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Live-streaming Selling Strategies for Competitive Firms

Quan Du, Zhixin Chen, Jie Wu, and Xiang Ji ✉

School of Management, University of Science and Technology of China, Hefei 230026, China

✉Correspondence: Xiang Ji, signji@mail.ustc.edu.cn

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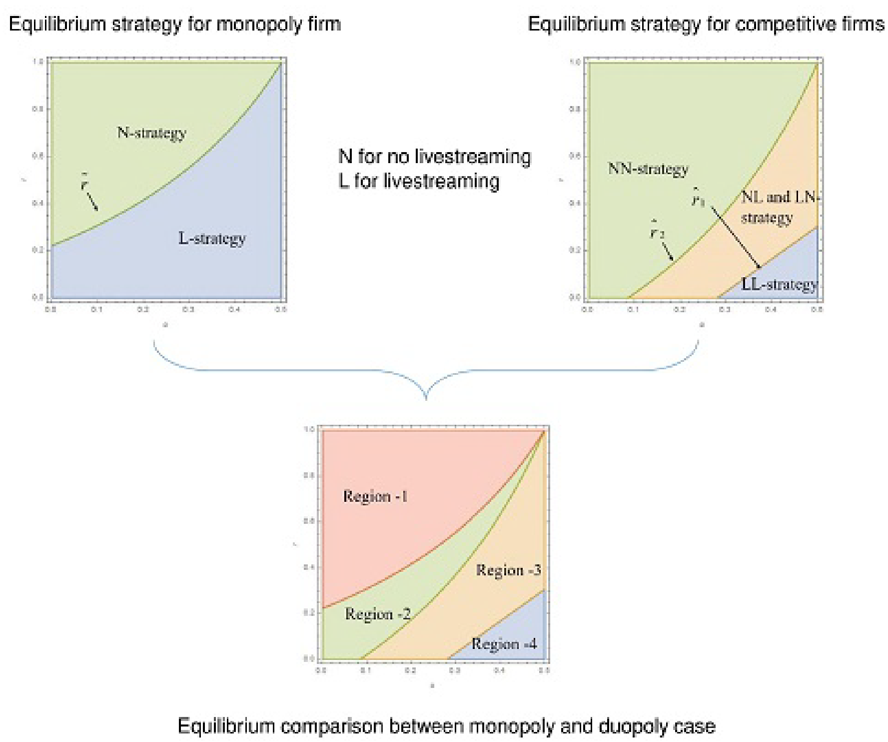
Graphical abstract



Public summary

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Abstract: The booming live-streaming commerce has significantly changed the traditional e-commerce model, thus attracting much attention from both industry and academia. In recent years, an increasing number of scholars have applied analytical models to explore live-streaming strategies for firms in different scenarios. However, the previous literature mainly considers monopolists, while in the real world, competition is not rare. To fill this gap between the literature and practical observations, this paper applies a game theoretical model to study live-streaming adoption and pricing strategy for firms under competitive environments. The results show that, for competitive firms, the equilibrium strategy depends on the relation between the commission rate and the intensity of the market expansion effect. Additionally, compared to the case in which no firm adopts live-streaming, competitive firms do not always benefit from the adoption of live-streaming selling. The paper also shows that competition plays a negative role in inducing a firm to adopt live-streaming.

Keywords: Live-Streaming Commerce; Live-Streaming Strategy; Game; Competition

CLC number: **Document code:** A

1 Introduction

The emergence of live-streaming sales has changed the traditional e-commerce model, and consumers are eager to purchase from live-streaming sales. Live-streaming marketing has now become a major force in retailing. According to *Bloomberg*, live-streaming marketing allows almost anyone (e.g., celebrities, influential people or local store owners) to quickly create their own shopping television channel that is also a social network and e-commerce platform for a tiny fraction of the cost①. From China to North America and Europe, an increasing number of firms, brand owners and retailers have joined the trend of live-streaming sales. According to a report jointly issued by the Alibaba Research Institute and KPMG Consulting②, the overall scale of live-streaming e-commerce exceeded one trillion in 2020.

Live-streaming sales are typically completed by manufacturers, retailers or firms who sign a contract with a celebrity, influential person or professional salesman. We collectively refer to these people as streamers. Streamers introduce the characteristics and functions of the firm's products to consumers through online live-streaming sale shows, and they show consumers how to use the products by displaying them. In addition, if consumers have any questions about the specific display product, then they can ask the streamer by sending a "bullet screen", and the streamer can give the consumer an instant response to this question; this quick interactive feedback process helps to reduce the uncertainty about the product.

① https://www.bloomberg.com/news/features/2020-09-14/what-is-livestream-shopping-it-s-the-future-u-s-e-commerce
② https://home.kpmg/cn/zh/home/insights/2020/10/live-streaming-e-commerce-toward-trillion-market.html

We summarize two key roles of live-streaming e-commerce. On the one hand, like commercial advertisements, celebrity streamers can attract more consumer traffic to watch and buy from live-streaming sales in a very short time; that is, celebrity streamers can expand the market size of products. According to *McKinsey*, in December 2020, Walmart piloted a live-stream fashion event on TikTok that netted seven times more viewers than expected and enabled it to add 25 percent to its TikTok follower base①. On the Tmall platform, L'ORÉAL invited over 60 top celebrities and famous influencers to attend a live stream. The live stream gained eight million views and over 800,000 new fans on Tmall in only four days②. Moreover, as a basic function of live-streaming sales, consumers can learn more details about the product by watching live-stream sales shows and interacting with streamers, which reduces consumers' uncertainty about the products[1]. Streamers display and demonstrate products, explain features, and answer consumers' questions on the "bullet screen" in real time, which may help enhance consumers' trust in the products and make them more likely to buy[2].

The booming live-streaming marketing has also attracted much attention from the academia. In recent years, an increasing number of scholars have applied analytical models to explore live-streaming strategies for firms in different scenarios.

① https://www.mckinsey.com/business-functions/mckinsey-digital/our-insights/its-showtime-how-live-commerce-is-transforming-the-shopping-experience
② https://www.loreal-finance.com/en/annual-report-2020/loreal-worldwide-2-3-0/loreal-in-mainland-China-the-era-of-accelerated-digitalisation-2-3-1/

For example, Pan et al.[3] show that, regardless of the switching demand, a live-streaming strategy is profitable only if the streamer's ability to sell is sufficiently high. Wang et al.[4] examine the live-streaming strategy for a firm selling through a platform with different distribution contracts. Jiang et al.[5] show that, when the seller offers a product line, the upward-line extension can incentivize the adoption of live-streaming. However, the above papers mainly consider monopolists, while in the real world, competition is not rare. To fill in this gap between the literature and practical observations, we apply a game theoretical model to study live-streaming adoption and pricing strategy for firms under competitive environments. We wish to answer the following research questions. First, how should competitive firms make their live-streaming strategy decisions? Second, how does the adoption of a live-streaming strategy affect firms' profit under a competitive environment? Can firms truly benefit from the adoption of live-streaming sales? Third, how does competition affect firms' live-streaming strategy?

To answer the above questions, we assume that consumers are uniformly distributed based on their ideal product taste and choose to buy from a firm or its competitor based on maximized utility. We assume that consumers can reduce their uncertainty about a particular firm's product; our model shows a unit mismatch cost reduction by watching the product's live-streaming sale show. Furthermore, we allow for some consumers who will always buy the product whether live streaming exists or not. However, another fraction of consumers buy the product only when live sales are available in the market. For this type of consumer, the role of live streaming is similar to informative advertising[6]; the only way that they become informed about a firm's product is through its live-streaming sales show. The main results are as follows. First, our paper shows that, for competitive firms, the equilibrium strategy depends on the relation between the commission rate and the intensity of the market expansion effect. Second, compared to the case when no firm adopts live sales or is not available in the market, we derive the profit implications for the firms that have the choice to use live-stream selling. When in a competitive environment, we demonstrate that the adoption of a live-streaming strategy may result in more intense competition between firms and further lead to a price war. Even when both firms adopt live-streaming sales as an equilibrium outcome, firms may suffer a loss of profit at the same time (i.e., a *prisoner's dilemma* occurs). Third, we find that a higher unit mismatch reduction level caused by live-streaming selling is beneficial for monopolists to adopt a live-stream strategy. However, under competition, a higher unit mismatch reduction level would restrain duopoly firms from simultaneously adopting a live-stream strategy to avoid fiercer price competition.

The rest of this paper is organized as follows: Section 2 reviews the related literature. Section 3 introduces the model, and Section 4 presents the main results. Section 5 concludes the paper.

2 Literature Review

Our paper is related to the emerging literature on live-streaming marketing. With the rapid development of livestream commerce in recent years, the literature has empirically studied livestream commerce from various perspectives. For example, Xu et al.[7] investigate three stimulus effects from livestreaming on viewers' cognitive and emotional states and the three subsequent responses. Wongkitrungrueng and Assarut[8] propose a comprehensive framework to explore the relationships among consumers' perceived value of livestreaming, consumer trust and engagement. Kang et al.[9] develop a research model using real-time data to study the dynamic effect of interactivity on customer engagement behavior through tie strength in livestream commerce. Meanwhile, there are also some analytical studies on livestreaming. For example, Sun et al.[10] build a theoretical model to show that factors such as the immediate visibility and availability of live shopping and shopping experiences can increase consumers' intention to purchase. Hou et al.[2] develop a model to study a firm's optimal livestream adoption strategy by incorporating influencers' characteristics and consumer perceptions of product quality. Qi et al.[11] investigate the capacity investment strategy of a manufacturer that sells a product on a livestream shopping platform and find that overcapacity can benefit the manufacturer by driving down the platform's commission fees. Li et al.[12] build a stylized model to study when the introduction of an influencer marketing channel can increase a retailer's profit and social welfare with or without a merchant live broadcast channel. Pan et al.[3] show that, regardless of the switching demand, a live-streaming strategy is profitable only if the streamer's ability to sell is sufficiently high. Wang et al.[4] examine the live-streaming strategy for a firm selling through a platform with different distribution contracts. Jiang et al.[5] show that, when the seller offers a product line, the upward-line extension can incentivize the adoption of live-streaming.

Our paper is also related to the literature on social operations, in which the effects of social influence on decisions have been explicitly modeled. Earlier attempts in this field mainly focus on measuring social influence intensity and examining differences between different types of social influence[13−14]. In recent years, scholars have paid more attention to exploring how social influence affects managerial decisions. For example, Godes[15] analyzes how social communications affect optimal product quality and shows that quality might either increase or decrease as social communication expands. Chong et al.[16] analyze the roles of online promotional marketing and online reviews in predicting consumer product demands via Big Data. Choi[17] studies how social media affects a quick-response fashion supply chain and shows that the manipulation of comments left on social media could benefit the manufacturer under a surplus sharing contract. Kuksov and Liao[18] analyze how a firm should adjust its product variety in the presence of opinion leaders in social networks and show that this adjustment might be either upward or downward. Orji et al.[19] study the critical success factors involved in using social media for supply chain social sustainability in the freight logistics industry. Ji et al.[20] examine how social communications affect an upstream firm's product line design in the platform economy and find that social communications

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can increase the product line length while decreasing the product price and quality.

Unlike existing studies, our work complements the literature in the following ways. First, this paper analyzes live-streaming selling strategies for competitive firms. By examining profit implications, this work uniquely reveals that competitive firms do not always benefit from the adoption of live-steaming selling. Second, as live-streaming selling is a special kind of social-commerce, our work highlights the role of competition in reducing firms' incentives to adopt social operations.

3 Model

We assume that consumers are uniformly distributed on the Hotelling line between $[0,1]$. A consumer's location x denotes his or her relative ideal preferences toward the product of a firm, parameter V indicates the basic value of the product, and t represents the unit mismatch cost of the product compared to consumers' ideal taste. As a brand new mode of retailing and marketing, we summarize two key features of live-stream sales as follows. First, the market expansion effect and live-streaming sales can attract consumer traffic and expand the demand of the potential market scale. Second, with the mismatch cost reduction effect, consumers can reduce the uncertainty of the product by watching both the product display and the introducing streamers.

To capture the market size expansion effect of live-stream selling, in a competitive duopoly case, we assume that the products offered by two competitive firms are completely homogeneous. Furthermore, live-streaming is endowed with preferences over product attributes, and then, consumers know the type of products that exist and their characteristics. That is, the role of live-streaming for products is to convey information about existing products and their attributes so that an originally uninformed consumer can evaluate his or her degree of preference for the products and choose between two competing homogeneous products to make a purchase. Specifically, we normalize the market size to 1 when both firms in the market adopt live selling, the market size is reduced to $1-\alpha$ when one firm adopts live selling and the other firm does not, and the market size is further reduced to $1-2\alpha$ when live selling is not available. Accordingly, the α fraction of consumers makes purchases only when a live stream exists, and a higher α means a stronger market expansion effect caused by the live stream.

Parameter t captures the unit mismatch cost of the consumer. When a firm decides to use live-streaming selling, by the sophisticated display of the streamer, consumers will obtain a unit mismatch reduction. Specifically, if one firm adopts live-streaming selling and consumers buy from this firm, then they will obtain a unit mismatch cost t_L; otherwise, they will obtain t_N. Without loss of generality, we assume that $t_L = 1$ and that $t_N > 1$. A higher unit mismatch cost t_N means a higher unfit cost to the consumer regarding how the product matches his or her personal ideal preference. That is, when the difference between t_N and t_L is greater, the reduction of consumer uncertainty by live-stream sales is greater.

We assume that there are two competitive firms in the market named A and B. They are horizontally differentiated on the Hotelling Line, with Firm A's location at $L_A = 0$ and Firm B's location at $L_B = 1$. Each firm i, $i = A, B$, has some basic value V_i of the product that it offers, and each firm charges a price p_i to consumers. Without loss of generality, we assume that $V_A = V_B = V$ means the homogeneity of their product, and V is large enough that all consumers will buy the product. Specifically, when consumers watch the live stream of firm i, they will obtain utility $U_i = V - t_L \times |L_i - x| - p_i$, where $t_L = 1$. Otherwise, they will obtain utility $U_i = V - t_N \times |L_i - x| - p_i$, where $t_N > 1$. Consumers will compare the utility of firms (A or B) to make a purchase.

The firms decide whether to open a live-stream shopping show for consumers. If the firms open live-stream shopping, then the live stream could increase brand attractiveness and reduce the uncertainty about the firms' product to consumers. However, the firms will pay extra costs for streamers. Typically, the cost for hiring a celerity streamer consists of two parts. Streamers hired by firms typically charge an *upfront fee* to secure a slot on a live-stream session and a *commission* from the total sales①. To simplify our analysis and obtain clear results, in our model, we assume that if the firms adopt live-stream selling, then streamers will take r percent from the firm's live-stream sales profit as a *commission*.

The sequences of events are as follows. In stage 1, the firms simultaneously decide to choose a live-stream strategy (L strategy) or a nonlive-stream strategy (N strategy). In stage 2, the firms then simultaneously decide the price of product p_i. In stage 3, consumers make purchase decisions, and they choose to buy a product from one of the firms. We summarize the main parameter and decision variables in Table 1.

4 Results

In this section, we first analyze the live-streaming strategy for competitive firms. Then we compare the results to the case where both firms do not adopt live-streaming to obtain the profit implications for firms. We accordingly investigate the benefits and disadvantages of live-streaming adoption. Additionally, we explore how competition affects live-streaming adoption by incorporating a monopoly case as a reference.

Table 1. Parameters and decision variables

Symbols	Definition
V	Consumer's base utility
t_N	Consumer's unit mismatch disutility without live-streaming
t_L	Consumer's unit mismatch disutility with live-streaming
x	Consumer's ideal taste location
α	Market expansion coefficient
r	Commission rate
p_i	Price set by firm i
D_i	Demand for firm i
Π_i	Profit of firm i

① https://kr-asia.com/chinas-e-commerce-merchants-eschew-livestreaming-as-costs-eat-into-profits

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4.1 Live-streaming Strategy for Competitive Firms

We first consider the subgame when no firm adopts live-stream sales. When no firm adopts the live-stream strategy (NN strategy), the total market size is only $1-2\alpha$, and the firms separately set their prices as p_i^{NN}. Superscript NN indicates the price when the firms adopt the nonlive-stream strategy. Under the NN-case, the utility of a consumer buying from firm i is as follows:

$$\begin{aligned} U_A^1 &= V - t_N x - p_A^{NN} \\ U_B^1 &= V - t_N(1-x) - p_B^{NN} \end{aligned} \tag{1}$$

By solving equation $U_A^1 = U_B^1$, we obtain $\widetilde{x_1}^{NN} = \dfrac{1}{2} + \dfrac{-p_A^{NN} + p_B^{NN}}{2t_N}$, which means that there is no difference between consumers buying Firm A's or Firm B's products at this point. Recall that the market size is $1-\alpha$ in this case; then, we obtain the demand of the two firms given by $D_A^{NN} = (1-2\alpha)\widetilde{x_1}^{NN}$, $D_B^{NN} = (1-2\alpha)(1-\widetilde{x_1}^{NN})$.

Therefore, the profit function of Firm A and Firm B under this subgame is as follows:

$$\begin{aligned} \Pi_A^{NN} &= D_A^{NN} * p_A^{NN} = (1-2\alpha) * \left(\frac{1}{2} + \frac{-p_A^{NN} + p_B^{NN}}{2t_N}\right) * p_A^{NN} \\ \Pi_B^{NN} &= D_B^{NN} * p_B^{NN} = (1-2\alpha) * \left(\frac{1}{2} + \frac{p_A^{NN} - p_B^{NN}}{2t_N}\right) * p_B^{NN} \end{aligned} \tag{2}$$

Each firm determines its price p_i^{NN} to maximize its own profit, and we summarize the equilibrium price and profit in the lemma below.

Lemma 1 *In the duopoly NN-subgame, each firm charges price* $p_i^{NN} = t_N$ *and obtains profit* $\Pi_i^{NN} = \dfrac{t_N}{2}(1-2\alpha)$.

Next, we analyze the subgame of both firms when they adopt a live-stream strategy (LL strategy). Since both firms choose to adopt live selling, the market size expands from $1-2\alpha$ to 1, and the firms separately set their prices as p_i^{LL}. Superscript LL indicates the price when both firms adopt the live-stream strategy. Under the LL-case, the utility of a consumer buying from firm i is as follows:

$$\begin{aligned} U_A^2 &= V - t_L x - p_A^{LL} \\ U_B^2 &= V - t_L(1-x) - p_B^{LL} \end{aligned} \tag{3}$$

By solving equation $U_A^2 = U_B^2$, we find that the marginal consumer is indifferent to buying from Firm A or B $\widetilde{x_2}^{LL} = \dfrac{1}{2} + \dfrac{-p_A^{LL} + p_B^{LL}}{2}$. Recall that the market size is 1 in this case; then, we obtain the demand of the two firms as given by $D_A^{LL} = \widetilde{x_2}^{LL}, D_B^{LL} = (1-\widetilde{x_2}^{LL})$.

Therefore, the profit function of Firms A and B under this subgame is as follows:

$$\begin{aligned} \Pi_A^{LL} &= D_A^{LL} * p_A^{LL} * (1-r) = \left(\frac{1}{2} + \frac{-p_A^{LL} + p_B^{LL}}{2}\right) * p_A^{LL} * (1-r) \\ \Pi_B^{LL} &= D_B^{LL} * p_B^{NN} * (1-r) = \left(\frac{1}{2} + \frac{p_A^{LL} - p_B^{LL}}{2}\right) * p_B^{LL} * (1-r) \end{aligned} \tag{4}$$

Each firm determines its price p_i^{LL} to maximize its own profit, and we summarize the equilibrium price and profit in the lemma below.

Lemma 2 *In the LL case, each firm charges price* $p_i^{LL} = 1$ *and obtains profit* $\Pi_i^{LL} = \dfrac{1}{2}(1-r)$.

Finally, we examine the asymmetric subgame in which only one firm adopts a live-stream strategy (LN and NL strategies). Without loss of generality, we assume that Firm A adopts a live-stream strategy and that Firm B does not. Since only one firm adopts live selling, the market size under this circumstance is $1-\alpha$, and the firms separately set their prices to p_A^{LN} and p_B^{LN}. Superscript LN indicates the price when Firm A adopts the live-stream strategy while Firm B does not. Under an asymmetric subgame, the live-streaming adoption firm could increase its market share by a low unit mismatch cost; meanwhile, it generates new sales by increasing the demand of the product category. Therefore, under the LN case, the utility of a consumer buying from firm i is as follows:

$$\begin{aligned} U_A^3 &= V - t_L x - p_A^{LN} \\ U_B^3 &= V - t_N(1-x) - p_B^{LN} \end{aligned} \tag{5}$$

By solving equation $U_A^3 = U_B^3$, we obtain the marginal consumer who is indifferent to buying from Firm A or B $\widetilde{x_3}^{LN} = \dfrac{-p_A^{LN} + p_B^{LN} + t_N}{1+t_N}$. Recall that the market size is $1-\alpha$ in this case; the newly informed consumers evaluate their degree of preference for the specific product category due to the live-streaming selling show by Firm A and then compare the products between the two competing firms to make a pur-

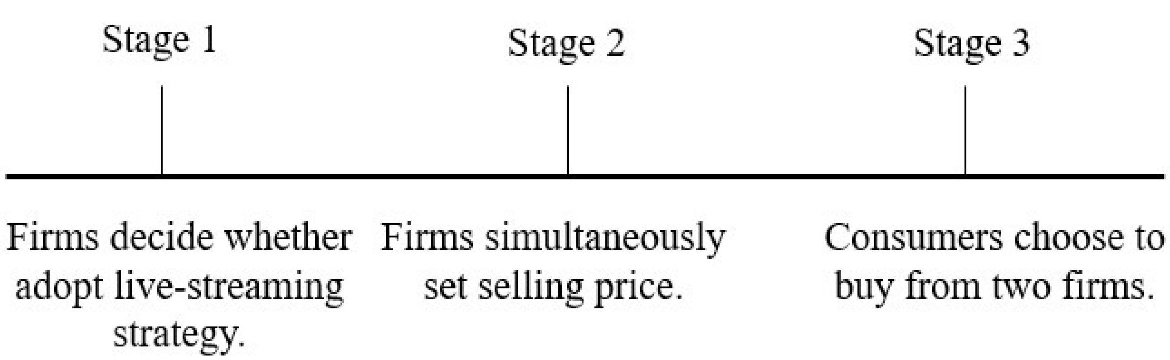

Fig. 1. Model timeline

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chase. Therefore, we obtain the demand of the two firms as $D_A^{LN} = (1-\alpha)\widetilde{x}_3^{LN}, D_B^{LN} = (1-\alpha)(1-\widetilde{x}_3^{LN})$.

Therefore, the profit function of Firm A and Firm B under this subgame is as follows:

$$\Pi_A^{LN} = D_A^{LN} * p_A^{LN} * (1-r) = (1-\alpha)\left(\frac{-p_A^{LN}+p_B^{LN}+t_N}{1+t_N}\right) * p_A^{LN} * (1-r)$$
$$\Pi_B^{LN} = D_B^{LN} * p_B^{LN} = (1-\alpha)\left(1-\frac{-p_A^{LN}+p_B^{LN}+t_N}{1+t_N}\right) * p_B^{LN} \tag{6}$$

Each firm determines its prices p_A^{LN} and p_B^{LN} to maximize its own profit, and we summarize the equilibrium price and profit in the lemma below.

Lemma 3 *In the LN case,*

Firm A charges price $p_A^{LN} = \frac{1+2t_N}{3}$ *and obtains profit* $\Pi_A^{LN} = \frac{(1+2t_N)^2}{9(1+t_N)}(1-\alpha)(1-r)$.

Firm B charges price $p_B^{LN} = \frac{2+t_N}{3}$ *and obtains profit* $\Pi_B^{LN} = \frac{(2+t_N)^2}{9(1+t_N)}(1-\alpha)$.

We have obtained the firms' equilibrium price decisions under each subgame (NN, LL and LN). Consumers usually believe that when they buy through online live-stream sales, they will always get lower prices in contrast to when they do not buy through the live-selling channel. However, the answer is negative; we summarize our findings in the Corollary below.

Corollary 1 *Under the competitive duopoly case, the firms' price is* $p_A^{NN} > p_A^{LN} > p_B^{LN} > p_A^{LL}$.

Corollary 1 answers the question of whether consumers will always obtain a lower price if they buy in a live-stream shopping show. On the one hand, compared to the NN-subgame, the existence of live streaming shows in the market can indeed result in a price reduction. The display and introduction of products by streamers is always beneficial to consumers. On the other hand, we notice that under the asymmetric subgame (LN and NL cases), when Firm A adopts live streaming, it can always set a higher price than Firm B. To some extent, live streaming acts as a special form of advertising, and it can help a firm both increase its market power and support a high price. Therefore, if Firm A adopts live streaming while Firm B does not, then Firm A has enough incentive to set a higher price due to the advantages brought by live selling in a competitive environment. Corollary 1 also warns consumers that the price of brands in a live sales room will be even higher under the asymmetric strategy. Instead of following the trend to buy in live-stream sales, rational consumers should give more attention to the products of competitors that have not adopted live sales.

In the above analysis, we studied the competitive firms' equilibrium price and profit under each subgame. Then, we explore the equilibrium strategy decision for duopoly firms by using backward induction. We summarize the equilibrium profits of Firm A and Firm B in each subgame and obtain the profit matrix of the duopoly case in Table 2 below.

Due to the symmetry of the firms, we need to compare only Π_A^{NN} with Π_A^{LN} and Π_A^{NL} with Π_A^{LL} to identify the equilibrium of the game. Define $\Delta_1 = \Pi_A^{LN} - \Pi_A^{NN}$ and $\Delta_2 = \Pi_A^{LL} - \Pi_A^{NL}$, where Δ_1 captures a firm's incentive to lead the adoption of a live-stream strategy if its competitor is not using live streaming; Δ_2 captures a firm's incentive to follow the adoption of a live-stream strategy if its competitor has already adopted live streaming. If neither Firm A nor Firm B can improve its profit via unilateral deviation, then the outcome will be a Nash equilibrium.

First, by solving the inequality $\Delta_1 > 0$, we obtain a threshold of the market expansion parameter $\hat{\alpha}_1 = \frac{t_N^2+t_N-2}{10t_N^2+10t_N-2}$ and a threshold of commission rate $\hat{r}_2 = \frac{2\alpha-10\alpha t_N^2+t_N^2-10\alpha t_N+t_N-2}{2(\alpha-1)(2t_N+1)^2}$. We show that in the duopoly case, only if α is higher than the threshold and r is lower than the threshold does the firm have enough incentive to take the lead to first make a live-stream decision when its competitor does not use live streaming. Then, by solving the inequality $\Delta_2 > 0$, we obtain two other thresholds of a market expansion parameter $\hat{\alpha}_2 = \frac{2t_N^2-t_N-1}{2t_N^2+8t_N+8}$ and a threshold of commission rate $\hat{r}_1 = \frac{8\alpha+2\alpha t_N^2-2t_N^2+8\alpha t_N+t_N+1}{9t_N+9}$. We show that in the duopoly case, only if α is higher than the threshold and r is lower than the threshold does the firm have enough incentive to follow its competitor to make a live-stream decision. Therefore, we derive the best reaction for the firm when it observes its rival's live-streaming adoption strategy.

Proposition 1 *(a) When a rival company adopts a nonlive-streaming strategy (N strategy), the firm will adopt a live-streaming strategy (L strategy) if and only if* $\alpha > \hat{\alpha}_1$ *and* $r < \hat{r}_2$*; otherwise, the firm will also adopt a nonlive-streaming strategy (N strategy).*

(b) When a rival company adopts a live-streaming strategy (L strategy), the firm will also adopt the live-streaming strategy (L strategy) if and only if $\alpha > \hat{\alpha}_2$ *and* $r < \hat{r}_1$*; otherwise, the firm will adopt the nonlive-streaming strategy (N strategy).*

Then, we calculate the general equilibrium of the game when two competitive firms adopt their strategy simultaneously. Notice that the equilibrium outcome of competitive firms depends on the values of Δ_1 and Δ_2. The NN strategy is

Table 2. Equilibrium profits for the competitive firms

	Firm B nonlive (N)	Firm B live (L)
Firm A nonlive (N)	$\Pi_A^{NN} = \frac{1}{2}t_N(1-2\alpha), \Pi_B^{NN} = \frac{1}{2}t_N(1-2\alpha)$	$\Pi_A^{NL} = \frac{(1-\alpha)(2t_N+4)^2}{36(t_N+1)}, \Pi_B^{NL} = \frac{(1-\alpha)(1-r)(4t_N+2)^2}{36(t_N+1)}$
Firm A live (L)	$\Pi_A^{LN} = \frac{(1-\alpha)(1-r)(4t_N+2)^2}{36(t_N+1)}, \Pi_B^{LN} = \frac{(1-\alpha)(2t_N+4)^2}{36(t_N+1)}$	$\Pi_A^{LL} = \frac{t_N(1-r)}{2}, \Pi_B^{LL} = \frac{t_N(1-r)}{2}$

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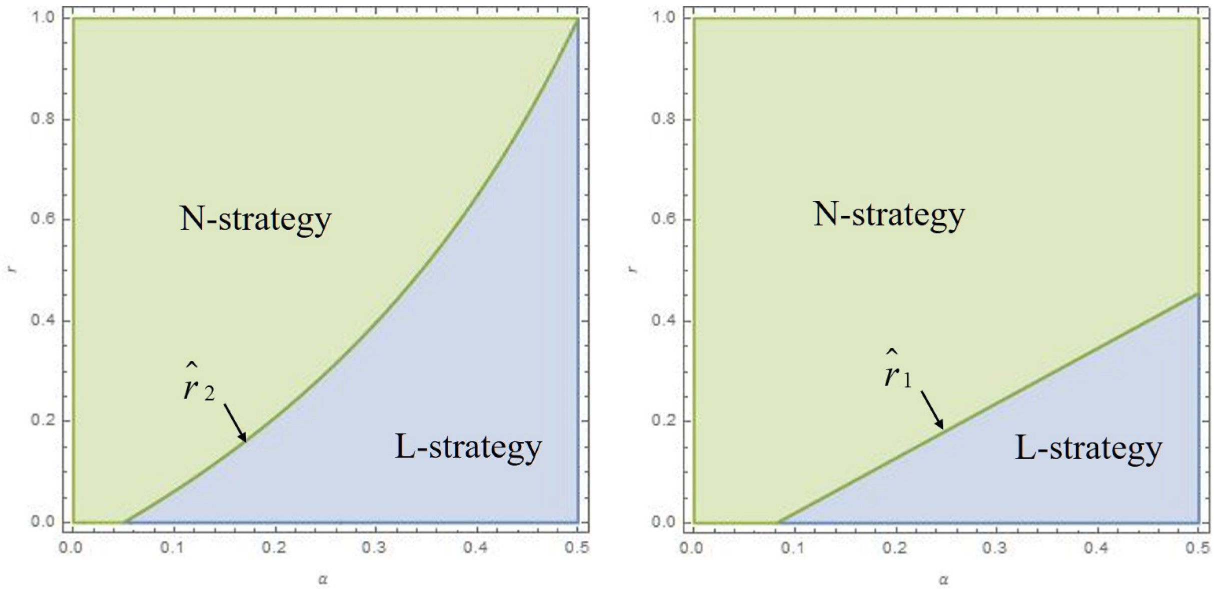

(a) rival adopts N-strategy (b) rival adopts L-strategy

Fig. 2. Firm's response to the rival's strategy

an equilibrium if and only if $\Delta_1 < 0$ and $\Delta_2 < 0$. In contrast, the LL strategy is the equilibrium combination if and only if $\Delta_1 > 0$ and $\Delta_2 > 0$. Otherwise, the equilibrium outcome will be an asymmetric strategy (LN and NL strategy).

By comparing the values above the thresholds, we find that the equilibrium outcome depends on the relation among r, t_N and α. Specifically, when α is rather small, the NN strategy is the single equilibrium outcome. With the increase in α, a symmetric equilibrium outcome (LN and NL strategy) will occur. Eventually, when α becomes sufficiently high, the equilibrium outcome contains all three situations: no firm adopts the live stream (NN strategy); both firms adopt the LL strategy; and only one firm adopts the LN and NL strategies. We summarize our findings and give the equilibrium strategy decision for duopoly firms in the proposition below.

Proposition 2 *The result of competitive firms' equilibrium outcome is as follows:*

(a) No firm adopts a live-stream strategy (NN strategy) in the following cases: (i) $0 < \alpha < \hat{\alpha}_1$; *and (ii)* $\hat{\alpha}_1 < \alpha < \frac{1}{2}$, $\hat{r}_2 \leqslant r < 1$.

(b) Only one firm adopts a live-stream strategy (LN or NL strategy) in the following cases: (i) $\hat{\alpha}_1 < \alpha \leqslant \hat{\alpha}_2$, $0 < r < \hat{r}_2$; *and (ii)* $\hat{\alpha}_2 < \alpha < \dfrac{1}{2}$, $\hat{r}_1 < r \leqslant \hat{r}_2$.

(c) Both firms adopt a live-stream strategy (LL strategy) only when $\hat{\alpha}_2 < \alpha < \frac{1}{2}$, $0 < r \leq \hat{r}_1$.

To build the intuition behind the strategic decision for competition, we first investigate the role of the market expansion effect. Note that parameter α captures the market scale, and an increase in α means that the live stream can attract more consumers. When α is sufficiently small (i.e., $\alpha < \hat{\alpha}_1$), the market scale effect by live stream is not very significant, and the live stream is not attractive to consumers, although they could enjoy a lower price compared to the NN case. The increase in additional sales opportunity cannot offset the profit loss caused by the sales price reduction. Therefore, the firm does not have an incentive to open a live-stream sales show; thus, the NN strategy is a single equilibrium combination. With an increase in α, the market scale gives firms an incentive to open a live-stream show. In addition, the expanded market leads to more intense price competition for the firms, and the price of Firm A or Firm B is lower than p_i^{NN}. When $\hat{\alpha}_1 < \alpha < \hat{\alpha}_2$ and $r < \hat{r}_2$ due to the advantage of the first move, the firm that adopts a live-stream can set a higher price than the other firm (i.e., $p_A^{LN} > p_B^{LN}$). That is, for the decision maker, as long as the streamer that they hired can attract enough consumer traffic, they have sufficient incentive to take the lead in adopting live sales. However, the firm has an incentive to be the first mover to adopt the live stream, but the second mover will not follow. If the second mover does follow the step, then the limited market scale is not enough to compensate for the reduction in revenue caused by price competition. In this circumstance, the equilibrium will be an asymmetric strategy outcome. If α is sufficiently high (i.e.,$\alpha > \hat{\alpha}_2$), then the commission is relatively low. For the second mover, when it notices that its competitor has already adopted live-streaming selling, it will still follow the step, although this action will lead to intensified price competition. When the additional market increase scale is sufficiently large, the increase in demand will offset the potential price downtrend, and the profit increase caused by market expansion will dominate the profit decrease caused by the unit price reduction. In the meantime, consumers could enjoy an extremely low price.

We next focus on the role of the mismatch reduction effect. In the monopoly case, we show that an increase in t_N will induce the monopolist to adopt live-streaming selling (i.e., $\dfrac{\partial \widetilde{r}}{\partial t_N} > 0$). However, in the duopoly case, we find that the impact of the unit mismatch cost is rather complicated. In fact, as shown in Figure 3, the thresholds of the commission rate in competition decrease with increasing t_N (i.e., $\dfrac{\partial \hat{r}_1}{\partial t_N} < 0$, $\dfrac{\partial \hat{r}_2}{\partial t_N} < 0$), and the thresholds of α decrease with increasing t_N

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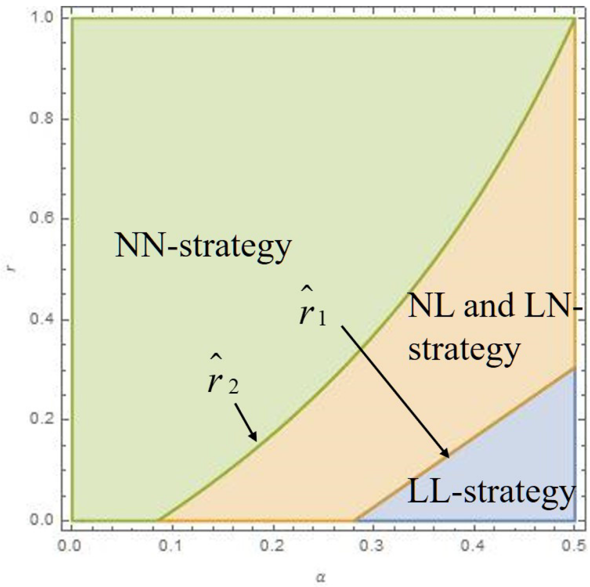

Fig. 3. Equilibrium strategy for competitive firms

(i.e., $\frac{\partial \hat{\alpha}_1}{\partial t_N} > 0$, $\frac{\partial \hat{\alpha}_2}{\partial t_N} > 0$). However, the impact of unit mismatch reduction on live-streaming adoption decisions is still unclear. We answer this question in the last section.

The intuition behind Proposition 2 is an alarm to the decision maker of competitive firms when he or she makes managerial decisions. This is the era of live streaming where it is becoming increasingly prevalent, and an increasing number of firms and manufacturers are embracing the trend of live-streaming sales. However, if all consumers are rational enough, then two competitive firms adopting live streaming are not always wise strategic actions. If all firms adopt a live-stream strategy, then live sales act as combative advertising, which can lead to intensified price competition. Moreover, under certain conditions, if one firm has noticed that its competitor has adopted a live-stream strategy, then at this time, "do not act" is the optimal decision. To avoid fierce price competition, this type of "do not act" is the best response for the decision maker of the second mover. Accordingly, in Proposition 2, we identify how the key factors work together to affect the strategy decision of duopoly firms to adopt live-stream selling in a competitive environment. Our findings provide clear guidelines for decision makers.

4.2 Profit Implications for Competitive Firms of Live-streaming Adoption

In the above subsection, we summarized the equilibrium strategy combination for firms. However, although the framework of the equilibrium strategy selection is shown in Proposition 2 and Figure 3, compared to the case in which no live streaming exists in the market (NN strategy), whether the firm's profit has been improved is still unclear. In this part, we want to answer the question concerning whether both firms always benefit from the adoption of live-stream sales in the LL case. In the asymmetric case, does a firm that adopts live-streaming always benefit, and does the other firm always lose? We hope to explore the impact of live-stream sales adoption on firms' profits.

Proposition 3 *In the duopoly case, compared to the NN strategy, when the equilibrium outcome is an LL strategy, thresholds* $\hat{\alpha}_3 = \frac{t_N - 1}{2t_N}$, $\hat{\alpha}_4 = \frac{7t_N^2 + t_N - 8}{16t_N^2 + 10t_N - 8}$, $\hat{r}_0 = 2\alpha t_N - t_N + 1$ *and* $\hat{t}_N$ *exist:*

(a) (Lose–Lose) Both firms adopt and lose from the adoption of the live-stream strategy when $t_N < \hat{t}_N$ *(i)* $\hat{\alpha}_2 < \alpha < \hat{\alpha}_3$, $0 < r < \hat{r}_1$ *and (ii)* $\hat{\alpha}_3 < \alpha < \hat{\alpha}_4$, $\hat{r}_0 < r < \hat{r}_1$.

(b) (Win–Win) Otherwise, both firms adopt and benefit from the live-stream strategy when (i) $t_N < \hat{t}_N$, $\hat{\alpha}_3 < \alpha < \hat{\alpha}_4$, $0 < r < \hat{r}_0$ *or (ii)* $t_N < \hat{t}_N$, $\alpha > \hat{\alpha}_4$, $0 < r < \hat{r}_1$, *and (iii)* $t_N > \hat{t}_N$, $\alpha > \hat{\alpha}_2$, $0 < r < \hat{r}_1$.

Proposition 3 and Figure 4 indicate that two competitive

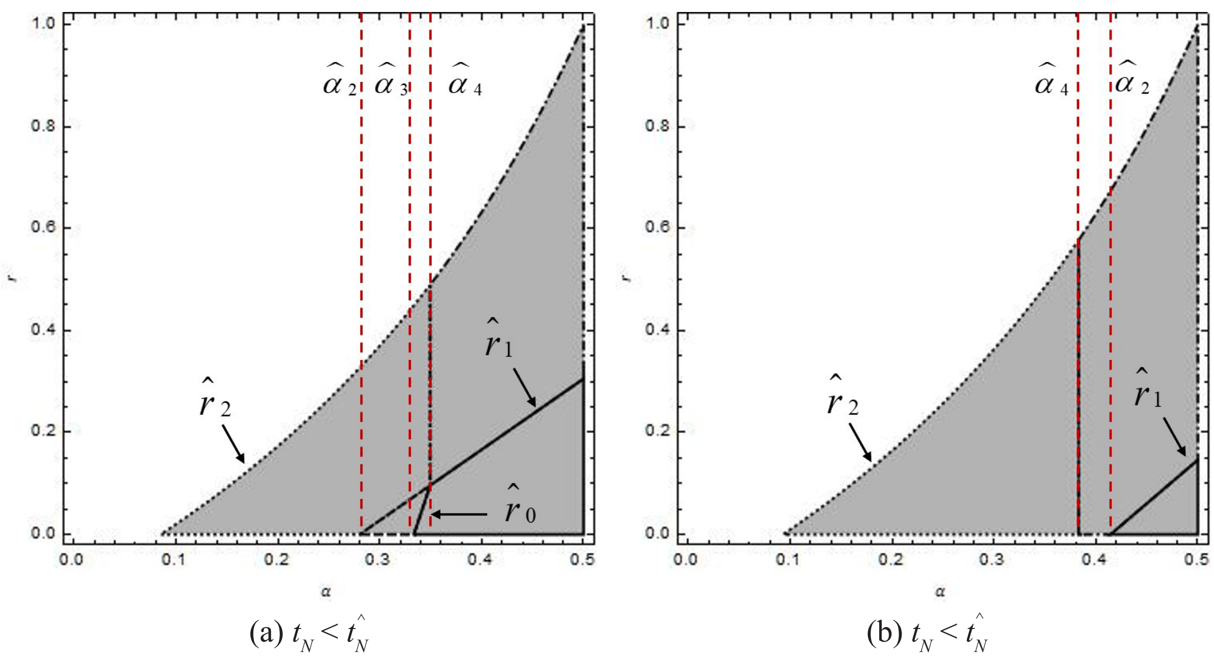

Fig. 4. Profit implication for competitive firms

firms will not always benefit from the adoption of live-stream sales. When the unit mismatch cost is not sufficient and the additional market increase scale is relatively medium, then the two firms can be trapped in a *prisoner's dilemma*. Specifically, when $\alpha \geq \hat{\alpha}_4$, for any value of t_N and $r < \hat{r}_1$, in equilibrium, both firms choose the LL strategy, and profit is better. However, if the unit mismatch is below $\hat{t}_N$, then the thresholds of market expansion parameters $\hat{\alpha}_3$ and $\hat{\alpha}_4$ exist, and both firms that choose the LL strategy may suffer a loss of profit at the same time.

The intuition behind Proposition 3 is the interaction between the effect of market expansion α and the effect of the price downtrend caused by t_N. Recall our conclusion of price comparison in Corollary 1; when both firms adopt the live-stream strategy, the price p_i^{LL} will be lower than the NN strategy p_i^{NN}. If the market expansion effect is strong enough, then the effect of the market plays a more significant role, and both firms can always obtain a profit through a win–win scenario. However, when the additional market increase scale is rather small and the unit mismatch cost is above the threshold, the two firms have to charge a lower price to consumers, and the profit increase caused by market expansion dominates the profit decrease caused by the unit price reduction. These two forces drive both firms to suffer losses with the adoption of live-stream sales, and a prisoner's dilemma occurs. That is, although the LL strategy is a Nash equilibrium, in fact, in this circumstance, the adoption of live-stream sales is detrimental for the two firms. If the live-streaming sales show is limited to attracting consumer traffic, then to some extent, live sales act as combative advertising, and the adoption of live-stream sales can intensify price competition such that an "advertising war" leads to a "price war" and eventually hurts the interest of the firms.

Our findings provide insights for decision makers. When firms join the trend of live-streaming sales, they need to consider both the market expansion effect and the price downtrend effect. Notably, When the consumer traffic attracted by live sales is not sufficiently large, live-stream sales could result in a profit reduction and lead to a lose–lose situation. In fact, it is a Pareto improvement for the two competing firms to simultaneously abandon the use of the live-stream strategy to avoid the intensified "price war" at this time.

We have summarized the profit implications of firms under a symmetric strategy combination. Next, we devote attention to the asymmetric strategy subgame to reveal the impact of live sales, and we compare the profits of the two firms when only one firm adopts live sales to the benchmark case. Our findings are concluded in the following proposition.

Proposition 4 *In the duopoly case, compared to the NN strategy, when the equilibrium outcome is an asymmetric strategy (NL and LN strategy), thresholds* $\hat{\alpha}_4 = \dfrac{7t_N^2 + t_N - 8}{16t_N^2 + 10t_N - 8}$ *and* $\hat{t}_N$ *exist:*

(a) (Win–Lose) The firm that adopts live-streaming benefits and the nonlive-streaming firm loses when (i) $t_N < \hat{t}_N$, $\hat{\alpha}_1 < \alpha < \hat{\alpha}_2$, $0 < r < \hat{r}_2$, *(ii)* $t_N < \hat{t}_N$, $\hat{\alpha}_2 < \alpha < \hat{\alpha}_4$, $\hat{r}_1 < r < \hat{r}_2$, *and (iii)* $t_N > \hat{t}_N$, $0 < \alpha < \hat{\alpha}_4$, $0 < r < \hat{r}_2$.

(b) (Win–Win) Otherwise, the firm that adopts live-streaming and the nonlive-streaming firm both benefit when (i) $t_N < \hat{t}_N$, $\alpha > \hat{\alpha}_4$, $\hat{r}_1 < r < \hat{r}_2$, *(ii)* $t_N > \hat{t}_N$, $\hat{\alpha}_4 < \alpha < \hat{\alpha}_2$, $0 < r < \hat{r}_2$, *and (iii)* $t_N > \hat{t}_N$, $\alpha > \hat{\alpha}_2$, $\hat{r}_1 < r < \hat{r}_2$.

Proposition 4 and Figure 4 indicate that under an asymmetric equilibrium, the firm that adopts live sales can always gain an advantage in the competition and thus obtain a profit increase compared to the NN strategy. Interestingly, if the effect of market expansion is sufficiently large and the commission rate is relatively medium, then the nonadopting firm can also obtain a better profit compared to the NN strategy.

To analyze and determine the above result, we isolate the impact of unit mismatch cost reduction and the market scale. Let us first consider the effect of parameter t_N; when only one firm adopts live sales, without loss of generality, we assume that Firm A adopts and Firm B does not adopt. Because of the advantages brought by live-stream sales, Firm A can obtain a much greater market share and can also charge consumers higher prices (i.e., $p_A^{LN} > p_B^{LN}$ and $D_A^{LN} > D_B^{LN}$). With an increase in t_N, Firm A obtains stronger market power p_A^{LN}, and D_A^{LN} increases monotonically. For Firm B's market demand D_A^{LN} decreases monotonically with t_N because of its weak status. However, with an increase in the unit mismatch cost, Firm B could follow the step of Firm A and charge a higher price to consumers (i.e., $\dfrac{\partial p_B^{LN}}{\partial t_N} > 0$). The increase in profit by the price increase effect is stronger than the decrease in demand, and we have $\dfrac{\partial \Pi_B^{LN}}{\partial t_N} > 0$.

Then, let us consider the effect of the market scale; parameter α captures the fraction of consumers who make purchases only with the existence of live sales. A larger α means a smaller-scale market in the NN case, and the role of consumer traffic attractiveness in live streaming is more prominent. For Firm B, the profit difference between the LN and NN cases monotonically increases with α (i.e., $\dfrac{\partial (p_B^{LN} - p_B^{NN})}{\partial \alpha} > 0$). Therefore, as shown in Proposition 5, a threshold of α exists when $\alpha > \hat{\alpha}_4$ for any t_N, and Firm B will always be better off. According to the aforementioned monotonicity of the two firms' profit functions, when t_N is sufficiently large, Firm A has enough incentive to set an extremely high price, and Firm B could follow and set a higher price (that is slightly lower than Firm A), although the demand will decrease. As a result, the equilibrium outcome will be a win–win situation compared to the NN case. Intuitively, people believe that the nonadopting firm will obtain a loss of profit due to its disadvantaged status in the competition. However, in certain specific parameter ranges, the nonadopting firm can also obtain a profit increase compared to the benchmark case. That is, the firm with the nonlive-stream strategy can free ride on its competitor.

4.3 Comparison and Analysis

In the above section, we derived the equilibrium outcome of the monopoly and duopoly cases. We now turn our attention to an investigation of how competition affects firms' live-

① See https://www.ceibs.edu/new-papers-columns/livestreaming-e-commerce-lasting-buzz-or-will-it-go-bust

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streaming strategies. Before proceeding, we incorporate the monopoly case as a benchmark reference.

In the monopoly case, we assume that there is only one Firm A in the market that sells one product to consumers. Located at $L_A = 0$, Firm A first decides whether to adopt a live-stream selling strategy (L strategy) or a nonlive-stream selling strategy (N strategy) and then decides the price that it charges consumers (p_A^N or p_A^L). If Firm A does not adopt live-stream selling, then consumers who buy from Firm A will obtain utility $U = V - t_N \times |L_A - x| - p_A^N$. If Firm A decides to use live-stream selling, then consumers who buy from Firm A will obtain utility $U = V - t_L \times |L_A - x| - p_A^L$. Consumers will make a purchase if $U > 0$. For the market size expansion effect, we normalize the market size to $1-\alpha$ when the monopolist adopts live-streaming selling; otherwise, the marker size is reduced to $1-2\alpha$. The sequences of the monopoly game are as follows. In stage 1, the firm chooses a live-stream strategy (L strategy) or a nonlive-stream strategy (N strategy). In stage 2, the firm then decides the sale price of the product (p_A^N or p_A^L). In stage 3, consumers make purchase decisions.

We first consider the subgame in which a monopoly firm does not adopt a live-streaming selling strategy (N strategy). Under this circumstance, the utility of the consumer who buys from the firm is $U = V - t_N x - p_N$. Let $U = 0$; we obtain the threshold of the consumers $\widetilde{x}^N = \dfrac{V - p_N}{t_N}$ who will or will not purchase. Then, we obtain the demand function of the monopoly firm $D^N = (1-2\alpha)min[\widetilde{x}^N, 1]$.

Therefore, the monopoly firm's profit function under strategy N is

$$\Pi^N = D^N p^N = (1-2\alpha)min\left[\frac{V-p_N}{t_N}, 1\right]p^N \tag{7}$$

The monopoly firm determines its price p^N to maximize its own profit, and we summarize the equilibrium price and profit in the lemma below.

Lemma 4 *In the monopoly case, when a firm adopts nonlive-streaming selling (N strategy),*

if $V \leqslant 2t_N$, then the firm charges price $p^N = \frac{V}{2}$ and obtains the profit.

If $V > 2t_N$, then the firm charges price $p^N = V - t_N$ and obtains the profit $\Pi^N = (V - t_N)(1-2\alpha)$.

Then, we consider the subgame in which a monopoly firm adopts a live-streaming selling strategy (L strategy). Under this circumstance, the utility of the consumer who buys from the firm is $U = V - t_L x - p_L$. Let $U = 0$; we obtain the threshold of consumers $\widetilde{x}^L = \dfrac{V - p_L}{t_L}$ who are indifferent between purchasing and not purchasing. Then, we obtain the demand function of the monopoly firm $D^L = (1-\alpha)min[\widetilde{x}^L, 1]$. The market size turn to $1-\alpha$ denotes the additional consumer traffic brought by live-stream selling.

Therefore, the monopoly firm's profit function under strategy L is

$$\Pi^L = D^L p^L(1-r) = (1-\alpha)(1-r)min\left[\frac{V-p_L}{t_L}, 1\right]p^L \tag{8}$$

The monopoly firm determines its price p^L to maximize its own profit, and we summarize the equilibrium price and the profit in the lemma below.

Lemma 5 *In the monopoly case, when a firm adopts live-streaming selling (L strategy),*

if $V \leqslant 2t_L$, then the firm charges price $p^L = \frac{V}{2}$ and obtains profit $\Pi^L = \dfrac{V^2}{4t_L}(1-\alpha)(1-r)$.

If $V > 2t_L$, then the firm charges price $p^L = V - t_L$ and obtains profit $\Pi^L = (V - t_L)(1-\alpha)(1-r)$.

Then, we investigate the equilibrium strategy decision for a monopoly firm. To avoid trivial cases and focus our main results, we assume $V \geqslant 2t_N$ to ensure that the product of the monopolist will cover the full market. To conclude, in the equilibrium strategy decision of a monopoly firm, we define $\Delta = \Pi^L - \Pi^N$. Δ captures the monopolist's incentive to choose a live-streaming selling strategy. By solving the inequality $\Delta \geqslant 0$, we obtain the threshold of commission rate r, and we summarize our finding in the proposition below.

Proposition 5 *A monopoly firm adopts live-streaming selling (L strategy) if and only if the commission rate is lower than the threshold $\widetilde{r} = \dfrac{\alpha(V - 2t_N + 1) + t_N - 1}{(1-\alpha)(V-1)}$. Specifically,*

when $r \leqslant \widetilde{r}$, the monopoly firm adopts live-streaming selling, and the firm sets an equilibrium price $p^* = V - t_L$ and obtains profit $\Pi^* = (V - t_L)(1-\alpha)(1-r)$.

When $r > \widetilde{r}$, the monopoly firm does not adopt live-streaming selling, and the firm sets an equilibrium price $p^* = V - t_N$ and obtains profit $\Pi^* = (V - t_N)(1-2\alpha)$.

Proposition 5 indicates that a monopoly firm will adopt live-stream selling when the commission rate paid to the streamer is not sufficiently high. As shown in Figure 5, we notice that $\dfrac{\partial \widetilde{r}}{\partial \alpha} > 0$ and that $\dfrac{\partial \widetilde{r}}{\partial t_N} > 0$, which implies that with the increase in the market expansion effect and the unit mismatch reduction effect of live-stream selling, monopoly firms have more incentive to adopt the L strategy.

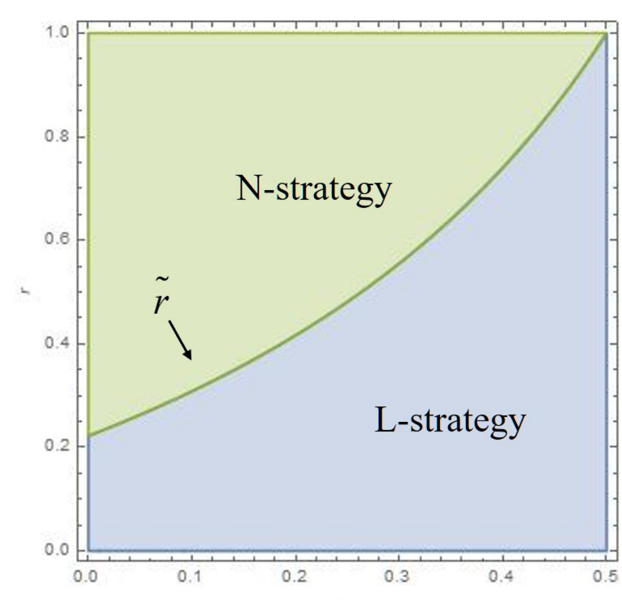

Fig. 5. Equilibrium strategy for a monopoly firm

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Note that in the monopoly case, the firm will adopt live streaming as long as the commission rate is lower than the threshold $\widetilde{r}$. However, in the duopoly case, live streaming can exist in the market only when the commission rate is lower than threshold $\hat{r}_2$ and the market size increment level is higher than $\hat{\alpha}_1$. Moreover, we show that the two thresholds of commission rate $\widetilde{r}$ are larger than $\hat{r}_2$, which implies that in the monopoly case, the firm has more incentive to adopt live-stream selling compared to the duopoly case, and we conclude our findings with the following proposition.

Proposition 6 *Competition reduces a firm's incentive to adopt live streaming compared to the monopoly case. Specifically, compared to monopoly live streaming, under a duopoly of competition,*

(a) No firm adopts a live-stream strategy (NN strategy) when (i) $0<\alpha<\hat{\alpha}_1$, $0\leqslant r<\widetilde{r}$ *or (ii)* $\hat{\alpha}_1<\alpha<\frac{1}{2}$, $\hat{r}_2\leqslant r<\widetilde{r}$.

(b) Only one firm adopts a live-stream strategy (LN or NL strategy) in the following cases: (i) $\hat{\alpha}_1<\alpha\leqslant\hat{\alpha}_2$, $0<r<\hat{r}_2$; *and (ii)* $\hat{\alpha}_2<\alpha<\frac{1}{2}$, $\hat{r}_1<r\leqslant\hat{r}_2$.

(c) Both firms adopt a live-stream strategy (LL strategy) only when $\hat{\alpha}_2<\alpha<\frac{1}{2}$, $0<r\leqslant\hat{r}_1$.

The result of Proposition 6 shows that competition reduces a firm's incentive to adopt a live-streaming strategy. We show that interestingly, it is possible for a firm to choose to live stream in a monopoly case but not in a competitive environment (for example, Region-2 in Figure 6). The intuition is that compared to the benchmark monopoly case, competitive firms' live-streaming decisions are more sensitive to the market expansion effect and commission rate. Although live streaming could enlarge the market share, it can also lead to a lower price in a competitive environment. Only when live streaming can attract enough consumer traffic and the commission rate is not too high will there be live-streaming selling in a competitive market.

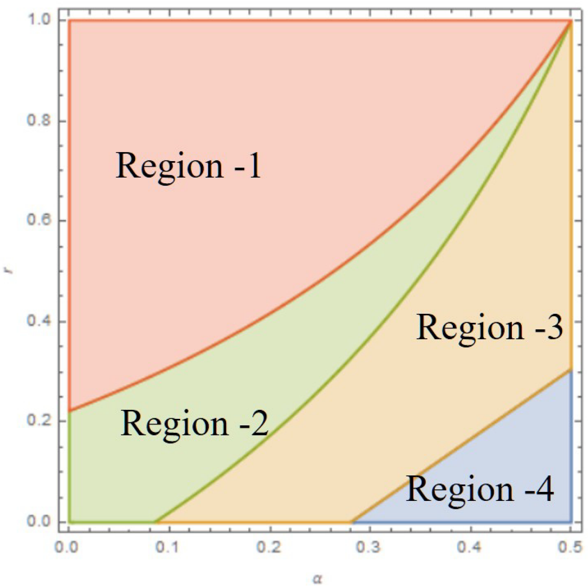

Fig. 6. Equilibrium comparison between the monopoly and duopoly cases

5 Conclusion and managerial implications

In this paper, we develop an analytic model to study live-streaming selling strategies for competitive firms. We capture the key characteristics of live-streaming selling, the effect of market scale expansion and the effect of unit mismatch cost reduction. Our main results are as follows. First, our paper clearly shows when firms should embrace live-streaming selling under a duopoly situation. That is, for competitive firms, the adoption of a live-streaming strategy is much more complicated and depends on the market expansion scale and the relative mismatch reduction level brought by live sales. Specifically, no firm adopts live sales when the additional market share brought by the streamer is limited and the commission rate is relatively high; both firms adopt a live-sale strategy only when the market expansion effect is strong enough and the commission rate is relatively low.

Second, our results also shed light on the profit implications of adopting a live-streaming strategy for firms. Compare the profit of competitive firms under different strategy combinations with the case in which live-stream sales do not exist or are not available in the market. We demonstrate that firms will not always benefit from the adoption of a live-stream sales strategy. When the unit mismatch cost is below a certain threshold and the additional market increase scale is medium, then the two firms can be trapped in a prisoner's dilemma. That is, when one firm has enough incentive to take the lead in adopting live sales, its opponent will follow, although this will result in a worse profit. As Professor Chen from CEIBS points out, high customer acquisition costs, including high commissions for live-streaming platforms, can squeeze brands' profits from live-streaming deals, and brands must not bet everything on live-streaming e-commerce[①]. Our finding alerts the advocates of live-streaming selling, although it is helpful in acquiring new customers and growing sales volume in a short time. Higher marginal costs, including a high commission rate for live-streaming streamers and platforms, can lead to a price war and eventually damage firms' profits.

Finally, by comparing the equilibrium outcome between the monopoly and duopoly cases, we show that competition reduces a firm's incentive to adopt live-streaming selling. That is, monopolists always have more incentives to adopt live streaming than duopolists have. Furthermore, we notice that a higher unit mismatch reduction level caused by live-streaming selling makes it beneficial for the monopolist to adopt a live strategy. However, under competition, with the increase in the unit mismatch reduction level, duopoly firms' decision makers would discourage the implementation of a live strategy at the same time to avoid fiercer price competition.

This research can be extended in several directions. First, in this paper, we assume that the products of the two firms are homogeneous. An interesting direction for future research is to study the case of two firms with product quality differentiation and consumers with heterogeneous preferences for quality. Second, in this paper, we weaken the role of the supply chain. In future research, one could provide a more systemat-

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ic study by taking upstream or downstream decision makers into consideration. Third, it is feasible to model a multiperiod live-streaming decision process under monopoly and duopoly situations, where consumers are repeat buyers, and firms can adopt live-streaming selling in several periods. Over such a long term, live sales may play a more sophisticated and complicated role in a multiperiod repeated game between monopolist and competitive firms.

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Conflict of interest

The authors declare no conflict of interest.

Biographies

Quan Du **Quan Du** is currently a graduate student under the tutelage of Prof. Jie Wu at University of Science and Technology of China. His research interests focus on operations management and marketing science.

Zhixin Chen **Zhixin Chen** is currently a postdoctoral research fellow at School of Management, University of Science and Technology of China (USTC). He received the PhD degree in management science and engineering from USTC in 2021. His research interests focus on operations management and marketing science.

Jie Wu **Jie Wu** is currently a professor at School of Management, University of Science and Technology of China (USTC). He received the PhD degree in management science and engineering from USTC in 2008. His research interests focus on operations research and decision science.

Xiang Ji **Xiang Ji** is currently an associate professor at School of Management, University of Science and Technology of China (USTC). He received the PhD degree in management science and engineering from USTC in 2017. His research interests focus on operations management and marketing science.

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Appendix

Proof of Lemma 1:

In the duopoly case, when no firm adopts a live-streaming strategy, the utility of consumer buying from firm i is $U_A^1 = V - t_N x - p_A^{NN}$ and $U_B^1 = V - t_N(1-x) - p_B^{NN}$. Let $U_A^1 = U_B^1$, and we obtain the consumer $\widetilde{x}_1^{NN} = \dfrac{1}{2} + \dfrac{-p_A^{NN} + p_B^{NN}}{2t_N}$ who is indifferent to buying from Firm A or B. Therefore, the profit functions of the two firms under this subgame are $\Pi_A^{NN} = (1-2\alpha)(\frac{1}{2}+\frac{-p_A^{NN}+p_B^{NN}}{2t_N})p_A^{NN}$ and $\Pi_B^{NN} = D_B^{NN}p_B^{NN} = (1-2\alpha)(\frac{1}{2}+\frac{p_A^{NN}-p_B^{NN}}{2t_N})p_B^{NN}$, and Firm A decides p_A^{NN}, and Firm B decides p_B^{NN} to maximize their profits. With the first-order conditions (FOCs) $\frac{\partial \Pi_A^{NN}}{\partial p_A^{NN}} = 0$ and $\frac{\partial \Pi_B^{NN}}{\partial p_B^{NN}} = 0$, we obtain the equilibrium price of the two firms $p_A^{NN} = p_b^{NN} = t_N$. Moreover, we have $\frac{\partial^2 \Pi_A^{NN}}{\partial (p_A^{NN})^2} < 0$ and $\frac{\partial^2 \Pi_B^{NN}}{\partial (p_B^{NN})^2} < 0$, and we also ensure that all second-order conditions (SOCs) are satisfied. Therefore, under the duopoly NN subgame, each firm charges price $p_A^{NN} = p_B^{NN} = t_N$ and obtains profit $\Pi_A^{NN} = \Pi_B^{NN} = \frac{t_N}{2}(1-2\alpha)$.

Proof of Lemma 2:

In the duopoly case, when both firms adopt a live-streaming strategy, the utility of consumer buying from the firms is $U_A^2 = V - t_L x - p_A^{LL}$ and $U_B^2 = V - t_L(1-x) - p_B^{LL}$. Let $U_A^2 = U_B^2 = 0$, and we obtain the consumer $\widetilde{x_2}^{LL} = \frac{1}{2}+\frac{-p_A^{LL}+p_B^{LL}}{2}$ who is indifferent to buying from Firm A or B. Therefore, the profit functions of the two firms under this subgame are $\Pi_A^{LL} = D_A^{LL} * p_A^{LL} * (1-r) = (\frac{1}{2}+\frac{-p_A^{LL}+p_B^{LL}}{2}) * p_A^{LL} * (1-r)$ and $\Pi_B^{LL} = D_B^{LL} * p_B^{NN} * (1-r) = (\frac{1}{2}+\frac{p_A^{LL}-p_B^{LL}}{2}) * p_B^{LL} * (1-r)$, and Firm A decides p_A^{LL}, and Firm B decides p_B^{LL} to maximize their profits. With the FOCs $\frac{\partial \Pi_A^{LL}}{\partial p_A^{LL}} = 0$ and $\frac{\partial \Pi_B^{LL}}{\partial p_B^{LL}} = 0$, we obtain the equilibrium price of the two firms $p_A^{LL} = p_B^{LL} = t_L$. Moreover, we have $\frac{\partial^2 \Pi_A^{LL}}{\partial (p_A^{LL})^2} < 0$ and $\frac{\partial^2 \Pi_B^{LL}}{\partial (p_B^{LL})^2} < 0$, and we also ensure that all SOCs are satisfied. Recall that we assume that $t_L = 1$; therefore, under the duopoly LL subgame, each firm charges price $p_A^{LL} = p_B^{LL} = 1$ and obtains profit $\Pi_A^{LL} = \Pi_B^{LL} = \frac{1}{2}(1-r)$.

Proof of Lemma 3:

In the duopoly case, when Firm A adopts a live-streaming strategy while Firm B does not, the utility of the consumer buying from the firms is $U_A^3 = V - t_L x - p_A^{LN}$ and $U_B^3 = V - t_N(1-x) - p_B^{LN}$, and we obtain the consumer $\widetilde{x_3}^{LN} = \frac{-p_A^{LN}+p_B^{LN}+t_N}{1+t_N}$ who is indifferent to buying from Firm A or B. Therefore, the profit functions of the two firms under this subgame are $\Pi_A^{LN} = D_A^{LN} * p_A^{LN} * (1-r) = (1-\alpha)(\frac{-p_A^{LN}+p_B^{LN}+t_N}{1+t_N}) * p_A^{LN} * (1-r)$ and $\Pi_B^{LN} = D_B^{LN} * p_B^{LN} = (1-\alpha)(1-\frac{-p_A^{LN}+p_B^{LN}+t_N}{1+t_N}) * p_B^{LN}$, and Firm A decides p_A^{LN}, and Firm B decides p_B^{LN} to maximize their profits. With the FOCs $\frac{\partial \Pi_A^{LN}}{\partial p_A^{LN}} = 0$ and $\frac{\partial \Pi_B^{LN}}{\partial p_B^{LN}} = 0$, we obtain the equilibrium price of the two firms $p_A^{LN} = \frac{1+2t_N}{3}$ and $p_B^{LN} = \frac{2+t_N}{3}$. Moreover, $\frac{\partial^2 \Pi_A^{LN}}{\partial (p_A^{LN})^2} < 0$ and $\frac{\partial^2 \Pi_B^{LN}}{\partial (p_B^{LN})^2} < 0$ when the SOCs are satisfied.

Proof of Proposition 1:

Define $\Delta_1 = \Pi_A^{NN} - \Pi_A^{LN}$, where Δ_1 captures a firm's incentive to lead the adoption of a live-streaming strategy when its rival does not adopt it. From the expression of Π_A^{NN} with Π_A^{LN}, we obtain $\Delta_1 = \frac{(-2\alpha+2(\alpha-1)r(2t_N+1)^2+(10\alpha-1)t_N^2+(10\alpha-1)t_N+2)}{18(1+t_N)}$. Note that $\frac{\partial(-2\alpha+2(\alpha-1)r(2t_N+1)^2+(10\alpha-1)t_N^2+(10\alpha-1)t_N+2)}{\partial r} = 2(\alpha-1)(2t_N+1)^2 < 0$, which implies that Δ_1 linearly decreases with r. We prove that when r approaches 1, $\lim_{r\to 1}\Delta_1 = 9(2\alpha-1)t_N(t_N+1) < 0$ always holds. When r approaches 0, $\lim_{r\to 0}\Delta_1 = -2\alpha+(10\alpha-1)t_N^2+(10\alpha-1)t_N+2$, this expression is positive if and only if α is larger than $\frac{t_N^2+t_N-2}{2(5t_N^2+5t_N-1)}$. If the condition of α is satisfied, then solve the equation of $\Delta_1 = 0$, and we then obtain the root of r. Therefore, there exists a threshold of $\hat{\alpha}_1 = \frac{t_N^2+t_N-2}{2(5t_N^2+5t_N-1)}$ and $\hat{r}_2 = \frac{2\alpha-10\alpha t_N^2+t_N^2-10\alpha t_N+t_N-2}{2(\alpha-1)(2t_N+1)^2}$; when a rival company does not adopt a live-streaming strategy, the firm will choose to adopt an L strategy if and only if $\alpha > \hat{\alpha}_1$ and $r < \hat{r}_2$; otherwise, the firm will also adopt an N strategy. This proves Proposition 2(a).

Define $\Delta_2 = \Pi_A^{LL} - \Pi_A^{NL}$, Δ_2 captures a firm's incentive to follow the adoption of a live-streaming strategy when noticing that its rival has already adopted it. From the expression of Π_A^{LL} with Π_A^{NL}, we obtain $\Delta_2 = \frac{8\alpha-9rt_N-9r+2\alpha t_N^2-2t_N^2+8\alpha t_N+t_N+1}{18(t_N+1)}$. Note that $\frac{\partial(8\alpha-9rt_N-9r+2\alpha t_N^2-2t_N^2+8\alpha t_N+t_N+1)}{\partial r} = -9(1+t_N) < 0$, which implies that Δ_2 linearly decreases with r. We prove that when r approaches 1, $\lim_{r\to 1}\Delta_2 = 2(\alpha-1)(t_N+2)^2 < 0$ always holds. When r approaches 0, $\lim_{r\to 0}\Delta_2 = 8\alpha+2(\alpha-1)t_N^2+8\alpha t_N+t_N+1$, this expression is positive if and only if α is larger than $\frac{2t_N^2-t_N-1}{2(t_N+2)^2}$. If the condition of α is satisfied, then solve the equation of $\Delta_1 = 0$, and we then obtain the root of r. Therefore, thresholds of $\hat{\alpha}_2 = \frac{2t_N^2-t_N-1}{2(t_N+2)^2}$ and $\hat{r}_1 = \frac{8\alpha+2(\alpha-1)t_N^2+8\alpha t_N+t_N+1}{9(t_N+1)}$ exist when the rival company has already adopted a live-streaming strategy, and the firm would choose to follow the L strategy if and only if $\alpha > \hat{\alpha}_2$ and $r < \hat{r}_1$; otherwise, the firm will adopt the N strategy. This proves Proposition 2(b).

Proof of Proposition 2:

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According to the analysis of Proposition 3, we obtain the thresholds of conditions $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{r}_1$ and $\hat{r}_2$, where for any $t_N > 1$, $\hat{\alpha}_2 - \hat{\alpha}_1 = (9(-1+t_N)^2(1+t_N)^2)/(2(2+t_N)^2(-1+5t_N+5t_N^2)) > 0$ is obvious, and we then check the value of $\hat{r}_1$ and $\hat{r}_2$. Because $\frac{\partial^2(\hat{r}_2-\hat{r}_1)}{\partial\alpha^2} < 0$, $\hat{r}_2-\hat{r}_1$ is a convex function of α. When α approaches zero, we have $\frac{\partial(\hat{r}_2-\hat{r}_1)}{\partial\alpha} < 0$, and when α approaches $\frac{1}{2}$, we have $\frac{\partial(\hat{r}_2-\hat{r}_1)}{\partial\alpha} > 0$. Therefore, a value of α exists where $\hat{r}_2-\hat{r}_1$ takes the global optimal minimum value. Take α such that $\frac{\partial(\hat{r}_2-\hat{r}_1)}{\partial\alpha} = 0$ into function, and algebraically show $\min\limits_{1<\alpha<1/2}(\hat{r}_2-\hat{r}_1) > 0$. Thus far, we prove that for any $t_L < t_N \leqslant \frac{V}{2}$, we have $\hat{\alpha}_2 > \hat{\alpha}_1$ and $\hat{r}_2 > \hat{r}_1$; then, we conclude the equilibrium outcome of the game.

When $\Delta_1 \geqslant 0$ and $\Delta_2 \geqslant 0$, the firm has enough to take the lead of live-streaming selling, and its rival would follow to adopt. The result of the game is that both firms adopt a live-streaming strategy. That is, when $\hat{\alpha}_2 < \alpha < \frac{1}{2}$ and $0 < r \leqslant \hat{r}_1$, the equilibrium outcome will be the LL strategy, and both firms set price $p_A^{LL} = p_B^{LL} = 1$ and obtain profit $\Pi_A^{LL} = \Pi_B^{LL} = \frac{1}{2}(1-r)$. In contrast, when $\Delta_1 < 0$ and $\Delta_2 < 0$, a firm does not have enough to take the lead of live-streaming selling, and its rival would not follow to adopt; the result of the game is that neither firm will adopt a live-streaming strategy. That is, when $0 < \alpha < \hat{\alpha}_1$ and when $\hat{\alpha}_1 < \alpha < \frac{1}{2}$, $\hat{r}_2 \leqslant r < 1$, the equilibrium outcome will be the NN strategy, and both firms charge price t_N and obtain profit $\frac{t_N}{2}(1-2\alpha)$. Otherwise, when $\Delta_1 \geqslant 0$ and $\Delta_2 < 0$ a firm would have enough to take the lead of live-streaming selling while its rival would not follow, and the result of the game is that only one firm adopts a live-streaming strategy and the other firm does not. That is, when $\hat{\alpha}_1 < \alpha \leqslant \hat{\alpha}_2$, $0 < r < \hat{r}_2$ and when $\hat{\alpha}_2 < \alpha < \frac{1}{2}$, $\hat{r}_1 < r \leqslant \hat{r}_2$, the equilibrium outcome will be an asymmetric outcome (LN or NL strategy). The firm that adopts live-streaming sets price $\frac{1+2t_N}{3}$ and obtains profit $\frac{(1+2t_N)^2}{9(1+t_N)}(1-\alpha)(1-r)$, and the other firm sets price $\frac{2+t_N}{3}$ and obtains profit $\frac{(2+t_N)^2}{9(1+t_N)}(1-\alpha)$.

Proof of Proposition 3:

Considering the equilibrium case when both firms choose live streaming, both firms will suffer a revenue loss when $\Pi_A^{LL} \leqslant \Pi_A^{NN}$, and this inequality is true only if $r > \hat{r}_0 = 2\alpha t_N - t_N + 1$. $\hat{r}_0$ is positive only if $\alpha > \hat{\alpha}_3 = \frac{t_N-1}{2t_N}$; recall the condition of the LL strategy combination of equilibrium in Proposition 3 (i.e., $\hat{\alpha}_2 < \alpha < \frac{1}{2}$ and $0 < r \leqslant \hat{r}_1$). When $\alpha \leqslant \hat{\alpha}_3$, the difference in $\hat{\alpha}_3 - \hat{\alpha}_2 = \frac{-(t_N-4)(t_N-1)(t_N+1)}{2t_N(t_N+2)^2}$ is positive only when $t_N < \hat{t_N} = 4$, where the upper bound of t_N equals $\frac{V}{2} = 5$, and the lower bound equals $t_L = 1$. Therefore, when $t_N < \hat{t_N}$ for any $\hat{\alpha}_2 < \alpha < \hat{\alpha}_3$ and $0 < r < \hat{r}_1$, both firms adopt and suffer losses from the adoption of live streaming. When $\alpha > \hat{\alpha}_3$, by inequality $\hat{r}_0 < \hat{r}_1$, we obtain the threshold of $\hat{\alpha}_4 = \frac{7t_N t^2 + t_N - 8}{2(8t_N^2 + 5t_N - 4)}$, and we have $\hat{\alpha}_4 - \hat{\alpha}_2 = (-9(t_N-4)(t_N-1)(t_N+1)^2)/(2(t_N+2)^2(8t_N^2+5t_N-4))$. The condition of $\hat{\alpha}_4 > \hat{\alpha}_2$ is also satisfied when $t_N < \hat{t_N}$. Therefore, when $t_N < \hat{t_N}$ for any $\hat{\alpha}_3 < \alpha < \hat{\alpha}_4$ and $\hat{r}_0 < r < \hat{r}_1$, both firms adopt and suffer losses from the adoption of live streaming. This proves part (a) of Proposition 4.

From the above analysis, we know that $r > \hat{r}_0$ and $t_N < \hat{t_N}$ are the necessary conditions for a prisoner dilemma to occur. Therefore, by contrast, when $r < \hat{r}_0$ or $t_N < \hat{t_N}$, the equilibrium will be a win–win result. Specifically, if $t_N < \hat{t_N}$, then both firms adopt and benefit from the live-stream strategy when $\hat{\alpha}_3 < \alpha < \hat{\alpha}_4$, $0 < r < \hat{r}_0$ and when $\alpha > \hat{\alpha}_4$, $0 < r < \hat{r}_1$. If $t_N > \hat{t_N}$, then for all parameters that are in the range of the LL equilibrium outcome, i.e.,$\alpha > \hat{\alpha}_2$, $0 < r < \hat{r}_1$, both firms adopt and benefit from the live-stream strategy. This proves part (b) of Proposition 4.

Proof of Proposition 4:

We consider the case when only one firm adopts live streaming in equilibrium, and without loss of generality, we assume that Firm A adopts while Firm B does not; then, we only need to focus on the LN strategy combination. Obviously, $\Delta_1 > 0$ is a necessary condition of asymmetric equilibrium, which implies that Firm A will always obtain better revenue compared to the NN subgame. We give our attention to the profit of Firm B, and we have $\Pi_B^{LN} - \Pi_B^{NN} = (-8\alpha + 16\alpha t_N^2 - 7t_N^2 + 10\alpha t_N - t_N + 8)/18(1+t_N)$. This expression is positive when $\alpha > \hat{\alpha}_4 = \frac{7t_N t^2 + t_N - 8}{2(8t_N^2 + 5t_N - 4)}$. In recalling the condition of Proposition 3(c), we need to compare the value between $\hat{\alpha}_2$ and $\hat{\alpha}_4$. According to the previous analysis, $\hat{\alpha}_4 \geqslant \hat{\alpha}_2$ is satisfied when $t_N \leqslant \hat{t_N}$, and $\hat{\alpha}_4 < \hat{\alpha}_2$ is satisfied when $t_N > \hat{t_N}$. Therefore, for the asymmetric equilibrium case when $t_N \leqslant \hat{t_N}$, Firm B will lose if $\hat{\alpha}_1 < \alpha < \hat{\alpha}_2$, $0 < r < \hat{r}_2$; $\hat{\alpha}_2 < \alpha < \hat{\alpha}_4$, $\hat{r}_1 < r < \hat{r}_2$, and Firm B will gain if $\alpha > \hat{\alpha}_4$, $\hat{r}_1 < r < \hat{r}_2$. When $t_N > \hat{t_N}$, Firm B will lose if $0 < \alpha < \hat{\alpha}_4$, $0 < r < \hat{r}_2$, and Firm B will gain if $\hat{\alpha}_4 < \alpha < \hat{\alpha}_2$, $0 < r < \hat{r}_2$; $\alpha > \hat{\alpha}_2$, $\hat{r}_1 < r < \hat{r}_2$. Proposition 5 is proved.

Proof of Lemma 4:

When the monopoly firm does not adopt a live-streaming strategy, the unit mismatch cost is t_N, and the market size is $1-2\alpha$. The utility function of the consumer is $U = V - t_N x - p_N$. In solving the equation of $U = 0$, we obtain the indifferent point of whether the consumer buys the product of firm $\widetilde{x}^N = \frac{V - p_N}{t_N}$. Because consumers are uniformly distributed between $[0,1]$, we obtain the profit function of the firm under this case $\Pi^N = D^N p^N = (1-2\alpha) min[\frac{V-p_N}{t_N}, 1]p^N$. If $\widetilde{x}^N < 1$, then in solving equality $\frac{\partial \Pi^N}{\partial p^N} = 0$, we obtain the optimal price of firm $p^N = \frac{V}{2}$ and profit $\Pi^N = \frac{V^2}{4t_N}(1-2\alpha)$, and we need $V \leqslant 2t_N$. Otherwise, if $\widetilde{x}^N >= 1$ (i.e.,

$V > 2t_N$), then the profit function is $\Pi^N = D^N p^N = (1-2\alpha)p^N$. Let $U = V - t_N - p^N = 0$; the firm will set price $p^N = V - t_N$ and obtain profit $\Pi^N = (V - t_N)(1-2\alpha)$ to extract all consumer surplus.

Proof of Lemma 5:

When the monopoly firm adopts a live-streaming strategy, the unit mismatch cost is t_N, and the market size is $1-\alpha$. The utility function of the consumer is $U = V - t_L x - p_L$. In solving the equation of $U = 0$, we obtain the indifferent point of whether the consumer buys the product of firm $\widetilde{x}^L = \dfrac{V - p_L}{t_L}$. The profit function of the firm under this case $\Pi^L = D^L p^L(1-r) = (1-\alpha)min[\dfrac{V-p_N}{t_N}, 1]p^N(1-r)$, if $\widetilde{x}^L < 1$, solves equality $\dfrac{\partial \Pi^L}{\partial p^L} = 0$, we obtain the optimal price of firm $p^L = \dfrac{V}{2}$ and profit $\Pi^L = \dfrac{V^2}{4t_L}(1-\alpha)(1-r)$, and we need $V \leqslant 2t_L$. Otherwise, if $\widetilde{x}^L >= 1$ (i.e., $V > 2t_L$), then the profit function is $\Pi^L = D^L p^L = (1-\alpha)p^L(1-r)$. Let $U = V - t_L - p^L = 0$, and the firm will set price $p^L = V - t_L$ and profit $\Pi^L = (V - t_L)(1-\alpha)(1-r)$ to extract all consumer surplus.

Proof of Proposition 5:

In the main text, we assume that $V \geq 2t_N$ to ensure full coverage of the market, and the monopolist compares $(V - t_N)(1-2\alpha)$ between $\Pi^L = (V - t_L)(1-\alpha)(1-r)$ to decide whether to adopt a live-streaming strategy. To capture the firm's incentive to adopt the L strategy, define $\Delta = (V - t_L)(1-\alpha)(1-r) - (V - t_N)(1-2\alpha)$. Obviously, Δ monotonically decreases with increasing commission rate r (i.e., $\frac{\partial \Delta}{\partial r} < 0$). In addition, when r approaches 0, we have $\lim\limits_{r\to 0} \Delta = (V - t_L)(1-\alpha) - (V - t_N)(1-2\alpha) > 0$, and when r approaches 1, we have $\lim\limits_{r\to 1} \Delta = -(V - t_N)(1-2\alpha) < 0$. Therefore, a threshold of the commission rate must exist. In solving the equation $\Delta = 0$, we obtain $\widetilde{r} = \dfrac{\alpha(V - 2t_N + 1) + t_N - 1}{(1-\alpha)(V-1)}$. When $r \leqslant \widetilde{r}$, the monopoly firm adopts live-streaming selling, and the firm sets equilibrium price $p^* = V - t_L$ and obtains profit $\Pi^* = (V - t_L)(1-\alpha)(1-r)$. Otherwise, when $r > \widetilde{r}$, the monopoly firm would not adopt live-streaming selling, and the firm sets an equilibrium price $p^* = V - t_N$ and obtains profit $\Pi^* = (V - t_N)(1-2\alpha)$.

Proof of Proposition 6:

For the threshold of the commission rate in the monopoly and duopoly cases, for any $0 < \alpha < \dfrac{1}{2}$ and $t_N \leqslant \dfrac{V}{2}$, the difference between $\widetilde{r} - \hat{r}_2 = \dfrac{(2\alpha-1)(t_N-1)(8t_N^2 + t_N(V+7) + 2v)}{2(\alpha-1)(2t_N+1)^2(v-1)}$ is positive and always holds. That is, in the monopoly case, a firm has more incentive to adopt live-streaming selling. Specifically, as shown in Figure 6, different thresholds of parameters divide the space into four regions. Region-1: When $r \geqslant \widetilde{r}$, whether under the competition or monopoly case, the firm does not adopt a live-streaming strategy. Region-2: When $0 < \alpha < \hat{\alpha}_1$, $0 \leqslant r < \widetilde{r}$ and $\hat{\alpha}_1 < \alpha < \dfrac{1}{2}$, $\hat{r}_2 \leqslant r < \widetilde{r}$, the firm adopts the live-streaming strategy under the monopoly case but not in the duopoly case. Region-3: When $\hat{\alpha}_1 < \alpha \leqslant \hat{\alpha}_2$, $0 < r < \hat{r}_2$ and $\hat{\alpha}_2 < \alpha < \dfrac{1}{2}$, $\hat{r}_1 < r \leqslant \hat{r}_2$, the firm adopts the live-streaming strategy in the monopoly case, and only one firm adopts it in the duopoly case. Region-4: When $\hat{\alpha}_2 < \alpha < \dfrac{1}{2}$, $0 < r \leqslant \hat{r}_1$, the firm adopts the live-streaming strategy in the monopoly case, and both firms adopt it in the duopoly case.

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