11] provided a review of the operational management literature on trade credit. Haley and Higgins[12] earliest introduced trade credit into the buyer's inventory policy. And then Goyal[13], Huang[14], Wu et al.[15] and Cárdenas-Barrón et al.[16] considered given trade credit terms and built economic order quantity (EOQ) models to decide the order policy. Abad and Jaggi[17], Wang et al.[18] and Pramanik et al.[19] developed models to find the optimal trade credit terms for vendors.

From the supply chain's point of view, Jaber and Osman[20] explored order quantity and trade credit to minimize the cost of the whole supply chain under constant demand. Luo[21] and Yang et al.[22] proved that trade credit is a new supply chain incentive mechanism. Sarkar et al.[6] studied the multi-level trade credit and single-setup multiple-delivery policy in a global sustainable supply chain. Lee and Rhee[23], Heydari et al.[24] and Tsao[5] investigated the role of the composite mechanism based on trade credit and other traditional coordinating contracts in supply chain coordination for uncertain demand. Zou and Tian[25] formulated an inventory model for a supply chain simultaneously considering flexible trade credit contract and two-level trade credit policy. Ren et al.[26] built a Bertrand model of a two-echelon supply chain consisting of two suppliers and a retailer and analyzed the impact of trade credit on supply chain parties' pricing decisions and profits.

Another stream of research has focused on trade credit under which the supplier allows the buyer to delay the payment as a common means of short-term financing. Seifert et al.[1] , Babich and Kouvelis[27] reviewed the existing literature on trade credit at the interface of operations and finance. Yang and Birge[28] investigated the interaction of firms' operations decisions and financial constraints and multiple financing channels (bank loans and trade credit), and explored the risk-sharing role of trade credit. Wu et al.[29] studied a supply chain consisting of a manufacturer and two asymmetric retailers, the manufacturer offered trade credit to the weak retailer that was capital-constrained and showed that the manufacturer used trade credit as a strategic response to the bargaining power of its dominant retailer. Kouvelis and Zhao[30] investigated the impact of trade credit ratings on the operational and financial decisions of a supply chain with a supplier and a retailer. Yan & He[31] examined a supply chain with a manufacturer and a capital-constrained retailer with and without bankruptcy costs, and showed that trade credit may encourage the retailer to increase order size.

2.2 Risk aversion and information asymmetry in trade credit

Risk aversion in the trade credit setting has received little attention with a few recent exceptions. Li et al.[32] investigated that the risk-averse supplier offered trade credit to the retailer with CVaR criterion and analyzed the impact of risk aversion on supply chain decisions. Yang et al.[10] built a mean-variance model to analyze the decisions of the supply chain with a risk-averse capital-constrained retailer and a supplier under both trade credit financing and bank credit financing, and found that risk aversion played an important role in determining the financing equilibrium. Zhang and Chen[33] considered a dyadic closed-loop supply chain consisting of one risk-averse supplier and one risk-neutral capital-constrained OEM and three financing modes-partial trade credit with bank loan (PTC-with-BL), full trade credit with bank loan (FTC-with-BL) and pure bank loan (PBL), and studied the optimal production and financing portfolio strategies.

Several articles have investigated the incentive effects of trade credit under asymmetric information. Luo and Zhang[34] derived the optimal trade credit periods when the buyer's capital cost was asymmetric. Wang et al.[3] investigated and compared the screening, checking, and insurance mechanisms to address credit default problems in a supplier-retailer-customers supply chain when the retailer's credit level was unobservable. Devalkar and Krishnan[35] studied how trade credit coordinated supply chains when the buyer cannot observe the supplier's exerting effort (moral hazard). Wang et al.[7] considered the setting in which a risk-neutral supplier offered

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trade credit to a risk-averse retailer, and explored the incentive effect of trade credit on the supply chain's decisions under adverse selection.

Unlike their models, we concentrate on trade credit contracting in a risk-averse supply chain under dual information asymmetry, namely, considering adverse selection and moral hazard simultaneously. Moreover, we analyze the influence of risk aversion on the optimal contracts and optimal decisions, thus providing strategic guidance for the operation of trade credit.

3 Model setup

In a supply chain consisting of a risk-neutral manufacturer and a risk-averse retailer, the manufacturer offers a single product to the retailer, and allows the latter to delay payment; that is, the retailer does not have to pay the manufacturer immediately when products are delivered but pays all purchased at the end of the credit period.

In this study, product quantity q demanded in the final market is given by the demand function $q = a - bp + \epsilon + f(e)$, where a is market size and b is demand responsiveness to sales price. The sales price is exogenous. We assume the market random factor ϵ to indicate market uncertainties, which follows a normal distribution such that ϵ $(0, \sigma^2)$[8,36]. $f(e)$ is the output function of the retailer's sales effort level e, following $f^{'}(e) > 0$ and $f^{''}(e) \leq 0$, which will affect the market demand. Referring to the literature[37], we assume that $f(e) = e$ and then obtain $q = a - bp + \epsilon + e$. The cost $C(e)$ to the retailer of exerting e units of sales effort is $c(e) = \frac{\mu}{2}e^2$, and $C(e)$ is increasing, convex, and differentiable in e ($c^{'}(e) > 0$, $c^{''}(e) \geqslant 0$), where μ is the effort cost coefficient and $\mu > 0$ and $e \geqslant 0$ [11,38]. The manufacturer often cannot directly observe the retailer's sales effort level, but can observe the demand (i.e., the value of q), and may then motivate the retailer based on realized demand.

The retailer holds private information about its unit sales cost C_r, which may not be fully observable to the manufacturer. The manufacturer only knows that C_r is random and $C_r \in [C_{r1}, C_{r2}]$, which has a probability density function $f(C_r)$ and a cumulative distribution function $F(C_r)$, and is continuous and conforms to increasing failure rate distribution (IFRD) characteristics[39,40] .

Other parameters used in this paper are as follows:

w-Manufacturer's unit product wholesale price; C_s-Manufacturer's unit production cost; p-Retailer's unit product sales price; C_r-Retailer's unit sales cost; i_s-Manufacturer's capital cost rate; i_r-Retailer's return on investment; t-Trade credit period; k-Risk aversion coefficient.

To avoid unrealistic and trivial cases, we assume that $w > C_s, p > C_r$, and $i_r > i_s$.

The problem facing the manufacturer is a mixture of moral hazard (post-contractual opportunism associated with the decision about effort level) and adverse selection (pre-contractual asymmetric information about sales cost). Under these circumstances, the manufacturer needs to offer a menu of contracts to motivate the retailer to report its real information about sales costs and encourage it to exert the appropriate effort expected by the manufacturer. The retailer decides whether to accept a contract, and if so, how much sales effort to exert. We consider an incentive contract $\{t, T\}$, where t means the credit period and T means the lump-sum transfer payment.

Based on the preceding parameters and assumption, the retailer's expected profit is $E[\pi_r] = (p - C_r - w + i_r tw)(a - bp + e) - \frac{\mu}{2}e^2 - T$.

The variance of the retailer's profit is $Var(\pi_r) = (p - C_r - w + i_r tw)^2 \sigma^2$.

The retailer's expected utility is

$$\begin{aligned} & E[U_r] = E[\pi_r] - k\sqrt{Var(\pi_r)} = \\ & (p - C_r - w + i_r tw)(a - bp + e) - \\ & \frac{\mu}{2}e^2 - k\sigma(p - C_r - w + i_r tw) - T = \\ & (p - C_r - w + i_r tw)(a - bp + e - k\sigma) - \frac{\mu}{2}e^2 - T \end{aligned} \tag{1}$$

Where $E[U_r] = E[\pi_r] - k\sqrt{Var(\pi_r)}$, similar to Lau [41], and k reflects the degree of risk aversion, i.e., a higher value implies higher risk aversion, and $k = 0$ denotes that the retailer is risk-neutral. To ensure that the retailer's utility is not negative, let $(p - C_r - w + i_r tw)(a - bp + e - k\sigma) > 0$, and we have $k \in [0, \frac{a-bp+e}{\sigma})$.

The manufacturer's expected profit is $E[\pi_s] = (w - C_s - i_s tw) \cdot (a - bp + e) + T$. Using these equations, we present the following analysis and discussion.

3.1 Models under moral hazard (single information asymmetry)

As a benchmark, we first study the case of moral hazard. After the manufacturer and retailer reach a trade credit agreement, as the manufacturer cannot observe the retailer's selling effort level e, the retailer may become lazy, resulting in the loss of the manufacturer's profits. In the case of moral hazard, the manufacturer needs to offer an incentive contract to induce the retailer to exert the intended sales effort. The contract must be acceptable to the retailer, in addition to having the objective of maximizing the manufacturer's own expected profit. Problem R1 of finding an optimal incentive contract can be stated as

$$\max_{t,T} E[\pi_s] = (w - C_s - i_s tw)(a - bp + e) + T. \tag{2}$$

s.t.

$$\mathrm{IC}: e^* = \operatorname*{argmax}_{e \geqslant 0} E[U_r]. \tag{3}$$

$$\mathrm{IR}: E[U_r] \geqslant \underset{-r}{\pi} \tag{4}$$

Constraint (3) is an incentive compatibility (IC) constraint indicating that the retailer can choose the optimal selling effort level to maximize its expected utility. Constraint (4) is an individual rationality (IR) constraint indicating that the retailer is better off participating relative to its reservation utility $\underset{-r}{\pi}$.

Proposition 1. Under moral hazard (single information asymmetry), the manufacturer's optimal contract configuration $\{t^*, T^*\}$ is as follows:

$$t^* = \frac{w - C_s}{(2i_s - i_r)w} + \frac{(i_r - i_s)[\mu(a-bp) + p - C_r - w]}{(2i_s - i_r)wi_r} - \frac{\mu k\sigma}{(2i_s - i_r)w}. \tag{5}$$

$$T^* = -\underline{\pi}_r + (p - C_r - w + i_r t^* w)(a - bp - k\sigma) + \frac{1}{2\mu}(p - C_r - w + i_r t^* w)^2. \tag{6}$$

The retailer's optimal selling effort level is as follows:

$$e^* = \frac{p - C_r - w + i_r t^* w}{\mu}, \tag{7}$$

where $i_S < i_r < 2i_S$ and $k \in [0, \frac{a-bp+e}{\sigma})$.

Proof. See Appendix A.

Furthermore, under moral hazard (single information asymmetry), we have the manufacturer's optimal expected profit $E^*[\pi_s]$, retailer's optimal expected utility $E^*[U_r]$, and system's optimal expected utility $E^*[U_{SC}]$, as follows:

$$E^*[\pi_s] = (w - C_s - i_s t^* w)\left(a - bp + \frac{p - C_r - w + i_r t^* w}{\mu}\right) + T^*. \tag{8}$$

$$E^*[U_r] = \underline{\pi}_r. \tag{9}$$

$$E^*[U_{SC}] = E^*[\pi_s] + E^*[U_r]. \tag{10}$$

The manufacturer's expected utility equals its expected profit $E^*[\pi_s]$ because it is risk-neutral.

Proposition 1 shows that the manufacturer can effectively motivate the retailer to exert its selling effort by appropriately adjusting incentive parameters.

3.2 Models under simultaneous adverse selection and moral hazard (dual information asymmetry)

The manufacturer needs to design a menu of contracts $\{t, T\}$ when it cannot observe the retailer's selling effort level e and sales cost C_r and only knows the probability density function $f(C_r)$and cumulative distribution function $F(C_r)$ of C_r. The menu must meet the three following requirements: The first is that the menu must include at least one acceptable contract $\{t(C_r), T(C_r)\}, C_r \in [C_{r1}, C_{r2}]$ for any $C_r \in [C_{r1}, C_{r2}]$ so that both retailer and manufacturer will reach an agreement; The second is that the manufacturer should ensure that the retailer chooses the appropriate contract corresponding to its real cost C_r[42, 43]; The third requirement is that the manufacturer can correctly anticipate the amount of selling effort under each type of retailer, and ensure that the intended selling effort level is acceptable to that type of retailer.

The sequence of contracting between the manufacturer and the retailer (essentially a Stackelberg game process) is as follows.

• The manufacturer offers a menu of contracts $\{t(\cdot), T(\cdot)\}$.

• The retailer chooses one of the contracts$\{t(C_r), T(C_r)\}$ and discloses its cost information to the manufacturer.

• The retailer determines its optimal selling effort level according to its sales cost type.

We first consider the retailer's effort decision. Due to information asymmetry, the retailer may choose a contract $\{t(C_r), T(C_r)\}$that is not intended for his real cost C_r. Then, the retailer's expected utility function $E[U_r](C_r, C_r \mid e)$ is

$$E[U_r](C_r, C_r \mid e) = [p - C_r - w + i_r t(C_r) w][a - bp + e(C_r) - k\sigma] - \frac{\mu}{2}e^2(C_r) - T(C_r). \tag{11}$$

Now, let $e^{**}(C_r)$ be the corresponding optimal sales effort level of the retailer; that is,

$$e^{**}(C_r) = \underset{e \geqslant 0}{\operatorname{argmax}} E[U_r](C_r, C_r \mid e) = \frac{p - C_r - w + i_r t(C_r) w}{\mu}. \tag{12}$$

Substituting $e^{**}(C_r)$of (12) into (11), we calculate the expected utility achieved by a retailer with cost C_r for choosing contract $\{t(C_r), T(C_r)\}$ as

$$E[U_r](C_r, C_r) = (p - C_r - w + i_r t(C_r) w)(a - bp - k\sigma) + \frac{1}{2\mu}(p - C_r - w + i_r t(C_r) w)^2 - T(C_r). \tag{13}$$

We also calculate the expected utility achieved by a retailer with cost C_r by choosing an appropriate contract $\{t(C_r), T(C_r)\}$ as

$$E[U_r](C_r) = E[U_r](C_r, C_r) = (p - C_r - w + i_r t(C_r) w)(a - bp - k\sigma) + \frac{1}{2\mu}(p - C_r - w + i_r t(C_r) w)^2 - T(C_r). \tag{14}$$

When a retailer that has sales cost C_r chooses contract $\{t(C_r), T(C_r)\}$, the manufacturer's expected profit is as follows.

$$E[\pi_s](C_r) = \int_{C_{r1}}^{C_{r2}} \{[w - C_S - i_S t(C_r) w] \left[a - bp + \tfrac{p - C_r - w + i_r t(C_r) w}{k}\right] + T(C_r)\} dF(C_r). \tag{15}$$

Based on the revelation principle[42, 44], the retailer will select an intended contract corresponding to its real sales cost C_r. Taking into account the retailer's incentive compatibility constraint of moral hazard and incentive compatibility constraint of adverse selection, and the individual rationality constraint, the manufacturer's problem of maximizing its expected profit is described by the following model R2:

$$\max_{t(C_r), T(C_r)} E[\pi_S](C_r). \tag{16}$$

s.t.

$$\mathrm{IC1}: E[U_r](C_r) \geqslant E[U_r](C_r, C_r). \tag{17}$$

$$\mathrm{IC2}: e^{**}(C_r) = \underset{e \geqslant 0}{\operatorname{argmax}} E[U_r](C_r). \tag{18}$$

$$\mathrm{IR}: E[U_r](C_r) \geqslant \underline{\pi}_r, \tag{19}$$

where $\forall C_r, C_r \in [C_{r1}, C_{r2}]$.

Function (16) is the manufacturer's objective function.

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Constraint (17) is the incentive compatibility constraint of adverse selection, ensuring that the retailer picks the contract intended for its type. Constraint (18) is the incentive compatibility constraint of moral hazard, stating that each retailer can choose the optimal effort level for its type. Constraint (19) is the individual rationality constraint, ensuring that each retailer earns at least its reservation utility $\underline{\pi}_r$. Denote $\{t^{**}(C_r), T^{**}(C_r)\}$ as an optimal solution of model R2.

Proposition 2. Under dual information asymmetry (when moral hazard and adverse selection coexist), the manufacturer's optimal contract configuration $\{t^{**}(C_r), T^{**}(C_r)\}$ is as follows:

$$t^{**}(C_r) = \frac{1}{(2i_s - i_r)w}\left[\frac{-F(C_r)}{f(C_r)} + w - C_s\right] + \frac{(i_r - i_s)[\mu(a-bp) + p - C_r - w]}{(2i_s - i_r)wi_r} - \frac{\mu k\sigma}{(2i_s - i_r)w}. \tag{20}$$

$$T^{**}(C_r) = [p - C_r - w + i_r t^{**}(C_r)w](a - bp - k\sigma) + \frac{1}{2\mu}[p - C_r - w + i_r t^{**}(C_r)w]^2 - \underline{\pi}_r - \int_{C_r}^{C_{r2}}\left\{(a - bp - k\sigma) + \frac{(p - \tau - w + i_r t^{**}(\tau)w)}{\mu}\right\}\mathrm{d}\tau. \tag{21}$$

The retailer's optimal selling effort level $e^{**}(C_r)$ is

$$e^{**}(C_r) = \frac{p - C_r - w + i_r t^{**}(C_r)w}{\mu}, \tag{22}$$

where $i_S < i_r < 2i_S$, $k \in [0, \frac{a - bp + e}{\sigma})$ and the retailer's true sales cost is $C_r, \forall C_r \in [C_{r1}, C_{r2}]$.

Proof: See Appendix B.

Furthermore, under dual information asymmetry, we have the manufacturer's optimal expected profit $E^{**}[\pi_s](C_r)$, retailer's optimal expected utility $E^{**}[U_r](C_r)$, and system's optimal expected utility $E^{**}[U\pi_{sc}(C_r)]$, as follows:

$$E^{**}[\pi_s](C_r) = \int_{C_{r1}}^{C_{r2}}\left\{\left[a - bp + \frac{p - C_r - w + i_r t^{**}(C_r)w}{\mu}\right][w - c_s - i_s t^{**}(C_r)w] + (p - C_r - w + i_r t^{**}(C_r)w)(a - bp - k\sigma) + \frac{1}{2\mu}(p - C_r - w + i_r t^{**}(C_r)w)^2 - \underline{\pi}_r - \int_{C_{r2}}^{C_r}[(a - bp - k\sigma) + \frac{1}{\mu}(p - C_r - w + i_r t^{**}(\tau)w)]\mathrm{d}\tau\right\}\mathrm{d}F(C_r). \tag{23}$$

$$E^{**}[U_r](C_r) = \underline{\pi}_r + \int_{C_{r2}}^{C_r}[(a - bp - k\sigma) + \frac{1}{\mu}(p - C_r - w + i_r t^{**}(\tau)w)]\mathrm{d}\tau, \tag{24}$$

$$E^{**}[U_{SC}](C_r) = E^{**}[\pi_s](C_r) + E^{**}[U_r](C_r). \tag{25}$$

The manufacturer's expected utility equals its expected profit $E^{**}[\pi_s](C_r)$ because it is risk-neutral.

4 Model analysis

4.1 Validity of the manufacturer's contracts

Theorem 1. Under dual information asymmetry, the incentive contract menu $\{t(C_r), T(C_r)\}$ has the characteristic of self-selection; that is, there is no motive for a retailer to disguise itself as another cost type, so it will self-select the appropriate contract.

Proof: See Appendix C.

Theorem 1 shows that facing dual information asymmetry, the manufacturer can design a reasonable contract menu and motivate the retailer to reveal her truthful sales cost information. Then by executing the contract menu, the manufacturer will be able to distinguish different-cost-type retailers.

Theorem 2. Whether under single or dual information asymmetry, trade credit incentive contracts enable the retailer to achieve the optimal effort level at which it obtains the optimal expected utility, and the manufacturer can obtain the net income from the retailer's efforts.

Proof: Under dual information asymmetry, the marginal cost of the retailer's effort is $c'(e^{**}) = p - C_r - w + i_r t^{**}(C_r)w$, and the marginal expected utility of the retailer's effort is $E^{'}U[pq - C_r q - wq + i_r t^{**}(C_r)wq] = p - C_r - w + i_r t^{**}(C_r)w$. Then we know that the marginal cost of the retailer's efforts and the marginal expected utility of its efforts are equal, which demonstrates that trade credit incentive contracts can enable the retailer to reach the effort level that maximizes its utility.

Moreover, at this point, the net income due to the retailer's efforts is $[p - C_r - w + i_r t^{**}(C_r)w]e^{**} - c(e^{**}) = \frac{1}{2\mu}[p - C_r - w + i_r t^{**}(C_r)w]^2$. From Equation (21), we can see that the net income can be completely transferred from the retailer to the manufacturer by skillfully designing incentive contracts. This can be proved under single information asymmetry using the same reasoning. Hence, we conclude the proof.

According to Theorems 1 and 2, in the practice of trade credit, the manufacturer can prevent the coincidence of adverse selection and moral hazard by designing incentive contracts, which can induce the retailer to disclose its true cost information and inspire the retailer to work hard enough.

4.2 Comparison and analysis of decisions under single and double information asymmetry

Theorem 3. The retailer only obtains the reserved utility $\underline{\pi}_r$ under single information asymmetry. Under dual information asymmetry, the retailer can obtain an extra strict non-negative information rent that is negatively correlated with C_r because it possesses private information C_r.

Proof. From Eqs. (24) and (9), the retailer's information rent M is as follows.

$$M = E^{**}[U_r](C_r) - E^{*}[U_r] = \int_{C_r}^{C_{r2}}\left\{(a - bp - k\sigma) + \frac{1}{\mu}(p - \tau - w + i_r t^{**}(\tau)w)\right\}\mathrm{d}\tau,$$

where $(a - bp - k\sigma) + \frac{1}{\mu}(p - \tau - w + i_r t^{**}(\tau)w) > 0$ when $C_{r1} \leqslant C_r \leqslant C_{r2}$. Hence, information rent is strictly non-negative. We

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have $\dfrac{\partial M}{\partial C_r}<0$ and $M=0$ where $C_r=C_{r2}$, so M monotonically decreases in C_r. Hence, we conclude the proof.

Theorem 3 indicates that the retailer with higher sales cost receives a lower information rent, and therefore obtains a lower profit than the retailer with lower sales cost. As a result, private information is more valuable to the manufacturer as sales cost decreases.

Theorem 4. The optimal trade credit period $t^{**}(C_r)$, optimal transfer payment $T^{**}(C_r)$, and retailer's optimal selling effort level $e^{**}(C_r)$ under dual information asymmetry are less than the optimal trade credit period t^*, optimal transfer payment T^*, and retailer's optimal selling effort level e^* under single information asymmetry.

Proof: Comparing Equation (20) with Equation (5), we have $t^{**}(C_r)\le t^*$. Following Theorem 1, under dual information asymmetry, the manufacturer needs to shorten the trade credit period to motivate the retailer to report its private information.

Comparing $T^{**}(C_r)$ with T^* according to $t^{**}(C_r)\le t^*$, we have $T^{**}(C_r)\le T^*$. Under dual information asymmetry, the transfer payment obtained by the manufacturer is less than that under single information asymmetry.

Lastly, comparing $e^{**}(C_r)$ with e^* according to $t^{**}(C_r)\le t^*$, we have $e^{**}(C_r)\le e^*$. Hence, we conclude the proof.

Theorem 4 suggests that compared with single information asymmetry, the manufacturer may offer a shorter trade credit period to the retailer under dual information asymmetry. And another observation is that the retailer's sales effort becomes smaller under dual information asymmetry than that under the single information asymmetry. In the presence of dual information asymmetry, due to the shorter credit period offered by the manufacturer, there is insufficient incentive for the retailer to work hard.

4.3 The effects of the retailer's risk aversion

Theorem 5. Whether under single or dual information asymmetry, the optimal trade credit period decreases as the degree k of the retailer's risk aversion increases, where $k\in[0,\dfrac{a-bp+e}{\sigma})$.

Proof: Under dual information asymmetry, we have $\dfrac{\partial t^{**}(C_r)}{\partial k}=-\dfrac{\mu\sigma}{(2i_s-i_r)w}<0$ from Equation (19), where $0\leqslant k<\dfrac{a-bp+e}{\sigma}$, so the optimal trade credit period $t^{**}(C_r)$ is a decreasing function of the risk-aversion coefficient k. Hence, we conclude the proof.

Theorem 5 shows that the risk-averse behavior of the retailer will weaken the incentive effect of trade credit. This can be proved under single information asymmetry using the same reasoning.

Theorem 6. Regardless of single or dual information asymmetry, the retailer's optimal selling effort level increases as the risk-aversion coefficient k decreases.

Proof: Under dual information asymmetry, we have $\dfrac{\partial e^{**}(C_r)}{\partial t^{**}(C_r)}=\dfrac{1}{\mu}i_r w>0$ from Equation (22); that is, the retailer's optimal sales effort level increases as the optimal trade credit period increases. We also have $\dfrac{\partial t^{**}(C_r)}{\partial k}=-\dfrac{\mu\sigma}{(2i_s-i_r)w}<0$ where $k\in[0,\dfrac{a-bp+e^{**}(C_r)}{\sigma})$ from Eq. (20); that is, the optimal trade credit period decreases with the risk-aversion coefficient. We thus obtain $\dfrac{\partial e^{**}(C_r)}{\partial k}=-\dfrac{\mu\sigma i_r}{(2i_s-i_r)\mu}<0$ where $k\in[0,\dfrac{a-bp+e^{**}(C_r)}{\sigma})$; that is, the retailer's optimal sales effort level is negatively correlated with the risk-aversion coefficient. Hence, we conclude the proof.

The proof under single information asymmetry is similar to that under dual information asymmetry.

4.4 The effects of the retailer's sales cost and other parameters

Theorem 7. Regardless of single or dual information asymmetry, the optimal credit period, optimal transfer payment and retailer's optimal effort level decrease as the retailer's unit sales cost C_r increases.

Proof. Under dual information asymmetry, we know $\dfrac{\partial t^{**}(C_r)}{\partial C_r}<0$ in Theorem 1. From Eq. (21), we also have

$$\begin{aligned}\frac{\partial T^{**}(C_r)}{\partial C_r}&=(a-bp-k\sigma)\,i_r w\frac{\partial t^{**}(C_r)}{\partial C_r}+\\&\frac{1}{\mu}[p-C_r-w+i_r t^{**}(C_r)w]\,i_r w\frac{\partial t^{**}(C_r)}{\partial C_r}=\\&i_r w\frac{\partial t^{**}(C_r)}{\partial C_r}(a-bp-k\sigma+e^{**}(C_r)).\end{aligned}$$

In addition, $(a-bp-k\sigma+e^{**}(C_r))>0$, so we obtain $\dfrac{\partial T^{**}(C_r)}{\partial C_r}<0$.

We have $\dfrac{\partial e^{**}(C_r)}{\partial t^{**}(C_r)}=\dfrac{1}{\mu}i_r w>0$ from Eq. (22); that is, the retailer's optimal sales effort level increases as the optimal trade credit period increases. We also know $\dfrac{\partial t^{**}(C_r)}{\partial C_r}<0$ from Theorem 1. Then we obtain $\dfrac{\partial e^{**}(C_r)}{\partial C_r}<0$. Hence, we conclude the proof.

Theorem 7 indicates that the higher the retailer's selling cost is, the shorter the credit period provided by the manufacturer and the lower the transfer payment received by the manufacturer. The higher-cost-type retailer obtains the shorter credit period, leading to insufficient incentives, which makes it reduce its selling effort level. In addition, this further demonstrates that the manufacturer's contracts can provide different appropriate trade credit periods and transfer payments to different-cost-type retailers.

The case of single information asymmetry can be proved using the same reasoning.

Theorem 8. Regardless of single or dual information asymmetry, the optimal credit period, optimal transfer payment and retailer's optimal effort level decrease as the retailer's unit sales cost C_r increases. The optimal credit period and retailer's optimal effort level increase as the market size a increases.

Proof: Under dual information asymmetry, from Equations (20) and (21) and (22), we obtain $\dfrac{\partial t^{**}(C_r)}{\partial a}=\dfrac{(i_r-i_s)\mu}{(2i_s-i_r)i_r w}>0$, $\dfrac{\partial e^{**}(C_r)}{\partial a}=\dfrac{(i_r-i_s)}{(2i_s-i_r)}>0$, $\dfrac{\partial t^{**}(C_r)}{\partial C_s}=-\dfrac{1}{(2i_s-i_r)w}<0$,

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$\frac{\partial e^{**}(C_r)}{\partial C_s} = -\frac{1}{(2i_s - i_r)w} < 0$ and $\frac{\partial T^{**}(C_r)}{\partial C_s} < 0$. Hence, we conclude the proof.

Theorem 8 shows that as the manufacturer's production costs increase, the manufacturer is unwilling or unable to pay more capital costs, and consequently the credit period provided by the manufacturer becomes shorter, the transfer payment obtained by the manufacturer decreases, and the retailer's effort level also decreases. The improvement of the market's basic demand situation will prompt the manufacturer to extend the length of the trade credit period to encourage the retailer to engage in cooperation and work harder. The case of single information asymmetry can be proved using the same reasoning.

5 Computational experiments

We now use numerical computations to further analyze the effects of sales cost and risk aversion on the supply chain's profit and utility. We assume that $a = 7900$, $b = 20$, $i_r = 0.2$, $i_s = 0.15$, $C_s = 150$, $w = 250$, $\underline{\pi}_r = 10000$; ϵ follows the normal distribution $\epsilon\left(0, 150^2\right)$, and C_r follows the uniform distribution with the probability distribution function $F(C_r) = \frac{C_r - 110}{20}$ and density function $f(C_r) = \frac{1}{20}$, where $[C_{r1}, C_{r2}] = [110, 130]$.

5.1 The effects of sales cost C_r on the supply chain's profit and utility

The results, as shown in Fig. 1, are as follows:

Under single information asymmetry (moral hazard), a retailer of any cost type receives only the reservation utility, i.e., $E^*[U_r] = \underline{\pi}_r = 10000$. However, under dual information asymmetry, the retailer benefits from a positive information rent, as it possesses private information, so the retailer's utility is not less than reservation utility $\underline{\pi}_r$. Moreover, the lower the sales cost C_r is, the larger the information rent gained by the retailer. Thus, the retailer's utility increases as its sales cost decreases. Therefore, under dual information asymmetry, only the retailer with the highest sales cost ($C_{r1} = 130$) obtains the reservation utility $\underline{\pi}_r$. These results are consistent with Theorems 3 and 8.

Under single information asymmetry (moral hazard), when the sales cost C_r increases, the manufacturer's profit decreases. However, under dual information asymmetry, the manufacturer's profit weakly increases with the increase of the retailer's selling cost. The manufacturer needs to pay information rent to the retailer for identifying the retailer's real sales cost under dual information asymmetry, so that the manufacturer's profit is less than under moral hazard.

The system utility decreases as the sales cost increases under both single and dual information asymmetry. When the sales cost is lower, the difference of the channel utility under single versus dual information asymmetry (moral adventure)

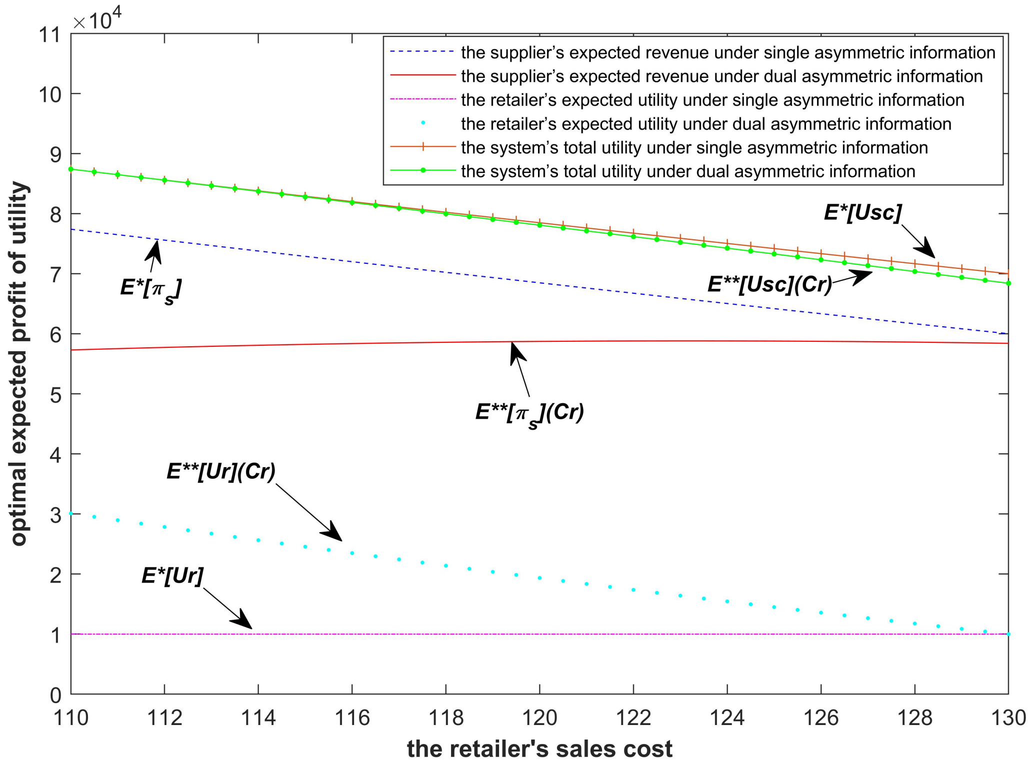

Fig. 1. The effects of sales cost C_r on the supply chain's profit and utility y ($k = 0.5, C_r \in [110, 130]$).

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is smaller, or even close to zero. As the retailer's sales cost increases, the system utility decreases under both single and dual information asymmetry, dropping at a faster rate under dual information asymmetry. This demonstrates that the overall performance of the supply chain will be to some extent weakened because the retailer possesses private information.

5.2 The effects of risk aversion k on the supply chain's profit and utility

From the horizontal point of view, Fig. 2 shows that under single information asymmetry (moral hazard), any cost-type retailer only receives the reservation utility (i.e., $E^*[U_r] = \bar{\pi}_r = 10000$) and is not affected by its risk aversion coefficient. However, the retailer's risk attitude has a greater impact on the manufacturer's profit and system's utility in the case of single information asymmetry, and on the manufacturer's profit, system's utility, and retailer's utility in the case of dual information asymmetry. In addition, the profit or utility decreases as the retailer's risk-aversion degree increases. Therefore, manufacturers are reluctant to cooperate with retailers with greater risk aversion.

From the vertical perspective, we see that the manufacturer's profit and system's utility under dual information asymmetry are smaller than under single information asymmetry, but the retailer's utility under dual information asymmetry is larger than under single information asymmetry, which is consistent with Fig. 1.

The preceding numerical analysis yields a number of managerial insights: regardless of single or dual information asymmetry, when manufacturers choose retailers to form a supply chain, they should pay attention to the retailers' sales cost information and risk attitude information. Assuming other factors do not take precedence, manufacturers should try their best to pick low-cost, low-risk retailers to cooperate with, which will help create value for manufacturers themselves and the entire supply chain system.

6 Conclusions

We have considered the case when a manufacturer offers trade credit to a downstream retailer; that is, the retailer does not need to pay the manufacturer immediately when the products are delivered, but completes all payments at the end of the sales period. Given that the retailer's sales cost is unknown to the manufacturer, its sales effort level is unobservable, and the retailer is risk averse, we have used double-objective incentive models to obtain several interesting insights:

(Ⅰ) When the degree of the retailer's risk aversion is within a certain range, the manufacturer can effectively identify the retailer's private information and motivate it to exert the optimal effort level by designing acceptable and appropriate trade credit contracts.

(Ⅱ) Under dual information asymmetry, the optimal trade credit period, optimal transfer payment, and retailer's optimal

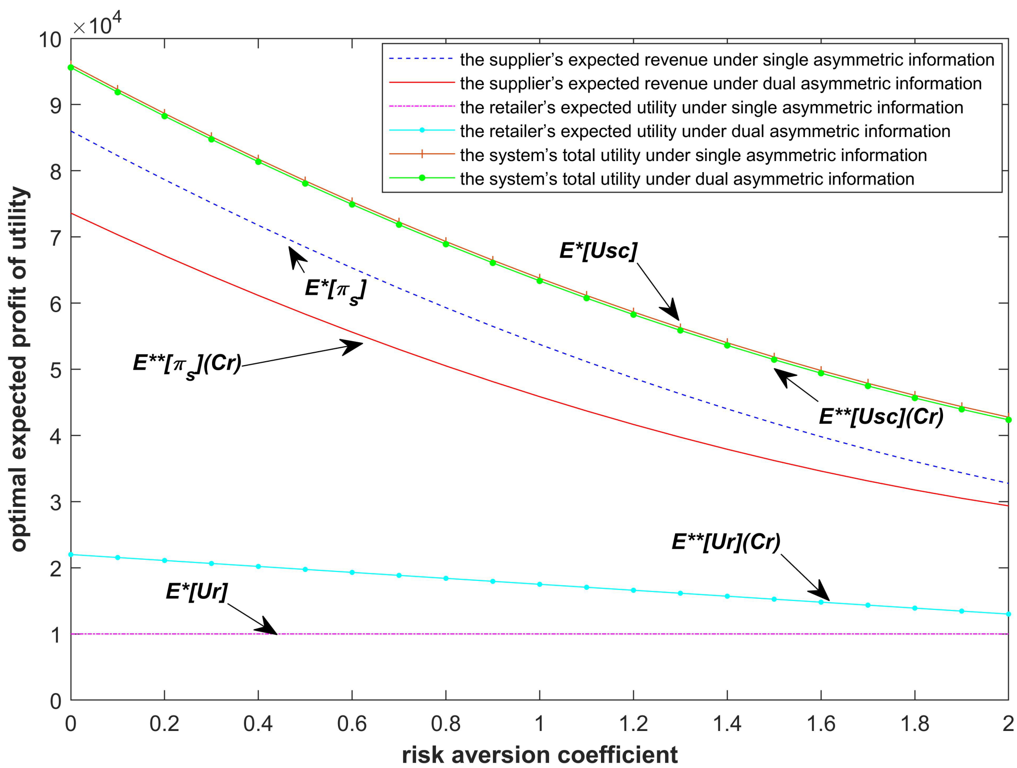

Fig. 2. The effects of risk aversion coefficient k on the supply chain's profit and utility ($C_r = 120, k \in [0, 2]$).

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sales effort level are less than under single information asymmetry. The retailer can receive a non-negative information rent due to its holding private cost information, where the information rent is negatively correlated with the cost.

(Ⅲ) The retailer's risk aversion will weaken the incentive of trade credit and the willingness of the retailer to exert effort.

(Ⅳ) Manufacturers prefer to offer trade credit to low-risk, low-cost retailers.

Our research can be extended in the following ways. First, this paper only considers that the retailer is risk-averse, but the provision of trade credit may make the manufacturer confront a number of risks, such as the retailer's demand risk and default risk, which may induce the manufacturer to exhibit risk-averse behavior. Therefore, we will consider the case that both the manufacturer and the retailer are risk-averse in the future research. In addition, our model considers the wholesale price to be exogenous. Offering trade credit will influence the wholesale price decision. Therefore, we will regard the wholesale price as a decision variable in the future work. Meanwhile, in the actual operation of trade credit, manufacturers and retailers conduct more multi-period cooperation. As a result, we will extend our future research to the case of multiple periods.

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Conflict of interest

The authors declare that they have no conflict of interest.

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Appendix

A.1 Proof of Proposition 1

From Eq. (1), we have the first-order and second-order partial derivatives of $E[U_r]$ with respect to e as follows:

$$\frac{\partial E[U_r]}{\partial e} = (p - C_r - w + i_r tw) - \mu e, \frac{\partial^2 E[U_r]}{\partial e^2} = -\mu \le 0.$$

Therefore, $E[U_r]$ is concave in e. We can solve $\frac{\partial E[U_r]}{\partial e} = 0$ to obtain $e^* = \frac{p - C_r - w + i_r tw}{\mu}$.

We can substitute e^* into model R1 and arrive at the following model:

$$\max_{t,T} E[\pi_s] = (w - C_s - i_s tw)\left(a - bp + \frac{p - C_r - w + i_r tw}{\mu}\right). \tag{26}$$

s. t.

$$\text{IR}: \ (p - C_r - w + i_r tw)(a - bp - k\sigma) + \frac{1}{2\mu}(p - C_r - w + i_r tw)^2 - T \geqslant \underset{-r}{\pi}. \tag{27}$$

Next, we construct a Lagrange function as follows, where λ is the Lagrange multiplier:

$$L = (w - C_s - i_s tw)\left(a - bp + \frac{p - C_r - w + i_r tw}{\mu}\right) + T + \lambda\left[-\underset{-r}{\pi} + (p - C_r - w + i_r tw)(a - bp - k\sigma) + \frac{1}{2\mu}(p - C_r - w + i_r tw)^2 - T\right]. \tag{28}$$

We have the following Karush–Kuhn–Tucker conditions:

$$\begin{cases} \dfrac{\partial L}{\partial T} = 1 - \lambda = 0, \\ \dfrac{\partial L}{\partial t} = 0, \\ \lambda\left[-\underset{-r}{\pi} + (p - C_r - w + i_r tw)(a - bp - k\sigma) + \frac{1}{2\mu}(p - C_r - w + i_r tw)^2 - T\right] = 0, \\ \lambda \geqslant 0. \end{cases} \tag{29}$$

From Model (A4), we obtain $\lambda = 1$ and $\left[-\underset{-r}{\pi} + (p - C_r - w + i_r tw)(a - bp - k\sigma) + \frac{1}{2\mu}(p - C_r - w + i_r tw)^2 - T\right] = 0$. Therefore, in

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constraint (A2), the equation is established; that is, the expected utility of the retailer equals the reserved utility.

Thus, we have $T=-\underline{\pi}_r+(p-C_r-w+i_r tw)(a-bp-k\sigma)+\frac{1}{2\mu}(p-C_r-w+i_r tw)^2$ and substitute T into the Lagrange function (A3).

From $\frac{\partial L}{\partial t}=0$, we have the retailer's optimal credit period t^*.

$$t^*=\frac{w-C_s}{(2i_s-i_r)w}+\frac{(i_r-i_s)\left[\mu(a-bp)+p-C_r-w\right]}{(2i_s-i_r)wi_r}-\frac{\mu k\sigma}{(2i_s-i_r)w}.$$

Obviously, when $i_S<i_r<2i_S$, we have $t^*>0$. When $2i_S<i_r$, we have $t^*<0$, which is not suitable for delayed payment; and when $2i_S=i_r$, t^* does not exist.

Substituting t^* into IR constraint (A2), we obtain the optimal transform payment T^*:

$$T^*=-\underline{\pi}_r+(p-C_r-w+i_r t^* w)(a-bp-k\sigma)+\frac{1}{2\mu}(p-C_r-w+i_r t^* w)^2.$$

We prove that (t^*,T^*) is a maximizer of (A1)-(A2) as follows. Conditioned on the Lagrangian multiplier $\lambda=1$, define the restricted Lagrangian function:

$$L=(w-C_s-i_s tw)\left(a-bp+\frac{p-C_r-w+i_r tw}{\mu}\right)+T+\left[-\underline{\pi}_r+(p-C_r-w+i_r tw)(a-bp-k\sigma)+\frac{1}{2\mu}(p-C_r-w+i_r tw)^2-T\right]. \quad (30)$$

It is straightforward to obtain

$$\frac{\partial^2 L}{\partial t^2}=-\frac{i_r w^2}{u}(2i_s-i_r),\ \frac{\partial^2 L}{\partial t\partial T}=\frac{\partial^2 L}{\partial T^2}=0.$$

Since $2i_s-i_r>0$, then $\frac{\partial^2 L}{\partial t^2}<0$ and the Hessian matrix is negative semidefinite.

According to Lemma 4.4.1 in Bazaraa et al.[45], we conclude that (t^*,T^*) is a maximizer of (A1).

Substituting t^* into $e^*=\frac{p-C_r-w+i_r tw}{\mu}$, we obtain the optimal sales effort level:

$$e^*=\frac{p-C_r-w+i_r t^* w}{\mu}.$$

Hence, we conclude the proof.

A.2 Proof of Proposition 2

Based on the revelation principle, the retailer with sales cost C_r receives its optimal expected utility when it chooses the appropriate contract intended for its sales cost C_r, i.e., let $E[U_r](C_r)=\max_{C_r} E[U_r]\left(C_r,C_r\right)$. According to the envelope theorem, we have $\frac{dE[U_r](C_r)}{dC_r}=-(a-bp-k\sigma)-\frac{1}{\mu}(p-C_r-w+i_r t(C_r)w)\leq 0.$

From $k\in[0,\frac{a-bp+e(C_r)}{\sigma})$, we have $\frac{dE[U_r](C_r)}{dC_r}\leq 0.$ Therefore, $E[U_r](C_r)$ decreases in C_r. Combining IR constraint (19), we have $E[U_r](C_{r2})=\min E[U_r](C_r)\geqslant\underline{\pi}_r$. Then, there is

$$E[U_r](C_r)=E[U_r](C_{r2})+\int_{C_{r2}}^{C_r}[(a-bp-k\sigma)+\frac{1}{\mu}(p-C_r-w+i_r t(\tau)w)]d\tau\geqslant\underline{\pi}_r+\int_{C_{r2}}^{C_r}[(a-bp-k\sigma)+\frac{1}{\mu}(p-C_r-w+i_r t(\tau)w)]d\tau,$$

where $C_{r1}\leq C_r\leq C_{r2}$.

Therefore, IR constraint (19) in model R2 is translated into the IR1 constraint as follows:

IR1: $E[U_r](C_r)\geqslant\underline{\pi}_r+\int_{C_{r2}}^{C_r}[(a-bp-k\sigma)+\frac{1}{\mu}(p-C_r-w+i_r t(\tau)w)]d\tau.$

Next, we obtain the following optimal model:

$$\max_{t(C_r),T(C_r)} E[\pi_s](C_r). \quad (31)$$

s. t.

$$\text{IC}: E[U_r](C_r)\geqslant E[U_r]\left(C_r,C_r\right). \quad (32)$$

$$\text{IR1}: E[U_r](C_r)\geqslant\underline{\pi}_r+\int_{C_{r2}}^{C_r}\left\{(a-bp-k\sigma)+\frac{1}{\mu}(p-C_r-w+i_r t(\tau)w)\right\}d\tau, \quad (33)$$

where $\forall C_r\in[C_{r1},C_{r2}]$.

We construct the Lagrange function as follows, where λ and β are the Lagrange multipliers:

$$L=\int_{C_{r1}}^{C_{r2}}\left\{\left[a-bp+\frac{p-C_r-w+i_r t(C_r)w}{\mu}\right][\mathrm{w}-\mathrm{c}_s-i_s t(C_r)w]+T(C_r)\right\}$$

$$f(C_r)dC_r+\beta\left\{(p-C_r-w+i_r t(C_r)w)(a-bp-k\sigma)+\frac{1}{2\mu}(p-C_r-w+i_r t(C_r)w)^2-T(C_r)-\left(p-C_r-w+i_r t\left(C_r\right)w\right)(a-bp-k\sigma)-\frac{1}{2\mu}\right.$$

$$\{(p-C_r-w+i_r t(C_r)w)(a-bp-k\sigma)+\frac{1}{2\mu}(p-C_r-w+i_r t(C_r)w)^2-T(C_r)-\underline{\pi}_r-\int_{C_{r2}}^{C_r}$$

$$\left\{(a-bp-k\sigma)+\frac{1}{\mu}(p-C_r-w+i_r t(\tau)w)\right\}d\tau\}. \tag{34}$$

We have the following Karush–Kuhn–Tucker conditions:

$$\begin{cases}\frac{\partial L}{\partial T(C_r)}=1-\beta-\lambda=0\,(\mathrm{B5})\\ \frac{\partial L}{\partial t(C_r)}=0\\ \beta\left\{\begin{array}{c}(p-C_r-w+i_r t(C_r)w)(a-bp-k\sigma)+\frac{1}{2\mu}(p-C_r-w+i_r t(C_r)w)^2-T(C_r)\\ -\left(p-C_r-w+i_r t\left(C_r\right)w\right)(a-bp-k\sigma)-\frac{1}{2\mu}\left(p-C_r-w+i_r t\left(C_r\right)w\right)^2+T\left(C_r\right)\end{array}\right\}=0\,(\mathrm{B6})\\ \lambda\left\{\begin{array}{c}(p-C_r-w+i_r t(C_r)w)(a-bp-k\sigma)+\frac{1}{2\mu}(p-C_r-w+i_r t(C_r)w)^2-T(C_r)\\ -\underline{\pi}_r-\int_{C_{r2}}^{C_r}[(a-bp-k\sigma)]d\tau-\int_{C_{r2}}^{C_r}\left[\frac{1}{\mu}(p-C_r-w+i_r t(\tau)w)\right]d\tau\end{array}\right\}=0\,(\mathrm{B7})\\ \lambda\geqslant 0,\ \beta\geqslant 0\end{cases}$$

From Equation (B5), we have $\beta+\lambda=1$. Furthermore, we can obtain $\beta=0$ by contradiction. In fact, if $\beta>0$,, then (B6) or equivalently (B2) is binding; that is, the retailer's utility of disclosing its real cost is equal to that of not disclosing its real cost, which violates the incentive compatibility constraint. Therefore, we obtain $\beta=0$.

As $\beta+\lambda=1$ and $\beta=0$, we have $\lambda=1$. Then, from Equation (B7), we have

$$(p-C_r-w+i_r t(C_r)w)(a-bp-k\sigma)+\frac{1}{2\mu}(p-C_r-w+i_r t(C_r)w)^2-T(C_r)-\underline{\pi}_r-\int_{C_{r2}}^{C_r}\left\{(a-bp-k\sigma)+\frac{1}{\mu}(p-C_r-w+i_r t(\tau)w)\right\}d\tau=0$$

That is, the equality in IR1 constraint (B3) holds. We obtain

$$T(C_r)=(p-C_r-w+i_r t(C_r)w)(a-bp-k\sigma)+\frac{1}{2\mu}(p-C_r-w+i_r t(C_r)w)^2-\underline{\pi}_r-\int_{C_{r2}}^{C_r}\left\{(a-bp-k\sigma)+\frac{1}{\mu}(p-C_r-w+i_r t(\tau)w)\right\}d\tau. \tag{35}$$

Substituting Equation (B8) into the Lagrange function (B4), we obtain

$$L=\int_{C_{r1}}^{C_{r2}}\{\left[a-bp+\frac{p-C_r-w+i_r t(C_r)w}{\mu}\right][\mathrm{w}-\mathrm{c}_s-i_s t(C_r)w]+(p-C_r-w+i_r t(C_r)w)$$
$$(a-bp-k\sigma)+\frac{1}{2\mu}(p-C_r-w+i_r t(C_r)w)^2-\underline{\pi}_r-\int_{C_{r2}}^{C_r}\{(a-bp-k\sigma)+$$
$$\frac{1}{\mu}(p-C_r-w+i_r t(\tau)w)d\tau\}dF(C_r). \tag{36}$$

From $F(C_{r1})=0, F(C_{r2})=1$, by exchanging the integral order, we obtain

$$\int_{C_{r1}}^{C_{r2}}\int_{C_r}^{C_{r2}}\left\{(a-bp-k\sigma)+\frac{1}{k}(p-C_r-w+i_r t(\tau)w)\right\}d\tau dF(C_r)=\int_{C_{r1}}^{C_{r2}}\left\{(a-bp-k\sigma)+\frac{1}{\mu}(p-C_r-w+i_r t(\tau)w)\right\}\bullet\frac{F(C_r)}{f(C_r)}dF(C_r) \tag{37}$$

Substituting Equation (B10) into Equation (B9), Equation (B9) becomes

$$L=\int_{C_{r1}}^{C_{r2}}R(t,\ C_r)dF(C_r)+(p-w)(a-bp-k\sigma)+(w-C_m)\left(a-bp-k\sigma+\frac{p-w}{k}\right)-\underline{\pi}_r, \tag{38}$$

where

$$R(t,\ C_r)=[w-C_m-i_m t(C_r)w]\left[\frac{-C_r+i_r t(C_r)w}{\mu}\right]-i_m t(C_r)w\left(a-bp-k\sigma+\frac{p-w}{\mu}\right)+[-C_r+i_r t(C_r)w](a-bp-k\sigma)+$$
$$\frac{1}{2\mu}(p-C_r-w+i_r t(C_r)w)^2-\left[(a-bp-k\sigma)+\frac{1}{\mu}(p-C_r-w+i_r t(C_r)w)\right]\frac{F(C_r)}{f(C_r)}.$$

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From $\frac{\partial L}{\partial t(C_r)}=0$, we have the retailer's optimal credit period.

$$t^{**}(C_r)=\frac{1}{(2i_s-i_r)w}\left[\frac{-F(C_r)}{f(C_r)}+w-C_s\right]+\frac{(i_r-i_s)[\mu(a-bp)+p-C_r-w]}{(2i_s-i_r)wi_r}-\frac{\mu k\sigma}{(2i_s-i_r)w}.$$

Obviously, when $i_S<i_r<2i_S$, we have $t^{**}(C_r)>0$. When $2i_S<i_r$, we have $t^{**}(C_r)<0$, which is not suitable for delayed payment; and when $2i_S=i_r$, $t^{**}(C_r)$ does not exist.

Substituting $t^{**}(C_r)$ into the IR1 constraint (B3), we obtain the optimal transform payment $T^{**}(C_r)$.

$$T^{**}(C_r)=[p-C_r-w+i_rt^{**}(C_r)w](a-bp-k\sigma)+\frac{1}{2\mu}[p-C_r-w+i_rt^{**}(C_r)w]^2-\pi_{-r}-\int_{C_r}^{C_{r2}}\left\{(a-bp-k\sigma)+\frac{(p-\tau-w+i_rt^{**}(\tau)w)}{\mu}\right\}d\tau.$$

Conditioned on the Lagrangian multipliers $\lambda=1$ and $\beta=0$, define the restricted Lagrangian function. We can prove that $(t^{**}(C_r),T^{**}(C_r))$ is a maximizer of (B1)-(B3) in a similar way in Appendix A.

Substituting $t^{**}(C_r)$ into Equation (13), we obtain the optimal sales level $e^{**}(C_r)$.

$$e^{**}(C_r)=\frac{p-C_r-w+i_rt^{**}(C_r)w}{k}.$$

Hence, we conclude the proof.

A.3 Proof of Theorem 1

When the retailer has cost C_r and chooses contract $\{t(C_r),T(C_r)\}$, the optimal expected utility is

$$E^{**}[U_r](C_r)=\pi_{-r}+\int_{C_{r2}}^{C_r}[(a-bp-k\sigma)+\frac{1}{\mu}(p-C_r-w+i_rt^{**}(\tau)w)]\mathrm{d}\tau.$$

When the retailer possesses private cost C_r and chooses contract $\{t\left(C_r\right),T\left(C_r\right)$, the optimal expected utility is

$$E^{**}[U_r]\left(C_r,C_r\right)=\left(p-C_r-w+i_rt^{**}\left(C_r\right)w\right)(a-bp-k\sigma)+\frac{1}{2\mu}\left(p-C_r-w+i_rt^{**}\left(C_r\right)w\right)^2-T^{**}\left(C_r\right)=(a-bp-k\sigma)\left(C_r-C_r\right)+$$
$$\frac{1}{\mu}\left(p-w+i_rt^{**}\left(C_r\right)w\right)\left(C_r-C_r\right)+\frac{1}{2\mu}\left(C_r^{\,2}-C_r^{\,2}\right)+\pi_{-r}+\int_{C_r}^{C_{r2}}\left\{(a-bp-k\sigma)+\frac{1}{\mu}(p-\tau-w+i_rt^{**}(\tau)w)\right\}d\tau.$$

Then, the difference between $E^{**}[U_r](C_r)$ and $E^{**}[U_r]\left(C_r,C_r\right)$ is

$$\Delta U\pi_r=E^{**}[U_r](C_r)-E^{**}[U_r]\left(C_r,C_r\right)=$$
$$\int_{C_r}^{C_r}\{\left[(a-bp-k\sigma)+\frac{1}{\mu}(p-w+i_rt^{**}(\tau)w)\right]-\left[(a-bp-k\sigma)+\frac{1}{\mu}\left(p-w+i_rt\left(C_r\right)w\right)\right]\}d\tau=\int_{C_r}^{C_r}\frac{1}{\mu}i_rw\left[t^{**}(\tau)-t^{**}\left(C_r\right)\right]\mathrm{d}\tau.$$

According to the assumption that C_r satisfies the increasing failure rate distribution, $\frac{f(C_r)}{\bar{F}(C_r)}$ increases in C_r, where $\bar{F}(C_r)=1-F(C_r)$, and so we obtain $\frac{d}{dC_r}\left[\frac{F(C_r)}{f(C_r)}\right]>0$. Thus, we have $\frac{\partial t^{**}(C_r)}{\partial C_r}=-\frac{1}{(2i_s-i_r)w}\cdot\frac{d}{dC_r}\left[\frac{F(C_r)}{f(C_r)}\right]+\frac{-(i_r-i_s)}{(2i_s-i_r)i_rw}<0$ from Eq. (20). Therefore, optimal trade credit period $t^{**}(C_r)$ monotonically decreases in the retailer's sales cost. Regardless of the size relationship between C_r and C_r, there is always $\Delta U\pi_r\geqslant 0$ for unit sales cost τ where $C_r\leqslant\tau\leqslant C_r$ or $C_r\leqslant\tau\leqslant C_r$. That is, the retailer will have the utility loss $\Delta U\pi_r$ if it does not report its true cost information.

Hence, we conclude the proof.

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