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Entry or not? Manufacturers' product sharing strategy when facing competition

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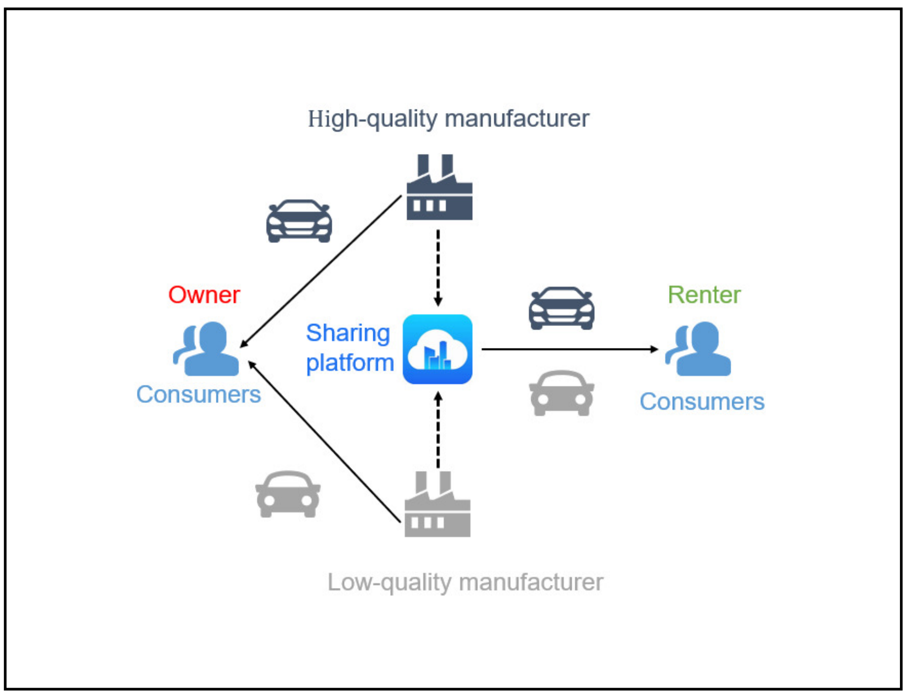
Graphical abstract



Manufacturers' product sharing strategies.

Public summary

- An analytical framework is developed to study the entry strategy of manufacturers to the sharing market facing competition.
- We identify whether and when manufacturers should enter the sharing market.
- The high-quality manufacturer should maintain a quality advantage once it chooses to share; otherwise, it is likely to enter the trouble of lose–lose.
- The low-quality manufacturer will be more interested in offering product sharing when product difference or the sharing platform's service level is low.

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Abstract: Traditional manufacturers can take part in the sharing economy by renting products to consumers through sharing platforms. We develop an analytical framework consisting of two manufacturers and a sharing platform to study the effect of product sharing on competing manufacturers' entry and pricing strategies. On the one hand, when the high-quality manufacturer works with the sharing platform, if the perceived quality of renting the high-quality product is larger than that of purchasing the low-quality product, it shows that the high-quality manufacturer will benefit and should enter the sharing market when the rental price is moderate. However, if the perceived quality of renting a high-quality product is smaller than that of purchasing a low-quality product, both manufacturers will always suffer losses; thus, the high-quality manufacturer should not provide sharing. Consequently, when the high-quality manufacturer chooses to share, the quality advantage should be maintained. On the other hand, when the low-quality manufacturer works with the sharing platform, it also finds that the low-quality manufacturer will always be better off from a moderate rental price. This implies that the low-quality OEM has more interest in offering product sharing if the perceived quality of renting high-quality product is smaller than that of purchasing low-quality product.

Keywords: product sharing; sharing economy; oligopoly competition; sharing mode
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1 Introduction

As the global economic recession and social focus on sustainability began, consumers and society began to explore more effective uses of resources and products. In recent years, progress on the Internet and mobile communication technology has made large-scale collaborative consumption possible[1–2]. The sharing economy is making changes in various industries, and emerging sharing companies, such as Airbnb and Uber, have made the sharing economy popular[3–5]. The sharing of products or services has developed rapidly in the past few years. It is estimated that global revenues of the sharing market have the potential to reach approximately $335 billion by 2025 from $15 billion now[6]. That is, the sharing economy has huge prospects.

In traditional sharing (peer-to-peer sharing market), product sharing occurs between consumers[7]. Although it provides convenience for consumers, original equipment manufacturers (OEMs) may suffer from a decrease in sales caused by the loss of some potential consumers. For example, new car ownership fell by 5% when Getaround built the car peer-to-peer sharing platform[8]. Similarly, in housing rental, the hotel room revenue is estimated to have fallen by 10% in the past five years due to Airbnb in Austin, Texas[9].

Faced with the emergence of the sharing economy, some traditional manufacturers are taking a certain strategy to cope with the sharing economy; for example, some traditional automobile manufacturers have provided product sharing. Some of them choose to rent products to consumers through their own sharing platform[10]. For instance, BMW and Daimler have launched ReachNow① and Car2go②, respectively. In reality, a self-built platform restricts consumers from renting products in many ways, including small scale and little service experience, leading to inconvenient and costly rental service[11]. As a result, many manufacturers actively cooperate with third-party B2C platforms, which charge a certain commission fee for helping OEMs rent out products. For example, JIA BEI, a third-party B2C car-sharing platform, provides operations and maintenance of car-sharing and charges OEMs commission fees③. Compared with a self-built platform, a third-party sharing platform is more feasible for most OEMs.

In this paper, we focus on the OEMs' sharing of products and examine how product sharing with a third-party platform affects competing manufacturers' entry and pricing strategies. Taking automobiles as an example, there are many OEMs in

① https://baike.baidu.com/item/ReachNow
② https://baike.baidu.com/item/car2go/16961433
③ https://www.joy-go.com/business.html

the market, but some of them cooperate with the sharing platform, such as Buick providing electric vehicles for Evcard①, while others do not. How do they make the choice? A few papers researched the B2C sharing model[12–14], and they considered only one OEM in the market, without competition factors. However, the impact of product sharing on competing OEMs is still unclear. Based on previous studies, we research the B2C sharing model in a competing environment. We aim to research the following questions in a competitive environment: (i) Under what conditions are consumers willing to rent products? (ii) How does product sharing affect different manufacturers' entry and pricing strategies?

We develop an analytical framework consisting of two manufacturers and a sharing platform to study how product sharing affects competing manufacturers. In the model, two OEMs produce alternative products with different qualities and sell them to consumers directly. When a third-party B2C sharing platform enters the market, OEMs can rent out products through the platform and pay a certain commission fee. According to different participants in sharing, we discuss two modes of B2C sharing, in which game theory is applied to research the competing OEMs' pricing strategies and then provide certain suggestions for manufacturers' entry strategies.

We contribute to the literature by exploring the B2C sharing model from the perspective of competing manufacturers. First, the conditions under which consumers are willing to rent products are identified. When the high-quality OEM cooperates with the third-party sharing platform, if the perceived quality of renting high-quality products is greater than that of purchasing low-quality products, consumers are willing to rent only if the use frequency and rental price are low. Otherwise, consumers are willing to rent only if the rental price is low enough. Moreover, when the low-quality OEM cooperates with the sharing platform, rental demand exists only when the rental price is low, and this rental price is much lower than in the former case.

Furthermore, we analyze the effect of product sharing on manufacturers' entry and pricing strategies. On the one hand, when the high-quality OEM cooperates with the third-party sharing platform, the high-quality OEM will benefit only if the rental price is moderate, and it should enter the sharing market at this time in the situation that the perceived quality of renting high-quality product is greater than that of purchasing low-quality product. However, the profit of a low-quality OEM is greater only with a high rental price. If the perceived quality of rental high quality is less than that of purchasing low quality product, the profits of both OEMs will be lower; accordingly, the high-quality OEM should not join in the sharing. The high-quality OEM is suggested to maintain a quality advantage if it enters the sharing market. On the other hand, when the low-quality OEM cooperates with the third-party sharing platform, it should share when the rental price is moderate. However, the profit of a high-quality OEM is higher only if the rental price is low. It deduces that the low-quality OEM has more interest in offering product sharing if the perceived quality of renting high-quality product is smaller than that of purchasing low-quality product.

The remainder of this paper is organized as follows. The related works are summarized in Section 2. In Section 3, we describe the model in detail, and in Section 4, we give the equilibrium results and conditions under which consumers are willing to rent. Through analysis, we show the effect of product sharing on OEMs' entry and pricing strategies in Section 5. Finally, we summarize the conclusions, limitations and future research directions of this study in Section 6.

2 Literature review

In the sharing economy, we study the entry strategies and pricing decisions of competing manufacturers. In this section, we review the literature related to sharing platforms and their impact on traditional manufacturers of product sharing.

2.1 Operations management of sharing platforms

The literature on the operation decisions of sharing platforms is extensive[15, 16]. Many studies focus on the service design and resource allocation of sharing platforms. Through trading off customer satisfaction against operation costs, He et al.[17] studied how to design a geographical service region for service providers to operate their service. Some car-sharing platforms, such as Uber or DiDi, provide pooled rides for customers, for which Jacob and Roet-Green[18] designed price-service menus and identified when the platform should offer ride-sharing services. Due to rapid expansion in early times of development, some platforms, such as ofo bicycles, have large redundancy in allocation. By analyzing platform companies' operations in China's sharing market, Xu et al.[19] proposed three main mechanisms for sharing economy platforms to coordinate external resources.

Many scholars study sharing platforms from an operational perspective. There are many practices on P2P sharing in the fashion industry. Motivated by this, Choi and He[20] employed stylized models and conducted empirical analysis in their research. It is proven that the platform should adopt the revenue sharing scheme rather than a fixed service fee scheme. However, Hu and Zhou[21] pointed out that the on-demand matching platform may prefer a fixed commission because of the simplicity of communication with independent contractors. Benjaafar et al.[22, 23] examined a sharing platform maximizing either profit or welfare in P2P and ride sharing, respectively. Likewise, focusing on prices and subsidies, Fang et al.[24] attempted to find the optimal prices of sharing platforms in revenue maximization and social welfare maximization. They show that supply shortages may lead to sharing platforms' limited profitability. Accordingly, platforms are motivated to encourage sharing via subsidies. Gal-Or[25] examined the effect of competition among peer-to-peer lodging platforms and finds that intensified competition may surprisingly lead to a decline in consumer surplus owing to platforms charging higher fees from peer suppliers. From other perspectives, Bian et al.[26] indicated that the platform company does not always opt to offer the B2C sharing service, which is heavily influenced by the fraction of P2P sharing product owners, the platform firm's customer approval level,

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and the related marginal production cost.

2.2 Impact of product sharing on traditional manufacturers

Many scholars research traditional manufacturers' response to changes in the market situation when the sharing economy emerges[23, 27, 28, 11]. For example, Bellos et al.[29] found that an OEM will improve the comfort of retail cars to raise their price and improve the fuel efficiency of rental vehicles to reduce operating costs when developing its car-sharing business. Roma et al.[30] examined incumbents' price response to emerging sharing economy platforms (e.g., Airbnb). They show that the influence of the expanding importance of the sharing economy on incumbents' prices is determined by the type of incumbents, the accommodation time, and the type of customers. Similarly, Dai and Nu[31] investigated the equilibrium price for a capacity-constrained manufacturer using various sharing modes. They discovered that B2C sharing results in a cheaper retail price for low-cost devices with a high limited capacity, creating a win–win situation for both the maker and the customer.

A set of studies focuses on whether a manufacturer should enter the sharing market and use a P2P or B2C sharing strategy. From the perspectives of ownership and the pooling effect, Yu et al.[32] found that selling items of relatively high (poor) quality in B2C sharing is extremely profitable for product categories with a substantial pooling effect or when firm profits highly from ownership. Tian et al.[28] employed a game-theoretic analytical model to research manufacturers' participation in the sharing economy, finding that the OEM will not join the product-sharing market at low transaction costs or not at a very high marginal cost of production. In contrast, based on a P2P sharing market, Wang et al.[33] found that only if the share of low-usage users is reasonably high will the traditional manufacturer enter the sharing market. According to the value perception factor and the marginal cost, Li et al.[34] examined the OEM's decision to collaborate with a third-party B2C platform or a third-party P2P platform. The self-built platform, however, has not been considered. To address this gap, Zhang et al.[11] investigated the effects of several factors, such as the consumers' inconvenient cost and the marginal cost of production, on the manufacturer's mode selection of establishing the sharing platform or cooperating with a third-party sharing platform.

The above literature expounds on the challenges and opportunities brought by the sharing economy and studies the impact of the sharing economy on manufacturers. However, they do not consider competing OEMs, which is common in reality. This research contributes to the literature by introducing competitive factors into the sharing market. Focusing on product sharing, we describe the utility of consumers' choices under various sharing modes from the perspective of consumers' preference for product quality and then establish a mathematical model using game theory to analyze the pricing decisions and entry strategies of manufacturers in a competitive market.

3 The model

Consider that there are two duopolistic original equipment manufacturers (OEMs) in the market who produce similar products with different qualities and sell them to customers directly, as shown in mode SN of Fig. 1. Suppose the high-quality OEM (OEM1) produces high-quality products (product 1) with quality q_1 and retails with price p_1, while the low-quality OEM (OEM2) produces low-quality products (product 2) with quality q_2 ($q_2 < q_1$) and retails with price p_2. Without loss of generality, we normalize the two OEMs' marginal costs to 0. This setting allows us to focus on the quality competition between the OEMs with clear presentations, and this setting is widely adopted in the literature[35, 36]. One may argue that producing high-quality products will cost more resources, including manpower, materials and financial support. Here, we focus on the marginal costs rather than investments for technology improvements. Therefore, it is reasonable to assume that both OEMs have the same marginal cost.

Each consumer cannot buy or rent more than one unit and can obtain usage value from the product. Consumer j's per usage value v_j depends on his own quality preference (θ_j) and the quality of product i (q_i), which means $v_j = \theta_j q_i$. We use the heterogeneous θ_j to describe consumers in the market who have different preferences for product quality, assuming that θ is uniformly distributed between 0 and 1. The use frequency of consumers is μ, which refers to the ratio of the time consumers spend using the product to the service life of the product, and we normalize the service life to one. μ is assumed to be a homogeneous parameter to focus on a certain market, such as the use of bicycles by office workers[37, 34]. Therefore, $\mu\theta_j q_i$ represents the total usage value for consumer j who buys product i. Without loss of generality, the total number of consumers is supposed to be one.

As shown in Fig. 1, in the sharing economy, OEMs can share products through a third-party sharing platform, which collects commission. According to different participants of sharing, we discuss two modes of B2C sharing (mode SBH and mode SBL). A consumer can either be owner by purchasing from OEM, or be renter by renting from sharing platform. It is noted that the commission rate $\alpha \in (0,1)$ is given. Different from previous studies in which sharing platforms make price decisions, the sharing price is considered to be exogenous in our research. On the one hand, an exogenous sharing price is in line with reality due to intense rivalry and governmental control①. For instance, in January 2018, the BMW 3 Series 2016 White model was priced at \$80 both on Car2go and Turo[28]. Furthermore, in China, the number of registered car-sharing firms has topped 1600, and key players, such as Gofun supported by Shou Qi Group and Morefun backed by BAIC Group, utilize similar sharing pricing②. On the other hand, an exogenous sharing price is widely adopted in the literature[23, 31]. This setting ensures that our study focuses on manufacturers facing competition. Accordingly, an exogenous sharing price is reasonably adopted in our research.

Consumers can choose to buy product 1, product 2, or an outside choice if there is no sharing in the market (mode SN), as shown in Fig. 1. To describe customer preferences, we em-

① http://www.gov.cn/xinwen/2019-11/21/content_5454186.htm
② https://www.iresearch.com.cn/Detail/report?id=3347&isfree=0

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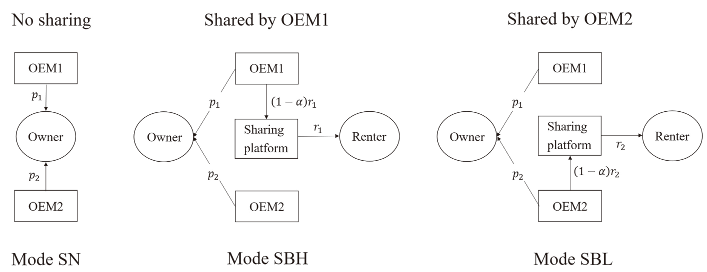

Fig. 1. Structure of different modes.

ploy a utility function. Because the consumer is the owner, he obtains gross usage value positively related to μ, θ and q and just needs to pay the purchase price p_i, which has no relation with use frequency μ. Considering that the utility of the external option is 0, which means $U_{0j}=0$, and U_{ij}^o is given by Eq. (1), where o and j means consumer j buys and owns a product, $i=1$ means product 1 and $i=2$ means product 2.

$$U_{ij}^o=\mu\theta_j q_i-p_i,\ i=1,2. \tag{1}$$

In the B2C sharing model, we discuss the situation where only one OEM works with the sharing platform. Accordingly, there are two sharing modes with different sharing participants. The OEM participating in the B2C sharing market puts its products on the sharing platform, which rents products to consumers at rental price (r_1 or r_2) and only needs to bear a certain maintenance cost. The OEM providing sharing product can earn rental revenue ($(1-\alpha)r_1$ or $(1-\alpha)r_2$) per transaction. β ($0<\beta<1$) is the quality discount coefficient of consumers using rented products, reflecting consumers' concerns about quality when renting products from the platform. β is assumed to be homogeneous to focus on a specific platform, which has a fixed service level. For the same product, the perceived quality from purchase will definitely be higher than that of rental owing to the platform's poor service level and the gap on psychological generated by nonownership. Every time the consumer uses product i, he obtains usage value and pays the rent. Thus, what he pays is related to use frequency μ.The utility functions of different types of owners (o) and renters (r) are given by Eqs. (1) and (2), respectively.

$$U_{ij}^r=\mu(\theta_j\beta q_i-r_i),i=1,2. \tag{2}$$

The following is the sequence of events. In mode SN, the OEMs first set the retail prices, and then consumers make their choices to buy which product. In mode SB, in a specific sharing mode (SBH or SBL), the OEMs choose their retail price in the meantime. Next, consumers make decisions on purchase, rental or outside options.

4 Analysis

In this section, we analyze the benchmark when there is no sharing economy (mode SN) and then research OEMs' equilibriums in mode SB. All the detailed analysis processes are presented in the appendix. To make it more understandable, we use subscript "1" (and "2") to denote the high- (and low-) quality manufacturer and use "o" and "r" for owner and renter. Furthermore, SN\SBH\SBL are used as superscripts to distinguish different modes.

4.1 Benchmark: no sharing mode (SN)

First, the no sharing mode is considered the benchmark case with the OEMs as sellers only. Consumers can choose to buy product 1, product 2 or outside options. According to Eq. (1), the indifference point $\theta_1=\dfrac{p_1-p_2}{\mu(q_1-q_2)}$ is obtained from $U_1^o=U_2^o$. Furthermore, we obtain the thresholds $\theta_0=\dfrac{p_2}{\mu q_2}$ through $U_2^o\geqslant 0$ and $\theta_0=\dfrac{p_1}{\mu q_1}$ through $U_1^o\geqslant 0$. Therefore, we have $U_1^o>U_2^o$ if $\theta>\theta_1$; $U_2^o>0$ if $\theta>\theta_0$; and $U_1^o>0$ if $\theta>\theta_0$. After considering all different conditions, the relation is reasonable only when $\theta_1>\theta_0>\theta_0$ to ensure that all possible market segments exist. Consumers with $\theta\in(\theta_1,1)$ prefer product 1 because $U_1^o>U_2^o>0$, while consumers with $\theta\in(\theta_0,\theta_1)$ prefer product 2 because $U_2^o>U_1^o$ and $U_2^o>0$. Thus, the retail demands for products 1 and 2 are $D_1^{SN}=1-\theta_1$ and $D_2^{SN}=\theta_1-\theta_0$, respectively. Accordingly, the manufacturers' profits are given by $\pi_1^{SN}=p_1\left(1-\dfrac{p_1-p_2}{\mu(q_1-q_2)}\right)$, $\pi_2^{SN}=p_2\left(\dfrac{p_1-p_2}{\mu(q_1-q_2)}-\dfrac{p_2}{\mu q_2}\right)$. In the following discussion, we will keep key steps and ignore some details for simplicity because they are similar to this process.

Note that there always exists purchase demand for product 1 and product 2 in mode SN. The OEMs' optimal pricing and profits are summarized in Lemma 4.1.

Lemma 4.1. When there is no sharing product in the market, the equilibria are as follows.

(1) The high-quality OEM's optimal retail price is $p_1^{SN^*}=\dfrac{2\mu q_1(q_1-q_2)}{4q_1-q_2}$, the retail demand is $D_1^{SN^*}=\dfrac{2q_1}{4q_1-q_2}$, and the optimal profit is $\pi_1^{SN^*}=\dfrac{4\mu q_1^2(q_1-q_2)}{(4q_1-q_2)^2}$.

(2) The low-quality OEM's optimal retail price is $p_2^{SN^*}=\dfrac{\mu q_2(q_1-q_2)}{4q_1-q_2}$, the retail demand is $D_2^{SN^*}=\dfrac{q_1}{4q_1-q_2}$, and the optimal profit is $\pi_2^{SN^*}=\dfrac{\mu q_1 q_2(q_1-q_2)}{(4q_1-q_2)^2}$.

To earn the most, the OEMs develop their optimal pricing strategy according to Lemma 4.1. Then, consumers make their purchase decisions. Finally, the OEMs make their profits.

4.2 B2C sharing mode (SB)

In this case, it is assumed that only one OEM works with the B2C sharing platform. Therefore, there are two conditions according to different sharing participants: (i) mode SBH: the high-quality OEM enters the sharing market; (ii) mode SBL: the low-quality OEM cooperates enters the sharing market.

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4.2.1 The high-quality OEM enters the sharing market (SBH)

Now, we examine when the third-party sharing platform cooperates with the high-quality OEM, consumers can either buy product 1 or product 2 from OEMs directly or rent product 1 on the third-party sharing platform. Several indifference points are obtained, $\theta_1=\dfrac{p_1-p_2}{\mu(q_1-q_2)}$, $\theta_2=\dfrac{p_1-\mu r_1}{\mu(1-\beta)q_1}$, and $\theta_3=\dfrac{p_2-\mu r_1}{\mu(q_2-\beta q_1)}$ from $U_1^o=U_2^o$, $U_1^o=U_1^r$ and $U_1^r=U_2^o$, respectively. We obtain the thresholds $\theta_4=\dfrac{r_1}{\beta q_1}$ and $\theta_0=\dfrac{p_2}{\mu q_2}$ from $U_1^r>0$, $U_2^o>0$.

In the sharing market, rental products compete with retail products. Nevertheless, product 1 is of better quality than product 2 ($q_1>q_2$), and the relationship between the perceived quality of renting product 1 and purchasing product 2 is still unknown due to the quality discount parameter β. For this reason, we research the SBH mode from two aspects according to the difference between the perceived quality of rental and purchase product, as shown in Fig. 2.

When $\beta q_1>q_2$, the perceived quality of renting product 1 is higher than that of buying product 2 due to the great quality difference or the large quality discount parameter. According to Fig. 2, with a similar derivation process in the benchmark model, we obtain the demands for product 1 and product 2, respectively: $D_{1o}^{SBH}=1-\theta_2$, $D_{1r}^{SBH}=\theta_2-\theta_3$ and $D_{2o}^{SBH}=\theta_3-\theta_0$. Moreover, the profits of the two OEMs can be expressed as follows: $\pi_1^{SBH}=p_1\left(1-\dfrac{p_1-\mu r_1}{\mu(1-\beta)q_1}\right)+(1-\alpha)\cdot r_1\left(\dfrac{p_1-\mu r_1}{\mu(1-\beta)q_1}-\dfrac{p_2-\mu r_1}{\mu(q_2-\beta q_1)}\right)$, $\pi_2^{SBH}=p_2\left(\dfrac{p_2-\mu r_1}{\mu(q_2-\beta q_1)}-\dfrac{p_2}{\mu q_1}\right)$.

Similarly, when $\beta q_1<q_2$, i.e., the perceived quality of renting product 1 is lower than that of buying product 2 to consumers as a result of a small quality difference or small quality discount parameter, the quantities demanded for product 1 and product 2 are as follows: $D_{1o}^{SBH}=1-\theta_1$, $D_{1r}^{SBH}=\theta_3-\theta_4$ and $D_{2o}^{SBH}=\theta_1-\theta_3$. The profit expressions of the OEMs are $\pi_1^{SBH}=p_1\left(1-\dfrac{p_1-p_2}{\mu(q_1-q_2)}\right)+(1-\alpha)r_1\left(\dfrac{p_2-\mu r_1}{\mu(q_2-\beta q_1)}-\dfrac{r_1}{\beta q_1}\right)$ and $\pi_2^{SBH}=p_2\left(\dfrac{p_1-p_2}{\mu(q_1-q_2)}-\dfrac{p_2-\mu r_1}{\mu(q_2-\beta q_1)}\right)$.

To further study the sharing economy, the conditions are concluded for the existence of rental demand in Proposition 4.1.

Proposition 4.1. When the high-quality OEM works with the B2C sharing platform:

(1) When $\beta q_1>q_2$, in the domain where $0<u\leqslant u^{SBH}$, or $0<r_1<r_1^{SBH1}$ and $u^{SBH}<u<1$, consumers willing to rent products exist;

(2) when $\beta q_1<q_2$, in the domain where $r_1<r_1^{SBH2}$, consumers willing to rent products exist;

(3) otherwise, there is no consumer willing to rent products.

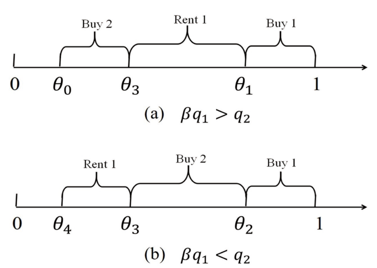

Fig. 2. Possible scenarios of consumer segmentations in mode SBH.

The most evident aspect of mode B2C, in comparison to mode SN, is that the sharing product directly competes with retail products. As shown in Fig. 3, consumers are willing to rent in the gray area. First, when $\beta q_1>q_2$, in the domain of $0<u\leqslant u^{SBH}$, due to consumers' low usage frequency and the high perceived quality of rental product 1, shared product 1 is particularly appealing to customers. Consequently, there exist consumers willing to rent. However, if consumers have a higher usage frequency ($u^{SBH}<u<1$), only product 1 with a low rental price ($0<r_1<r_1^{SBH1}$) can attract them. In addition, in the remaining area, consumers only purchase products because of the high use frequency and high rental price. In addition, when $\beta q_1<q_2$, in the domain of $r_1>r_1^{SBH2}$, no consumer is willing to rent product 1 because of the high rental price. Moreover, the feasible region of r_1 in this case is far smaller than the former. This can be explained by the small quality difference and the small quality discount parameter, which means that rental product 1 brings less usage value than retail product 2. Consequently, consumers decline in willingness to pay for rental and willing to rent exist only in the domain of small enough r_1 ($r_1<r_1^{SBH2}$) when $\beta q_1<q_2$.

In the domain where there exists consumer willing to rent, we can obtain the following equilibria in Lemma 4.2. In the remaining area, it is the same as the SN case, as there is no rental demand.

Lemma 4.2. When the high-quality OEM works with the B2C sharing platform and there exists rental demand, there exists a pair of unique solutions that maximizes the two OEMs' profits.

Lemma 4.2 shows that OEMs have unique optimal prices leading to the highest profits in the SBH mode. The analysis process is presented in detail in the appendix. We will compare OEMs' equilibrium profits in modes SBH and SN in Section 5 to further understand the impact of the sharing economy.

4.2.2 The low-quality OEM enters the sharing market (SBL)

In this part, the background is that the low-quality OEM not only sells product 2 to consumers but also shares it through the sharing platform, while the high-quality OEM only retails product 1. Consumers can rent product 2 at r_2 in the sharing market and purchase products directly from the two OEMs. Moreover, the sharing platform charges the low-quality OEM commission for providing shared service and bearing maintenance cost. The indifference points $\theta_1=\dfrac{p_1-p_2}{\mu(q_1-q_2)}$ and $\theta_6=\dfrac{p_2-\mu r_2}{\mu(1-\beta)q_2}$ are obtained by $U_1^o=U_2^o$ and $U_2^o=U_2^r$. Meanwhile, we obtain $\theta_7=\dfrac{r_2}{\beta q_2}$ from $U_2^r>0$. With a similar derivation process in the benchmark model, the demands for different products are expressed as $D_{1o}^{SBL}=1-\theta_1$, $D_{2o}^{SBL}=\theta_1-\theta_6$ and

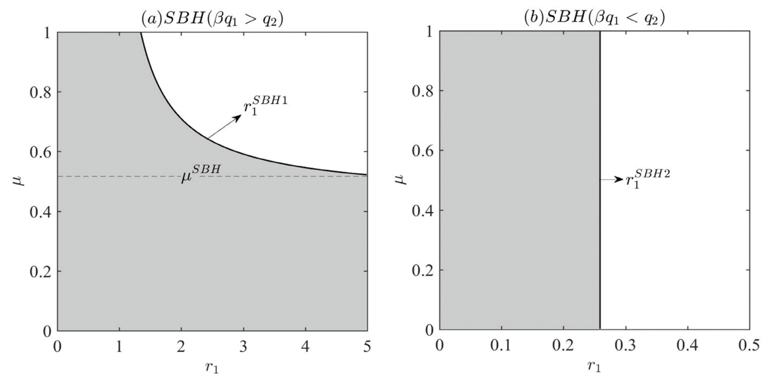

Fig. 3. Conditions for the existence of rental demand in mode SBH.

$D_{2r}^{SBL} = \theta_6 - \theta_7$. The profit expressions of the OEMs are $\pi_1^{SBL} = p_1\left(1 - \dfrac{p_1 - p_2}{\mu(q_1 - q_2)}\right)$ and $\pi_2^{SBL} = p_2\left(\dfrac{p_1 - p_2}{\mu(q_1 - q_2)} - \dfrac{-p_2 + \mu r_2}{\mu(-1+\beta)q_2}\right) + (1-\alpha)r_2\left(\dfrac{-p_2 + \mu r_2}{\mu(-1+\beta)q_2} - \dfrac{r_2}{\beta q_2}\right)$, where the high-quality OEM derives its income only from retail and the low-quality OEM from retail and rental. We know that product 1 is of better quality than product 2 ($q_1 > q_2$), and the quality discount parameter is smaller than 1. It is clear that $q_1 > \beta q_2$. The conditions under which rental demand exists are summarized in Proposition 4.2.

Proposition 4.2. When the low-quality works with OEM the B2C sharing platform:

(1) In the domain where $r_2 \leqslant r_2^{SBL}$, there exist consumers willing to rent products;

(2) otherwise, there is no consumer willing to rent products.

From Proposition 4.2, it is known that only in the part under $r_2 \leqslant r_2^{SBL}$ exist consumers willing to rent, as shown in the gray area of Fig. 4. When the low-quality OEM shares its product in the market, consumers' willingness to rent is low because of the low quality of product 2.

In the domain where $r_2 \leqslant r_2^{SBL}$, we can obtain the following lemma. In the remaining area, it degenerates to mode SN as there is no rental demand.

Lemma 4.3. When the low-quality OEM works with the B2C sharing platform, in the domain of $r_2 \leqslant r_2^{SBL}$, there exists a pair of unique solutions that maximizes the two OEMs' profits.

Lemma 4.3 shows that OEMs can reach an equilibrium in the sharing market with the participation of the low-quality OEM. More details of the analysis are presented in the appendix. In Section 5, we will analyze OEMs' equilibrium results in modes SBL and SN to make the impact of the sharing economy more understood.

In this section, we have answered the question: Under what conditions are consumers willing to rent products? Furthermore, there is a question to be answered: Should OEMs enter the sharing market?

5 Discussion

With the development of the sharing economy, an increasing number of manufacturers choose to cooperate with sharing platforms, and can sharing definitely bring benefits to OEMs? If not, under what environment should OEMs choose to share? In this section, the no sharing mode (mode SN) is used as the basic mode to research OEMs' entry strategies in the sharing market in a competitive environment. Similarly, it is divided into three parts according to the different sharing participants: (i) mode SBH vs mode SN; (ii) mode SBL vs mode SN; (iii) mode SBH vs mode SBL. To focus on the impact of product sharing, we only study the domains from Section 4, where consumers willing to rent in the SBH and SBL modes exist in the following analysis.

5.1 Mode SBH vs mode SN

We first compare the profits and prices under mode SBH and mode SN. The profit and retail price differences are defined as $\Delta\pi_i^{SBH} = \pi_i^{SBH^*} - \pi_i^{SN^*}$, $\Delta p_i^{SBH} = p_i^{SBH^*} - p_i^{SN^*}$, $i = 1, 2$, respectively. Through analysis of the profit difference of the two OEMs under different modes in the two situations, Proposition 5.1 is derived.

Proposition 5.1. When the high-quality OEM works with the B2C sharing platform and there exists rental demand:

(1) When $\beta q_1 > q_2$:

a) $\Delta\pi_1^{SBH} > 0$ if $r_1^{SBH3} < r_1 < r_1^{SBH4}$; and $\Delta\pi_1^{SBH} \leqslant 0$ if $r_1 \leqslant r_1^{SBH3}$ or $r_1 \geqslant r_1^{SBH4}$.

b) $\Delta\pi_2^{SBH} > 0$ if $r_1 > r_1^{SBH5}$; and $\Delta\pi_2^{SBH} \leqslant 0$ if $r_1 \leqslant r_1^{SBH5}$.

c) $\Delta p_1^{SBH} > 0$ if $r_1 > r_1^{SBH6}$; and $\Delta p_1^{SBH} \leqslant 0$ if $r_1 \leqslant r_1^{SBH6}$.

d) $\Delta p_2^{SBH} > 0$ if $r_1 > r_1^{SBH7}$; and $\Delta p_2^{SBH} \leqslant 0$ if $r_1 \leqslant r_1^{SBH7}$.

(2) When $\beta q_1 < q_2$:

a) $\Delta\pi_1^{SBH} < 0$ and $\Delta\pi_2^{SBH} < 0$.

b) $\Delta p_1^{SBH} < 0$ and $\Delta p_2^{SBH} < 0$.

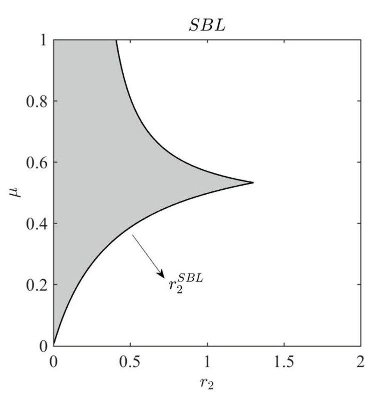

Fig. 4. Conditions for the existence of rental demand in mode SBL.

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Note that the profit differences are discussed in the condition with the existence of consumers willing to rent. We set $q_1 = 8.95$, $q_2 = 5.54$, $u = 0.598$, $\alpha = 0.231$, and $\beta = 0.855$ (0.5) to draw Fig. 5 (Fig. 6), which helps us comprehend Proposition 5.1 more easily. From the above proposition, it is known that in the SBH mode, product sharing affects the two OEMs in different ways. In the condition when the perceived quality of rental product 1 is larger than that of retail product 2 ($\beta q_1 > q_2$), first, if r_1 is too low ($r_1 \leqslant r_1^{SBH3}$), the emergence of rental products will erode the purchasing demand for the retail products. For this reason, the two OEMs have to lower retail prices to reduce the decrease in purchase demand. For the high-quality OEM, its total profit is lower than that in mode SN because the new rental profit cannot completely compensate for the decrease in retail profit. Second, as the rental price increases, the price for product 1 increases significantly, but the demand for retail and rental product 1 is inversely related to the rental price. It induces that if r_1 is in the middle range ($r_1^{SBH3} < r_1 < r_1^{SBH4}$), compared to the no-sharing market, the high-quality OEM makes more profits because of the revenue from rental exceeding the reduction in retail revenue. Third, if r_1 is high enough ($r_1 \geqslant r_1^{SBH4}$), although the retail price is much higher than that in mode SN, the demand for retail and rental products 1 is relatively low, so the total profit of the high-quality OEM is lower than that in mode SN.

In other words, the high-quality OEM should join the B2C sharing economy if r_1 is in the middle range ($r_1^{SBH3} < r_1 < r_1^{SBH4}$), while it should not if r_1 is too low ($r_1 \leqslant r_1^{SBH3}$) or too high ($r_1 \geqslant r_1^{SBH4}$).

For the low-quality OEM, if r_1 is low ($r_1 \leqslant r_1^{SBH5}$), it has to lower its retail price to reduce the decrease in purchase demand but still profits less than when there is no sharing. With r_1 higher ($r_1 > r_1^{SBH5}$), the demand and price for retail of product 2 will rise, even more than that in mode SN, which leads to greater profit. In summary, the low-quality OEM will be better off if r_1 is high and worse off if r_1 is low.

Additionally, the case is analyzed in detail when the perceived quality of rental product 1 is smaller than that of retail product 2 ($\beta q_1 < q_2$). Compared with the no sharing economy, the equilibrium prices of the two OEMs are lower, and the purchase demand for product 1 is lower. In contrast, the purchase demand for product 2 is positively related to the rental price of product 1 and finally higher than that without a sharing economy. In contrast, the rental demand for product 1 negatively correlates with the rental price. Inevitably, the appearance of rental products results in more competition between the OEMs, causing a price war. Hence, the low-quality OEM is worse off without a doubt because of the large loss of sale. For the high-quality OEM, rental revenue cannot make up for the loss of sale, leading to a decline in total profit compared to when there is no sharing.

In conclusion, if the high-quality OEM decides to join the sharing economy, it needs to maintain the quality advantage to ensure $\beta q_1 > q_2$. Once the quality gap with a low-quality OEM is narrowed, it is likely to enter the trouble of lose–lose. Under this condition, the low-quality OEM may be better off

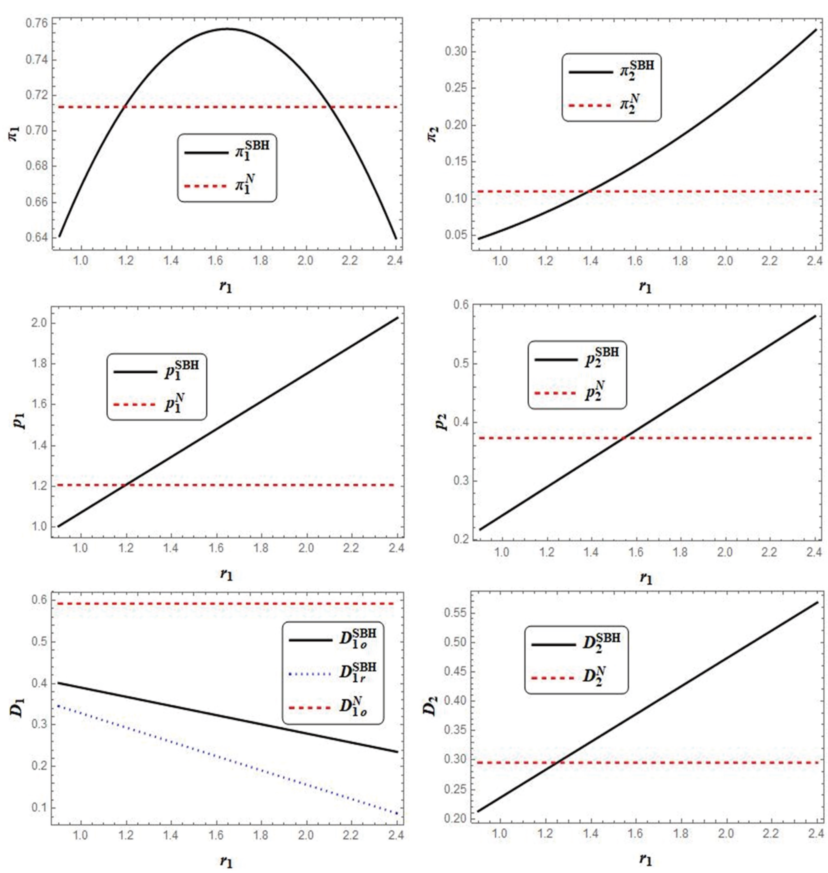

Fig. 5. Impact of sharing in mode SBH ($\beta q_1 > q_2$).

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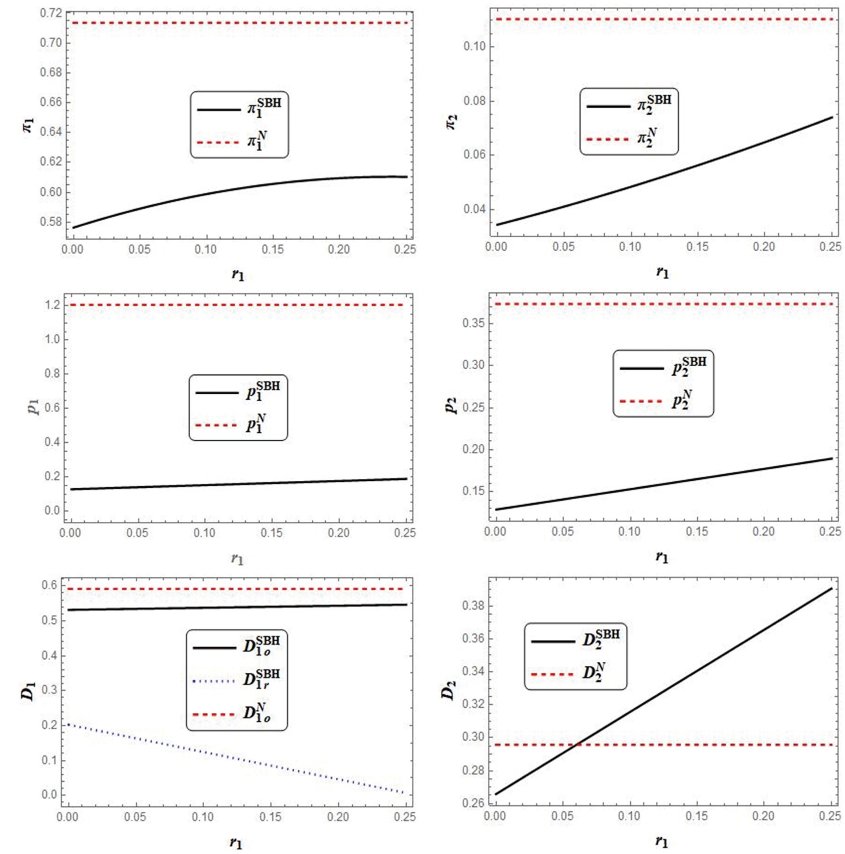

Fig. 6. Impact of sharing in mode SBH ($\beta q_1 < q_2$).

or worse. If worse, it can improve its product quality to force the high-quality OEM to give up sharing.

5.2 Mode SBL vs mode SN

Moreover, the equilibria of the two manufacturers are compared under mode SBL and mode SN. The profit and price differences are expressed as $\Delta\pi_i^{SBL} = \pi_i^{SBL^*} - \pi_i^{SN^*}$, $\Delta p_i^{SBH} = p_i^{SBH^*} - p_i^{SN^*}$, $i = 1, 2$, respectively. After analysis, the results are as shown in Proposition 5.2.

Proposition 5.2. When the low-quality OEM works with the B2C sharing platform and there exists rental demand, the rental price of product 2 affects the two OEMs in different ways:

(1) $\Delta\pi_1^{SBL} > 0$, if $r_2 > r_2^{SBL3}$; and $\Delta\pi_1^{SBL} \leqslant 0$ if $r_2 \leqslant r_2^{SBL3}$.

(2) $\Delta\pi_2^{SBL} > 0$, if $r_2^{SBL4} < r_2 < r_2^{SBL5}$; and $\Delta\pi_2^{SBL} \leqslant 0$ if $r_2 \leqslant r_2^{SBL4}$ or $r_2 \geqslant r_2^{SBL5}$.

(3) $\Delta p_1^{SBL} > 0$, if $r_2 > r_2^{SBL6}$; and $\Delta p_1^{SBL} \leqslant 0$ if $r_2 \leqslant r_2^{SBL6}$.

(4) $\Delta p_2^{SBL} > 0$, if $r_2 > r_2^{SBL7}$; and $\Delta p_2^{SBL} \leqslant 0$ if $r_2 \leqslant r_2^{SBL7}$.

Note that the profit differences are compared under the condition that consumers are willing to rent; otherwise, the profit difference is 0. To observe Proposition 5.2 more intuitively, we draw Fig. 7 by setting $q_1 = 8.95$, $q_2 = 5.54$, $u = 0.598$, $\alpha = 0.231$, and $\beta = 0.855$. To begin with the low rental price of product 2, the emergence of rental products erodes the purchase demand for the two products. To reduce the decrease in purchase demand, the OEMs will take the initiative to lower the retail price. For the low-quality OEM, its total profit is less than when there is no sharing, although it can obtain rental revenue that cannot completely compensate for the decrease in retail profit. Second, the demand for purchases and rentals is negatively related to rental prices. Thus, if r_2 is in the middle range ($r_2^{SBL3} < r_2 < r_2^{SBL4}$), the rental revenue exceeds the reduction in sales profit, which means that the low-quality OEM makes more profits in the SBL mode than in the SN mode. Third, if r_2 is high enough ($r_2 \geqslant r_2^{SBL4}$), although the retail price is higher, the low-quality OEM is worse off, resulting from very little demand for rental and purchase of product 2. To conclude, the low-quality OEM should enter the B2C sharing economy if r_2 is in a middle range ($r_2^{SBL3} < r_2 < r_2^{SBL4}$), while it should not if r_2 is too low ($r_2 \leqslant r_2^{SBL3}$) or too high ($r_2 \geqslant r_2^{SBL4}$).

For the high-quality OEM, when r_2 is low, the manufacturer earns less, although it tries to reduce the decline of purchase demand by price cut. If r_2 is high, the high-quality OEM will be better off because of the increase of the retail demand and retail price compared with the no-sharing market. In summary, the high-quality OEM will be better off if r_2 is high ($r_2 > r_2^{SBL5}$) and worse off if r_2 is low ($r_2 \leqslant r_2^{SBL5}$).

5.3 Mode SBH vs mode SBL

In this part, mode SBH is compared with mode SBL in the condition when there exist consumers willing to rent product 1 and product 2. Considering the complexity of the analytic formula, this part adopts a numerical experiment for analysis. We set $q_1 = 8.95$, $q_2 = 5.54$, $u = 0.598$, and $\alpha = 0.231$. Similarly, our discussion consists of two parts according to the difference between the perceived quality of purchase product 2 and rental product 1.

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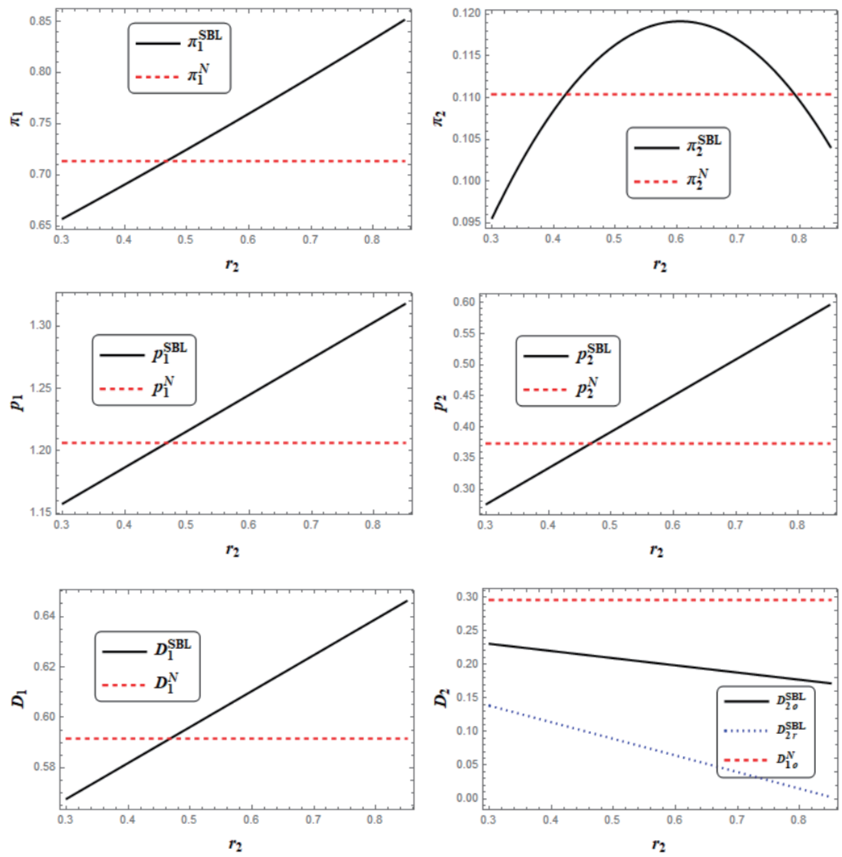

Fig. 7. Impact of sharing in mode SBL.

First, β is set as 0.855 to satisfy that the perceived quality of rental product 1 is larger than that of purchase product 2 ($\beta q_1 > q_2$), and different entry strategies are illustrated from the manufacturers' perspective. As shown in Fig. 8a, we can obtain four parts. First, in part Ⅰ, $\Delta\pi_1^{SBH} < 0$ and $\Delta\pi_1^{SBL} > 0$, which implies that the high-quality OEM is better off in the SBL mode and should not offer rental service. In contrast, $\Delta\pi_1^{SBH} > 0$ and $\Delta\pi_1^{SBL} < 0$ in part Ⅱ, which shows that the high-quality OEM should enter the sharing market. In parts Ⅲ and Ⅳ, $\Delta\pi_1^{SBL} > 0$ and $\Delta\pi_1^{SBH} > 0$. In contrast, if r_1 is given, the high-quality OEM is better off in mode SBH when r_2 is high (part Ⅲ) and better off in mode SBL when r_2 is low (part Ⅳ). As above, the high-quality OEM is suggested to enter the sharing market in part Ⅱ and part Ⅳ, that is, r_1 is moderate and r_2 is low.

Four parts can also be observed from Fig. 8b. Similarly, $\Delta\pi_2^{SBH} > 0$ and $\Delta\pi_2^{SBL} < 0$ in part Ⅴ; accordingly, the low-quality OEM should not cooperate with the sharing platform. In part Ⅵ, it is better to offer rental service for the low-quality OEM because $\Delta\pi_2^{SBH} < 0$ and $\Delta\pi_2^{SBL} > 0$. In part Ⅶ and part Ⅷ, the low-quality OEM is better off in modes SBH and SBL, respectively. This happens because if r_2 is given, the low-quality OEM's profit is positively related to r_1. Consequently, the low-quality OEM should cooperate with the platform when r_2 is moderate and r_1 is low.

From Proposition 5.1, we know that both OEMs are worse

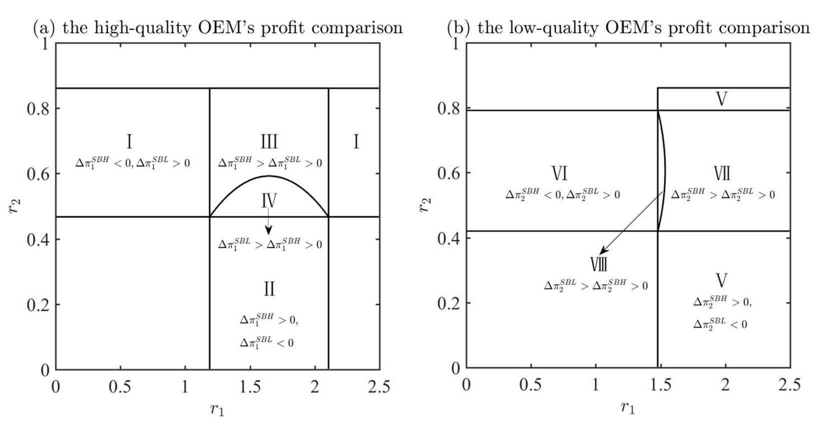

Fig. 8. OEMs' profit comparisons in modes SBH and SBL when $\beta q_1 > q_2$.

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off in mode SBH than in mode SN when the perceived quality of rental product 1 is larger than that of purchase product 2 ($\beta q_1 < q_2$), which leads to a low quality-difference or small quality discount parameter. However, there always exists a region such that OEMs are better off in mode SBL than in mode SN, according to Proposition 5.2. Consequently, both OEMs may prefer SBL, which means that only the low-quality OEM is interested in product sharing when $\beta q_1 < q_2$.

6 Conclusions

The sharing economy is displacing the traditional business mode as the Internet and mobile communication technology advance. Traditional companies provide product sharing in various forms to win back customers. At present, most studies on the decision-making of manufacturers in the sharing economy consider a complete monopoly market, while competition factors in real life have a significant influence. For this purpose, we innovatively add competition factors when considering the impact of the sharing economy on manufacturers. In this paper, we consider a market consisting of a sharing platform and two manufacturers, which produce alternative products of different quality. Furthermore, we propose a new sharing model in which two OEMs compete with each other. Two modes are researched according to different sharing participants. The manufacturer, joining the sharing market, not only sells products but also rents out through the sharing platform, and consumers make decisions on purchase or rental. We address the condition under which there exist consumers with rental willingness and how OEMs make their entry and pricing strategies in product sharing. Our findings are summarized below.

First, in mode SBH, where the high-quality OEM works with the sharing platform, we research two aspects according to the difference between the perceived quality of rental product 1 and retail product 2 because rental products compete with retail products. On the one hand, when the perceived quality of renting product 1 is greater than the perceived quality of purchasing product 2, if use frequency is low or use frequency is high but the rental price is low, consumers with rental willingness exist in the market. On this premise, the high-quality OEM profits less if the rental price of product 1 is too low or too high, resulting from encroachment of the retail market or loss of potential customers. Thus, it is suggested to share when rental prices are moderate. On the other hand, when the perceived quality of renting a high-quality product is lower than the perceived quality of purchasing a low-quality product, consumers are willing to rent only if the rental price is low enough, and this rental price is much lower than in the former case. Under this condition, the profits of both OEMs will be lower; accordingly, the high-quality OEM should not enter the sharing market.

Second, in mode SBH, if the high-quality OEM decides to join the sharing economy, it is suggested to maintain quality advantage because once the quality gap with low-quality OEM is narrowed, it is likely to enter the trouble of lose–lose. A low-quality OEM may be better off or worse. If worse, it can improve its product quality to force the high-quality OEM to give up sharing.

Third, in mode SBL, where the low-quality OEM works with the sharing platform, there exists no consumer willing to rent only if the rental price is high. Based on this, if the rental price is moderate, the low-quality OEM should enter the sharing market because of higher profit. When compared with mode SBH, we find that the low-quality OEM has more interest in offering product sharing if the perceived quality of renting high-quality product is smaller than that of purchasing low-quality product.

Finally, the retail prices of the OEMs are positively related to rental prices. At the beginning, when the rental price is low, the retail price is lower than when there is no sharing caused by fierce competition. Generally, once the rental price is above a certain value, the retail price in the sharing market is higher than no sharing. Differently, when the perceived quality of renting a high-quality product is lower than the perceived quality of purchasing a low-quality product, the retail prices in mode SBH are always lower than no sharing because the rental price is kept low enough to ensure the existence of consumers willing to rent.

Against the background of the sharing economy, this paper studies the influence of product sharing on the entry and pricing strategies of competing manufacturers and obtains some innovative research results. However, there are still some limitations in this study, and future directions can be expanded. For instance, it is more realistic to consider the heterogeneity of consumers both in quality preference and usage frequency. It would be interesting to research the scenario when both manufacturers enter the sharing market. In addition, we use fixed rental prices to study manufacturers' operations but do not take platform pricing into account. To research further, the platform's operation should be included.

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Conflict of interest

The authors declare that they have no conflict of interest.

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Appendix

Proof of Proposition 4.1.

(1) When $\beta q_1 > q_2$

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To ensure that there exist consumers willing to rent product 1, it should satisfy $\theta_5 < \theta_3 < \theta_2$. Therefore, we obtain $r_1 > 0$ if $0 < u \leqslant u^{SBH}$ and $0 < r_1 < r_1^{SBH1}$ if $u^{SBH} < u < 1$.

$$u^{SBH} = \frac{q_1q_2 - q_2^2 - q_1q_2\alpha + q_2^2\alpha - q_1^2\beta + 2q_1q_2\beta + q_1^2\alpha\beta - 2q_1q_2\alpha\beta - q_1\beta^2 + q_1^2\alpha\beta^2}{-q_1^2 + 3q_1q_2 - q_2^2 - 2q_1^2\beta + q_1^2\beta^2},$$

$$r_1^{SBH1} = \frac{q_1^2q_2u - q_1q_2^2u - q_1^3u\beta + q_1^2q_2u\beta + q_1q_2^2u\beta - 2q_1^2q_2u\beta^2 + q_1^3u\beta^3}{-q_1q_2 + q_2^2 - q_1^2u + 3q_1q_2u - q_2^2u + q_1q_2\alpha - q_2^2\alpha + q_1^2\beta - 2q_1q_2\beta - 2q_1^2u\beta - q_1^2\alpha\beta + 2q_1q_2\alpha\beta + q_1^2\beta^2 + q_1^2u\beta^2 - q_1^2\alpha\beta^2}.$$

(2) When $\beta q_1 < q_2$

Similarly, to ensure that consumers willing to rent product 1 exist, it should satisfy $\theta_4 < \theta_3 < \theta_1$. We obtain $0 < r_1 < r_1^{SBH2}$ if $0 < u < 1$.

$$r_1^{SBH2} = \frac{-q_1^2q_2\beta + q_1q_2^2\beta + q_1^3\beta^2 - q_1^2q_2\beta^2}{-4q_1q_2 + q_2^2 + 2q_1^2\beta + q_1q_2\beta}.$$

Proof of Lemma 4.2.

(1) When $\beta q_1 > q_2$

The analysis process is the same as the former, and finally, we have

$$p_1^{SBH^*} = \frac{1}{2}((u - u\beta)q_1 + (1 + u - \alpha)r_1),\ p_2^{SBH^*} = \frac{uq_1r_1}{2(1+\beta)q_1 - 2q_2},\ D_{1o}^{SBH^*} = \frac{1}{2} - \frac{(-1+u+\alpha)r_1}{2u(-1+\beta)q_1},$$

$$D_{2o}^{SBH^*} = \frac{r_1}{2\beta q_1 - 2q_2},\ D_{1r}^{SBH^*} = \frac{1}{2u}\left(u + \frac{(-1+u+\alpha)r_1}{(-1+\beta)q_1} - \frac{2u\left(-1 + \frac{q_1}{2(1+\beta)q_1 - 2q_2}\right)r_1}{-\beta q_1 + q_2}\right),$$

$$\pi_1^{SBH^*} = \frac{2u(1-\beta)(1-\alpha)r_1^2q_1 + (\beta q_1 - q_2)[u(1-\beta)q_1 + (1+u-\alpha)r_1][u(1-\beta)q_1 - (1-u-\alpha)r_1]}{4u(1-\beta)q_1(\beta q_1 - q_2)},\ \pi_2^{SBH^*} = \frac{q_1r_1^2u}{4(q_2 - q_1\beta)(q_2 - q_1(1+\beta))}.$$

(2) When $\beta q_1 < q_2$

First, we know that $\pi_1^{SBH} = p_1\left(1 - \frac{p_1 - p_2}{u(q_1 - q_2)}\right) + (1-\alpha)r_1\left(\frac{p_2 - ur_1}{u(q_2 - \beta q_1)} - \frac{r_1}{\beta q_1}\right)$, $\pi_2^{SBH} = p_2\left(\frac{p_1 - p_2}{u(q_1 - q_2)} - \frac{p_2 - ur_1}{u(q_2 - \beta q_1)}\right)$. Then, we can obtain $p_1^{SBH^*} = \frac{u(q_1 - q_2)(2(1-\beta)q_1 + r_1)}{(4-3\beta)q_1 - q_2}$ and $p_2^{SBH^*} = \frac{u(q_1 - q_2)(q_2 + 2r_1 - \beta q_1)}{(4-3\beta)q_1 - q_2}$ through $\frac{\partial \pi_1^{SBH}}{\partial p_1} = 0$ and $\frac{\partial \pi_2^{SBH}}{\partial p_2} = 0$. By substituting $p_1^{SBH^*}$ and $p_2^{SBH^*}$, we have $D_{1o}^{SBH^*} = \frac{2(-1+\beta)q_1 - r_1}{(-4+3\beta)q_1 + q_2}$, $D_{2o}^{SBH^*} = \frac{(-1+\beta)q_1(\beta q_1 - q_2 - 2r_1)}{(\beta q_1 - q_2)((-4+3\beta)q_1 + q_2)}$, $D_{1r}^{SBH^*} = \frac{(q_1 - q_2)(\beta q_1 - q_2 - 2r_1)}{((-4+3\beta)q_1 + q_2)(-\beta q_1 + q_2)} - \frac{q_2r_1}{\beta q_1(-\beta q_1 + q_2)}$, $\pi_1^{SBH^*} \frac{u(q_1 - q_2)(-2(-1+\beta)q_1 + r_1)^2}{((4-3\beta)q_1 - q_2)^2} + (1-\alpha)r_1\left(-\frac{r_1}{\beta q_1} + \frac{\frac{u(q_1 - q_2)(\beta q_1 - q_2 - 2r_1)}{(-4+3\beta)q_1 + q_2} - ur_1}{u(-\beta q_1 + q_2)}\right)$, and $\pi_2^{SBH^*} = \frac{u(1-\beta)q_1(q_1 - q_2)(q_2 + 2r_1 - \beta q_1)^2}{(q_2 - \beta q_1)((-4+3\beta)q_1 + q_2)^2}$.

Proof of Proposition 4.2.

To ensure that consumers are willing to rent product 2, from $\theta_7 < \theta_6$, we obtain $0 < r_2 < 1$ if $0 < u \leqslant u^{SBL1}$ and $0 < r_2 < r_2^{SBL1}$ if $u^{SBL1} < u < 1$. To ensure that the purchase demand of product 2 exists, from $\theta_6 < \theta_1$, we have $0 < r_2 < r_2^{SBL2}$ if $0 < u \le u^{SBL2}$ and $r_2 > 0$ if $u^{SBL2} < u < 1$. Because $u^{SBL1} < u^{SBL2}$ and $r_2^{SBL1} < r_2^{SBL2}$, we obtain $0 < r_2 < r_2^{SBL2}$ if $0 < u \leqslant u^{SBL2}$ and $0 < r_2 < r_2^{SBL1}$ if $u^{SBL2} < u < 1$ to ensure that both rental and purchase demands exist. In other words, there exist consumers willing to rent and buy product 2 only if $0 < r_2 < r_2^{SBL}$.

$$u^{SBL1} = \frac{2\beta q_1 - 2\alpha\beta q_1 - 2\beta q_2 + 2\alpha\beta q_2}{4q_1 - 2\beta q_1 - q_2 - \beta q_2 - \beta q_1q_2 + \beta^2q_1q_2 + \beta q_2^2 - \beta^2q_2^2},$$

$$r_2^{SBL1} = \frac{-\beta q_1q_2u + \beta^2q_1q_2u + \beta q_2^2u - \beta^2q_2^2u}{2\beta q_1 - 2\alpha\beta q_1 - 2\beta q_2 + 2\alpha\beta q_2 - 4q_1u + 2\beta q_1u + q_2u + \beta q_2u},$$

$$u^{SBL2} = \frac{-2q_1 + 2\alpha q_1 + q_2 - \alpha q_2 + \beta q_2 - \alpha\beta q_2}{-2q_1 + 2\beta q_2 - q_1q_2 + \beta q_1q_2 + \beta q_2^2 - \beta^2q_2^2},$$

$$r_2^{SBL2} = \frac{-q_1q_2u + \beta q_1q_2u + \beta q_2^2u - \beta^2q_2^2u}{-2q_1 + 2\alpha q_1 + q_2 - \alpha q_2 + \beta q_2 - \alpha\beta q_2 + 2q_1u - 2\beta q_2u},$$

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$$r_2^{SBL} = \begin{cases} r_2^{SBL1}, \ u^{SBL1} < u < 1; \\ r_2^{SBL2}, \ 0 < u \leqslant u^{SBL1}. \end{cases}$$

Proof of Lemma 4.3.

First, we know that $\pi_1^{SBL} = p_1\left(1-\dfrac{p_1-p_2}{u(q_1-q_2)}\right)$, $\pi_2^{SBL} = p_2\left(\dfrac{p_1-p_2}{u(q_1-q_2)} - \dfrac{-p_2+ur_2}{u(-1+\beta)q_2}\right)+(1-\alpha)r_2\left(\dfrac{-p_2+ur_2}{u(-1+\beta)q_2}-\dfrac{r_2}{\beta q_2}\right)$. Then, we can obtain $p_1^{SBL^*} = \dfrac{(q_1-q_2)(2uq_1-2u\beta q_2+(1+u-\alpha)r_2)}{4q_1-(1+3\beta)q_2}$ and $p_2^{SBL^*} = \dfrac{(q_1-q_2)(u(1-\beta)q_2+2(1+u-\alpha)r_2)}{4q_1-(1+3\beta)q_2}$ through $\begin{cases} \dfrac{\partial \pi_1^{SBL}}{\partial p_1} = 0, \\ \dfrac{\partial \pi_2^{SBL}}{\partial p_2} = 0. \end{cases}$

By substituting, we have

$$D_{1o}^{SBL^*} = \frac{2uq_1-2u\beta q_2+(1+u-\alpha)r_2}{u(4q_1-(1+3\beta)q_2)},\ D_{2o}^{SBL^*} = \frac{q_1((u-u\beta)q_2-2(1-u-\alpha)r_2)+q_2(-u(1-\beta)\beta q_2+(1-\alpha+\beta-2u\beta-\alpha\beta)r_2)}{u(1-\beta)q_2(4q_1-(1+3\beta)q_2)},$$

$$D_{2r}^{SBL^*} = \frac{(q_1-q_2)(u(1-\beta)q_2+2(1+u-\alpha)r_2)-ur_2(4q_1-(1+3\beta)q_2)}{uq_2(1-\beta)(4q_1-(1+3\beta)q_2)} - \frac{r_2}{\beta q_2},$$

$$\pi_1^{SBL^*} = \frac{(q_1-q_2)(2uq_1-2u\beta q_2+(1+u-\alpha)r_2)^2}{u(-4q_1+(1+3\beta)q_2)^2},$$

and

$$\pi_2^{SBL^*} = \frac{1}{u(1-\beta)\beta q_2(-4q_1+(1+3\beta)q_2)^2}(\beta(q_1-q_2)(u(-1+\beta)q_2-2(1+u-\alpha)r_2)(q_1(u(-1+\beta)q_2-2(-1+u+\alpha)r_2)+ q_2(-u(-1+\beta)\beta q_2+(-1+\alpha-\beta+2u\beta+\alpha\beta)r_2))-(1-\alpha)(4q_1-(1+3\beta)q_2)r_2(q_1(u(-1+\beta)\beta q_2-2(u(-2+\beta)+\beta-\alpha\beta)r_2)- q_2(u(-1+\beta)\beta q_2+(u-2\beta+u\beta+2\alpha\beta)r_2))).$$

Proof of Proposition 5.1.

(1) When $\beta q_1 > q_2$

a) The profit gaps of OEMs between mode SBH and mode SN are as follows:

$$\Delta\pi_1^{SBH} = \pi_1^{SBH^*} - \pi_1^{SN^*} = \frac{2u(1-\beta)(1-\alpha)r_1^2q_1+(\beta q_1-q_2)[u^2((1-\beta)q_1+r_1)^2-(1-\alpha)^2r_1^2]}{4u(1-\beta)q_1(\beta q_1-q_2)} - \frac{4uq_1^2(q_1-q_2)}{(4q_1-q_2)^2},$$

$$\Delta\pi_2^{SBH} = \pi_2^{SBH^*} - \pi_2^{SN^*} = \frac{q_1r_1^2u}{4(q_2-q_1\beta)(q_2-q_1(1+\beta))} - \frac{uq_1(q_1-q_2)q_2}{(4q_1-q_2)^2}.$$

Let us describe function A as $2u(1-\beta)(1-\alpha)r_1^2q_1+(\beta q_1-q_2)[u^2((1-\beta)q_1+r_1)^2-(1-\alpha)^2r_1^2](4q_1-q_2)^2-16(1-\beta)u^2q_1^3\cdot(q_1-q_2)(\beta q_1-q_2)=0$. We obtain $\Delta\pi_1^{SBH}>0$ from $2u(1-\beta)(1-\alpha)r_1^2q_1+(\beta q_1-q_2)[u^2((1-\beta)q_1+r_1)^2-(1-\alpha)^2r_1^2](4q_1-q_2)^2 > 16(1-\beta)u^2q_1^3(q_1-q_2)(\beta q_1-q_2)$. Otherwise, $\Delta\pi_1^{SBH}\leqslant 0$. Furthermore, r_1^{SBH3} and r_1^{SBH4} are a pair of solutions of function A.

b) Taking the first derivative of $\Delta\pi_2^{SBH}$ with respect to μ, we obtain $\dfrac{\partial\Delta\pi_2^{SBH}}{\partial\mu}>0$ if $r_1>r_1^{SBH5}$ and $\dfrac{\partial\Delta\pi_2^{SBH}}{\partial\mu}<0$ if $r_1<r_1^{SBH5}$, where $r_1^{SBH5}=\dfrac{2\sqrt{q_2(q_1-q_2)(\beta q_1-q_2)((1+\beta)q_1-q_2)}}{4q_1-q_2}$. Moreover, when $u=0$, $\Delta\pi_2^{SBH}=0$; thus, $\Delta\pi_2^{SBH}<0$ if $r_1<r_1^{SBH5}$ and $\Delta\pi_2^{SBH}>0$ if $r_1>r_1^{SBH5}$.

c) $\Delta p_1^{SBH} = p_1^{SBH^*}-p_1^{SN^*} = \dfrac{1}{2}((u-u\beta)q_1+(1+u-\alpha)r_1)-\dfrac{2uq_1(q_1-q_2)}{4q_1-q_2}$ from Lemmas 4.1 and 4.2. Through calculation, $\Delta p_1^{SBH}>0$ if $r_1>r_1^{SBH6}$; otherwise, $\Delta p_1^{SBH}\leqslant 0$ if $r_1\leqslant r_1^{SBH6}$, where $r_1^{SBH6}=\dfrac{4uq_1(q_1-q_2)-(u-u\beta)q_1(4q_1-q_2)}{(1+u-\alpha)(4q_1-q_2)}$.

d) $\Delta p_2^{SBH} = p_2^{SBH^*}-p_2^{SN^*} = \dfrac{uq_1r_1}{2(1+\beta)q_1-2q_2}-\dfrac{uq_2(q_1-q_2)}{4q_1-q_2}$. Similarly, $\Delta p_2^{SBH}>0$ if $r_1>r_1^{SBH7}$; otherwise, $\Delta p_1^{SBH}\leqslant 0$ if $r_1\leqslant r_1^{SBH7}$, where $r_1^{SBH7}=\dfrac{2q_2(q_1-q_2)((1+\beta)q_1-q_2)}{(4q_1-q_2)q_1}$.

(2) When $\beta q_1 < q_2$

The profit gaps of the OEMs between mode SBH and mode SN are as follows:

$$\Delta\pi_1^{SBH} = \pi_1^{SBH^*} - \pi_1^{SN^*} = \frac{u(q_1-q_2)(-2(-1+\beta)q_1+r_1)^2}{((4-3\beta)q_1-q_2)^2}+(1-\alpha)r_1\left(-\frac{r_1}{\beta q_1}+\frac{\dfrac{u(q_1-q_2)(\beta q_1-q_2-2r_1)}{(-4+3\beta)q_1+q_2}-ur_1}{u(-\beta q_1+q_2)}\right)-\frac{4uq_1^2(q_1-q_2)}{(4q_1-q_2)^2},$$

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$$\Delta\pi_2^{SBH}=\pi_2^{SBH^*}-\pi_2^{SN^*}=\frac{u(1-\beta)q_1(q_1-q_2)(-\beta q_1+q_2+2r_1)^2}{(q_2-\beta q_1)((4-3\beta)q_1-q_2)^2}-\frac{uq_1(q_1-q_2)q_2}{(4q_1-q_2)^2}.$$

a) Let us take the derivative of $\Delta\pi_1^{SBH}$ with respect to u. We find that $\frac{\partial\Delta\pi_1^{SBH}}{\partial\mu}>0$ if $r_1^{SBH8}<r_1$ and $\frac{\partial\Delta\pi_1^{SBH}}{\partial\mu}<0$ if $r_1^{SBH8}>r_1$, where $r_1^{SBH8}=\frac{2q_1{}^2\beta-2q_1q_2\beta}{4q_1-q_2}$. From Proposition 4.2, we know that rental demand exists only if $r_1<r_1^{SBH2}$. Therefore, $\frac{\partial\Delta\pi_1^{SBH}}{\partial\mu}<0$ always holds in the range of $r_1<r_1^{SBH2}$ as $r_1^{SBH2}<r_1^{SBH8}$. When $\mu=0$, $\Delta\pi_1^{SBH}=(1-\alpha)r_1\left(-\frac{r_1}{\beta q_1}+\frac{\frac{(q_1-q_2)(\beta q_1-q_2-2r_1)}{(-4+3\beta)q_1+q_2}-r_1}{(-\beta q_1+q_2)}\right)<0$; accordingly, $\Delta\pi_1^{SBH}<0$ always holds if $0<\mu<1$.

b) We find that $\frac{\partial\Delta\pi_2^{SBH}}{\partial\mu}>0$ if $r_1^{SBH9}<r_1$ and $\frac{\partial\Delta\pi_2^{SBH}}{\partial\mu}<0$ if $r_1^{SBH9}>r_1$, where $r_1^{SBH9}=\frac{\sqrt{q_2(q_2-\beta q_1)}}{2(4q_1-q_2)\sqrt{1-\beta}}-(q_2-\beta q_1)$. From Proposition 4.2, we know that rental demand exists only if $r_1<r_1^{SBH2}$. Based on this relation, $\frac{\partial\Delta\pi_2^{SBH}}{\partial\mu}<0$ in the range of $r_1<r_1^{SBH2}$ because $r_1^{SBH2}<r_1^{SBH9}$. Moreover, $\Delta\pi_2^{BSH}=0$ at the point of $\mu=0$, which implies $\Delta\pi_2^{BSH}<0$ in the range of $r_1<r_1^{SBH2}$.

c) $\Delta p_1^{SBH}=p_1^{SBH^*}-p_1^{SN^*}=\frac{u(q_1-q_2)(2(1-\beta)q_1+r_1)}{(4-3\beta)q_1-q_2}-\frac{2uq_1(q_1-q_2)}{4q_1-q_2}$ from Lemmas 4.1 and 4.2. Through calculation, $\Delta p_1^{SBH}>0$ if $r_1>r_1^{SBH10}$; otherwise, $\Delta p_1^{SBH}\leqslant 0$ if $r_1\leqslant r_1^{SBH10}$, where $r_1^{SBH10}=\frac{2q_1{}^2\beta-2q_1q_2\beta}{4q_1-q_2}$. From Proposition 4.2, we know that rental demand exists only if $r_1<r_1^{SBH2}$. However, it always holds that $r_1^{SBH2}<r_1^{SBH10}$, which means $\Delta p_1^{SBH}<0$ in the range of $r_1<r_1^{SBH2}$.

d) $\Delta p_2^{SBH}=p_2^{SBH^*}-p_2^{SN^*}=\frac{u(q_1-q_2)(q_2+2r_1-\beta q_1)}{(4-3\beta)q_1-q_2}-\frac{uq_2(q_1-q_2)}{4q_1-q_2}$. Similarly, $\Delta p_2^{SBH}>0$ if $r_1>r_1^{SBH11}$; otherwise, $\Delta p_1^{SBH}\leqslant 0$ if $r_1\leqslant r_1^{SBH11}$. From Proposition 4.2, we know that rental demand exists only if $r_1<r_1^{SBH2}$. However, it always holds that $r_1^{SBH2}<r_1^{SBH11}$, which means $\Delta p_2^{SBH}<0$ in the range of $r_1<r_1^{SBH2}$.

Proof of Proposition 5.2

(1) Simplifying $\Delta\pi_1^{SBL}=\pi_1^{SBL^*}-\pi_1^{SN^*}=\frac{(q_1-q_2)(2uq_1-2u\beta q_2+(1+u-\alpha)r_2)^2}{u(4q_1-(1+3\beta)q_2)^2}-\frac{4uq_1^2(q_1-q_2)}{(4q_1-q_2)^2}$, we obtain $(1+u-\alpha)r_2(4q_1-q_2)>2u\beta q_2(q_1-q_2)$. Consequently, $\Delta\pi_1^{SBL}>0$ if $r_2>r_2^{SBL3}$ and $\Delta\pi_1^{SBL}<0$ if $r_2<r_2^{SBL3}$, where $r_2^{SBL3}=\frac{2u\beta q_2(q_1-q_2)}{(1+u-\alpha)(4q_1-q_2)}$.

(2) It is known that

$$\begin{aligned}\Delta\pi_2^{SBL}=&\pi_2^{SBL^*}-\pi_2^{SN^*}=\frac{1}{u(1-\beta)\beta q_2(-4q_1+(1+3\beta)q_2)^2}(\beta(q_1-q_2)(u(-1+\beta)q_2-2(1+u-\alpha)r_2)(q_1(u(-1+\beta)q_2-2(-1+u+\alpha)r_2)+\\&q_2(-u(-1+\beta)\beta q_2+(-1+\alpha-\beta+2u\beta+\alpha\beta)r_2))-(1-\alpha)(4q_1-(1+3\beta)q_2)r_2(q_1(u(-1+\beta)\beta q_2-2(u(-2+\beta)+\beta-\alpha\beta)r_2)-\\&q_2(u(-1+\beta)\beta q_2+(u-2\beta+u\beta+2\alpha\beta)r_2)))-\frac{uq_1(q_1-q_2)q_2}{(4q_1-q_2)^2}.\end{aligned}$$

From $\Delta\pi_2^{SBL}=0$, we obtain $b(4q_1-q_2)^2-(1-\beta)\beta u^2q_2^2q_1(q_1-q_2)(-4q_1+(1+3\beta)q_2)^2=0$, where

$$\begin{aligned}b=&\beta(q_1-q_2)(u(-1+\beta)q_2-2(1+u-\alpha)r_2)(q_1(u(-1+\beta)q_2-2(-1+u+\alpha)r_2)+q_2(-u(-1+\beta)\beta q_2+(-1+\alpha-\beta+2u\beta+\alpha\beta)r_2))-\\&(1-\alpha)(4q_1-(1+3\beta)q_2)r_2(q_1(u(-1+\beta)\beta q_2-2(u(-2+\beta)+\beta-\alpha\beta)r_2)-q_2(u(-1+\beta)\beta q_2+(u-2\beta+u\beta+2\alpha\beta)r_2)).\end{aligned}$$

As a result, $\Delta\pi_2^{SBL}>0$ if $b(4q_1-q_2)^2-(1-\beta)\beta u^2q_2^2q_1(q_1-q_2)(-4q_1+(1+3\beta)q_2)^2>0$, otherwise, $\Delta\pi_2^{SBL}\leqslant 0$. r_2^{SBL4} and r_2^{SBL5} are a pair of solutions of function B, and B is $b(4q_1-q_2)^2-(1-\beta)\beta u^2q_2^2q_1(q_1-q_2)(-4q_1+(1+3\beta)q_2)^2=0$.

(3) $\Delta p_1^{SBL}=p_1^{SBL^*}-p_1^{SN^*}=\frac{(q_1-q_2)(2uq_1-2u\beta q_2+(1+u-\alpha)r_2)}{4q_1-(1+3\beta)q_2}-\frac{2uq_1(q_1-q_2)}{4q_1-q_2}$ from Lemmas 4.1 and 4.3. Through calculation, $\Delta p_1^{SBH}>0$ if $r_1>r_1^{SBH6}$; otherwise, $\Delta p_1^{SBH}\leqslant 0$ if $r_1\leqslant r_1^{SBH6}$, where $r_1^{SBH6}=\frac{2uq_1(4q_1-(1+3\beta)q_2)}{4q_1-q_2}-2uq_1+2u\beta q_2$.

(4) $\Delta p_2^{SBL}=p_2^{SBL^*}-p_2^{SN^*}=\frac{(q_1-q_2)(u(1-\beta)q_2+2(1+u-\alpha)r_2)}{4q_1-(1+3\beta)q_2}-\frac{uq_2(q_1-q_2)}{4q_1-q_2}$. Similarly, $\Delta p_2^{SBH}>0$ if $r_1>r_1^{SBH7}$; otherwise, $\Delta p_1^{SBH}\leqslant 0$ if $r_1\leqslant r_1^{SBH7}$, where $r_1^{SBH7}=\frac{uq_2(4q_1-(1+3\beta)q_2)}{2(1+u-\alpha)(4q_1-q_2)}-\frac{u(1-\beta)q_2}{2(1+u-\alpha)}$.

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