Socially responsible donation decision analysis in a supply chain under government subsidy

Dongxu Du, Feng Yang, and Manman Wang

International Institute of Finance, School of Management, University of Science and Technology of China, Hefei 230026, China

Correspondence: Manman Wang, E-mail: wmm2016@mail.ustc.edu.cn
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Graphical abstract

The research framework of socially responsible donation supply chain under different models.

Public summary

- The study investigates a specific CSR activity that links donation behavior with consumer purchase under government subsidy.
- To balance corporate income and social welfare, we explore how to design donation and product pricing effectively using the Stackelberg game model.
- The study provides decision supports for firms and governments to improve firm performance and social welfare by choosing appropriate policies.

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Abstract: As an important form for firms to demonstrate social responsibility, socially responsible donation (SRD) is becoming increasingly widespread and attracting more attention. It is important to encourage firms to effectively undertake social responsibilities and improve social welfare. Recently, it has become very popular for firms to demonstrate their social responsibility through SRD campaigns. With the aim of solving the decision-making problem of a retailer-led socially responsible supply chain considering government subsidy, this study established a three-stage Stackelberg game model. By analyzing the impact of pricing and donation decisions on operations and management, either with or without government subsidy, we found that to achieve better performance, retail firms will choose to implement donation activities. However, it is not the case that the higher the donation, the better the performance. In addition, there is a gap: without government subsidy, social welfare is not optimal when members’ performance in the supply chain is maximized, and more donations are needed to maximize social welfare. This study proves that government subsidy can encourage supply chain members to generate more donations, while improving the performance of the supply chain and social welfare. We propose an optimal subsidy rate to eliminate the gap and maximize social welfare. We also analyze the impact of external parameters, including the cost parameter and consumer preference level, on the equilibrium results. Finally, we extend this research to provide management insights for businesses under uncertain market sizes and centralized decision-making scenarios.

Keywords: corporate social responsibility; government subsidy; socially responsible donation; consumer preference

CLC number: F252

1 Introduction

Corporate social responsibility (CSR) has become a hot topic in all sectors of society. Recent researches show that consumers tend to buy products with social responsibility attributes[3, 5]. Most managers hold that CSR can not only improve the competitiveness of firms but also be crucial to the success of future business[9]. Stakeholders also expect firms to incorporate social responsibility into their business processes[9]. Moreover, implementing of CSR activities has become a feasible strategy for firms to improve corporate brand image and enhance their marketing ability[6, 8]. This study focuses on socially responsible donation (SRD) campaigns, a specific practical form of CSR whereby firms can donate a certain amount of money for social welfare projects when consumers buy a product or service. Socially responsible donation campaigns connect corporate donation activities with consumers’ purchases of products, and this has become a popular way for firms to participate in CSR[1, 9]. For example, in 2008, Starbucks donated 5 cents from each RED exclusive beverage sold to the Global Fund to fight AIDS, tuberculosis, and malaria[6]. Recently, Apple announced its donation of part of the proceeds from purchases of each iPhone RED to the Global Fund for COVID-19 Response[1]. In 2019, firms’ spending on SRD only in the United States reached $2.23 billion, increasing by 4.6% as compared to 2018[9]. Although many enterprises in supply chain networks demonstrate their social responsibility image through SRD, the successful implementation of this donation strategy has not been fully studied.

According to existing research, the implementation of SRD is mainly based on two possible motivations. One is profit-driven, expecting that SRD can increase profits[9], while the other is altruistic motivation, pursuing the maximization of social welfare to improve society through social responsibility activities[19]. According to their different motivations, firms can be divided into profit-oriented and welfare-oriented firms. In the retail industry, welfare-oriented retailers always choose to implement more donation activities to significantly improve social welfare. Profit-oriented retailers are more cautious in decision-making regarding donation size[19] and often contribute less to social welfare than welfare-oriented firms. Therefore, this study is to provide decision-making support for profit-oriented firms when they undertake social responsibility and encourage them to make more contributions to social welfare.

There are conflicting views on whether SRD campaigns can optimize corporate economic performance. On the one hand, consumers may perceive CSR attributes from purchase-
triggered donation and thus exhibit favorable purchasing behavior, which tends to be positively correlated with the amount of donation\textsuperscript{(10, 11)}. On the other hand, other studies indicate that this SRD campaigns may have the opposite effect because more donations will lead to higher costs and reduce enterprise profitability\textsuperscript{(12)}. Excessive focus on social responsibility may harm stakeholders’ interests. Some researchers have made efforts to solve this puzzle through empirical studies, finding that the key elements in SRD campaigns for strategic success are retail price and the amount of SRD per unit. These two factors are important because they directly correlate with the total amount of donation, expenditure on public welfare activities, and the degree of improvement in corporate benefits, and they can be controlled in the managers’ decision-making processes\textsuperscript{(13)}. Our contribution is the use of the operation management method to support enterprise decision-making in this regard. In addition, an increase in the indirect investment cost of donation may be another important factor that hinders profit-oriented enterprises from increasing donations. From the perspective of corporate practice, a complete SRD strategy must consider additional processing and post-maintenance costs related to social responsibility projects\textsuperscript{(14)}. The money required for each social responsibility project has a certain limit. Excessive donation amounts lead to unreasonable allocation and waste of resources. With an increase in donation amounts, firms need to extend the scales of their social responsibility projects and pay greater preparation and maintenance costs.

Firms may increase retail prices to cover costs. However, existing research shows that this may not be a good solution because higher product prices increase the cost of consumers’ participation in social responsibility\textsuperscript{(10)} and reduce profits. Government subsidy is often considered as an effective strategy for improving the contribution of enterprises to social performance. For example, Chinese government subsidizes the costs of donations based on tax incentives for firms that donate property to charitable activities\textsuperscript{(6)}. The successful implementation of SRD to achieve a win-win situation between profit-oriented firms’ economic performance and social welfare is a common concern and expectation of governments, managers, and researchers. Therefore, it is of great management significance to explore the optimal donation and pricing decisions in the supply chain considering SRD campaigns under government subsidy. However, few studies use the operation management method to provide decision support for profit-oriented enterprises to implement such social responsibility activities. There have also been a few discussions about government subsidy for corporate donations. The specific impact of government subsidy on the SRD behavior of profit-oriented enterprises and social welfare remains to be further explored.

To address these gaps, this study aims to explore whether and how profit-oriented retailers integrate SRD campaigns into their business strategies in the context of government subsidy by establishing a Stackelberg game model. More precisely, we attempt to address the following key issues: First, does social responsibility donation provide economic benefits for profit-oriented firms? Second, is government subsidy effective in raising the donations of profit-oriented enterprises to a level similar to that of social welfare firms? Third, how do governments and enterprises make optimal decisions when considering donation subsidy? In addition, unlike previous studies that mainly focus on the production donation of firms, this study pays attention to the donation behavior related to sales volume and takes it as a basis to solve the abovementioned problems.

This study finds that SRD can realize profits only when consumers have high social responsibility preferences. Optimal pricing and donation decisions are available, but profit-oriented retailers’ decisions do not maximize social welfare. The existence of donation subsidy increases corporate profits, and ultimately, social welfare. However, excessive subsidy damage social welfare, and the government must set an appropriate donation subsidy rate. Extending studies have shown that larger market sizes and centralized decision-making are conducive toward increasing the positive effect of government subsidy. The remainder of this paper is organized as follows: Section 2 summarizes the relevant literature. Section 3 puts forward the basic assumptions of SRD behavior in the supply chain under the government subsidy and the related notation description, followed by Section 4, which gives more specific economic consequences when retailers adopt SRD campaigns with and without government subsidy. Section 5 presents an equilibrium analysis and discusses the impact of government subsidy on the supply chain. A few extensions are explored in Section 6. The final section concludes the paper and proffers management implications.

2 Literature review

Three main streams of research are closely related to our work. The first stream focuses on CSR supply chains, while the second stream is related to the SRD campaigns of firms, and the last stream concentrates on government donation subsidy policies.

Our work is based on the previous literature on CSR supply chains and contributes to this accordingly. Many studies have attempted to clarify the benefits of firms taking CSR activities in a supply chain. For example, Flammer\textsuperscript{(19)} proved through empirical research that the economic performance of firms was significantly affected by social responsibility performance. Albuquerque et al.\textsuperscript{(20)} conducted an empirical study on the causal relationship between CSR and corporate financial performance, and proved their positive relationship. With the deepening research on social responsibility, the social responsibility dimension has been gradually introduced into supply chain management in recent decades. Ni et al.\textsuperscript{(21)} analyzed a two-stage supply chain model with social responsibility and discussed the distribution of social responsibility among members of the supply chain. By including consumer surplus in the profit function, Panda\textsuperscript{(22)} proved that firms with a sense of social responsibility in the supply chain could achieve higher performance by adopting a game-theoretic model. These studies only regarded social responsibility as a general concern for consumer surplus or as a type of ex-
penditure cost. Instead, we focus on specific CSR behaviors.

Research that considers SRD campaigns has been attracting increasing attention in recent years. Servaes and Tamayo[23] pointed out that consumers’ perceptions of firms’ participation in social responsibility activities have a significantly positive impact on their purchasing decisions. As a manifestation of CSR, SRD can often build goodwill, reputation, image, or status in a market[24], thus becoming an emerging field of research. In most of the current literature, studies on SRD have focused on the impact of different factors on the effectiveness of SRD campaigns through empirical research, such as price discounts[25] and unethical behavior of others[26]. Xu and Li[27] used a game-theoretic model to study an e-retailer’s incentives to share demand information with its supplier, who may implement SRD. In contrast to these studies, the focus of our research is to examine the efficiency of product pricing, the amount of SRD per unit, and donation subsidy through a tractable theoretical framework.

This study also involves the literature on the design of government incentive policies. Many studies have proven that government subsidy can achieve the desired effect while adjusting firm strategy[28, 29]. Aiming at the relationship and influence between government subsidy and CSR, Chu and Sappington[30] studied the relationship between the design of additional fiscal policies and the willingness of power firms to provide maximum energy efficiency services and social undertakings. Liu et al.[31] studied the government’s decision to subsidize CSR investments in the supply chain. However, these studies did not explore the relationship between government subsidy and specific social responsibility activities. In terms of socially responsible behavior, Arya and Mittendorf[32] studied the impact of government subsidy on product donations by retail firms and found that the subsidy could reduce the double marginalization of the supply chain and improve corporate performance. Chu et al.[33] constructed a two-phase discount newspaper vendor model to investigate the effect of tax incentives on optimal pricing and quantity decisions in a supply chain that used inventory donations as a CSR tool. These publications focus on firms using part of their outputs for social welfare, assuming that total donations have no direct relationship with product sales. In summary, the design of a reasonable subsidy mechanism from the perspective of maximizing social welfare and motivating members of the supply chain to effectively implement sales-related SRD remains to be studied.

Compared to the abovementioned studies, this study first reflects the real CSR activities of firms and contributes to the implementation of CSR activities. Second, we establish a model to analyze the relationship between social welfare and SRD campaigns. Third, the previous literature on CSR donation behaviors mainly focus on output donation, while that on SRD mainly focuses on empirical research. Thus, the previous literature cannot provide support for enterprises to effectively implement SRD campaigns and the government to implement incentive policies. This study also fills this research gap in the literature. Moreover, to better understand the relationship between government subsidy and CSR, we use a three-stage Stackelberg game model to analyze and obtain optimal supply chain decisions. The research results can provide a reference for the government to formulate subsidy policies and long-term strategic decisions for leading firms. Finally, the implications of government policies and firm management are discussed.

3 Model setup

In this section, we describe the basic setup of the model and list the assumptions about the products, retailers, suppliers, and consumers. On this basis, we consider two models: one without government subsidy and the other with government subsidies. Notably, this study only focuses on government subsidy for the indirect costs involved in the implementation of SRD campaigns. For example, the government provides publicity channels and funds to subsidize the costs of firms implementing social responsibility. In the decision system, we consider a three-stage Stackelberg game formed by a single retailer, supplier, and government. Only one commodity is produced in a supply chain. The retailer first declares donations and the supplier supplies the retailer with goods in one lump sum at a wholesale price. The retailer sells the products to customers at a fixed price. The government provides subsidy to socially responsible enterprises to encourage them to undertake social responsibility. Consistent with previous studies, in the game between donation firms and the government, we assume that the government is in an absolutely dominant position. In other words, the government occupies an absolute optimal position in the tripartite game. Table 1 summarizes the notation for the decision variables and parameters.

We assume that $d$ is a function of the indirect costs of SRD campaigns and has the form $d = \sqrt{\beta/\mu}$, where $k$ is a cost parameter. Consistent with the assumption commonly used in the existing literature[24, 29], the indirect cost of social responsibility increases with SRD per unit. This is because SRD campaigns have a positive impact on the product demand of consumers with socially responsible preferences, thus increasing the total amount of the final donation. An excessive donation

<table>
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<tr>
<th>Table 1. Notations of the model.</th>
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<tr>
<td><strong>Parameters</strong></td>
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<tr>
<td>$\beta$ The degree of consumer preference for social responsibility</td>
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<tr>
<td>$k$ Cost parameter for the investment of highlighting CSR activities</td>
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<tr>
<td>$q$ Market demand</td>
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<tr>
<td>$\alpha$ Market size</td>
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<td>$I_d$ Indirect costs of socially responsible donation</td>
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<td>$\pi$ Profit of the player</td>
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<td><strong>Decisions</strong></td>
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<tr>
<td>$d$ Socially responsible donation (SRD) amount per unit sold</td>
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<tr>
<td>$p$ Retail price per unit product</td>
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<tr>
<td>$w$ Supplier’s unit wholesale price</td>
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<tr>
<td>$\tau$ The rate of government subsidy for the cost of SRD campaigns</td>
</tr>
<tr>
<td><strong>Superscripts/Subscripts</strong></td>
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<tr>
<td>$i$ Different models, no subsidy ($i = N$) and indirect cost subsidy ($i = I$)</td>
</tr>
<tr>
<td>$j$ Different players, the government ($j = G$), the supplier ($j = s$), and the retailer ($j = r$)</td>
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</table>
amount for a social responsibility project will lead to unreasonable allocation and waste of resources, which is not conducive to the improvement of social welfare. In this case, the scale of the corresponding social responsibility projects will expand, and the firm will need to pay more preparation and maintenance costs. For instance, Apple RED previously only participated in disease-fighting social responsibility programs in the United States, but with the increase in donation amounts, its social responsibility programs expanded to a global scale. Therefore, the human resource, financial, management, post-project evaluation, and subsequent maintenance costs required in the implementation process due to geographical distance language communication and management of donation implementation will naturally increase with the expansion of the program scope. \( I_r \) represents the cost of organizing an activity related to SRD, and it meets the law of marginal cost, namely, \( I_r' > 0, I_r' < 0 \).

As CSR is the focus of this study, we suppose no shortage and unsalable products in the supply chain. Donation, as a tool of social responsibility, can increase brand loyalty, by making consumers feel warm, as well as brand awareness\(^8\), which further increases consumer demand for products. Therefore, the retailer uses SRD as a corporate social practice and highlights its CSR by arranging investments related to social work to increase consumer demand. Specifically, the demand for products is \( q = a - p + \beta d \), where \( \beta \) represents consumers’ preference for socially responsible products\(^8,\(^1\). The impact of \( \beta \) will be discussed further later. Members of the decision-making system, following the Stackelberg game of full information, are risk-neutral and rational.

Regarding the government, we mainly discuss the subsidy policy that subsidizes the indirect cost of implementing the SRD campaign, which is expressed as \( \Phi = \tau I_r \). \( \tau \) is the rate of government subsidy. The government pursues the maximization of social welfare; therefore, its profit is expressed as social welfare, namely, \( \pi_\Phi = SW = \pi_r + \pi_s + CS - \Phi \). Its profit function includes the profit of the retail firm (item 1), profit of the supplier (item 2), consumer surplus under linear demand (item 3), and loss of government subsidy (item 4), where the consumer surplus is \( CS = q^2/2 \). Maintaining consistency with the hypothesis of previous studies\(^9\),\(^10\), we assume that the elastic parameters are sufficiently large \( k > \left[ 3(\beta - 1)^2 \right] / 8 \) to ensure that the profit function of firm is concave and that the expression has certain economic feasibility.

4 Model formulation and solution

4.1 Model N: Social responsibility supply chain without subsidy

In this section, we consider the basic situation of a CSR supply chain without government subsidy. As previously mentioned, only a single product is produced in the supply chain. The retailer first declares SRDs and makes the corresponding indirect investment to prepare and implement its social responsibility behavior to gain consumers’ preferences and increase demand. The supplier supplies goods to the retailer at a wholesale price in lump sum, and the retailer sells the products to customers to meet their demands. This situation can be better understood using a real example from Wal-Mart China. As a large retailer, Wal-Mart supermarket decided to upgrade its assistance to children in poor areas of China in 2011. Wal-Mart donated 5 cents to charity when customers bought a unit of designated products at their stores. In this situation, the retailer and supplier’s profits are as follows:

\begin{equation}
\pi_r^\Phi = (p - w - d)(a - p + \beta d) - kd^2, \tag{1}
\end{equation}

\begin{equation}
\pi_s^\Phi = (w - c)(a - p + \beta d). \tag{2}
\end{equation}

The social welfare without government subsidy is as follows:

\begin{equation}
SW^\Phi = \pi_r^\Phi + \pi_s^\Phi + CS. \tag{3}
\end{equation}

Based on the goal of maximizing corporate profits, we solve this game using backward induction, and the equilibrium results are presented as follows:

**Lemma 4.1.** Without subsidy, in Model N, the equilibrium results under the retailer profit maximization are as follows:

\begin{align*}
p^\Phi &= \frac{a(12k + \beta - 1) + c(4k + \beta - \beta^2)}{16k - (\beta - 1)^2}; \\
w^\Phi &= \frac{8ak + c(8k - (\beta - 1)^2)}{16k - (\beta - 1)^2}; \\
d^\Phi &= \frac{(a - c)(\beta - 1)}{16k - (\beta - 1)^2}; \\
\pi_r^\Phi &= \frac{32k^2(a - c)^2}{16k - (\beta - 1)^2}; \\
\pi_s^\Phi &= \frac{k(a - c)^2}{16k - (\beta - 1)^2}.
\end{align*}

Under this condition, optimal decisions exist for the amount of SRD per unit and pricing. In addition, the decision result \( d^\Phi \) increases with the degree of consumers’ social responsibility preference \( \beta \).

**Corollary 4.1.** The equilibrium amount of SRD per unit is as follows: when \( \beta > 1, d^\Phi > 0 \); when \( \beta < 1, d^\Phi = 0 \).

Corollary 4.1 shows that in the absence of donation subsidy, the retailer uses SRD as a CSR tool only when consumers have a high degree of preference for social responsibility. In this case, the retailer’s profit first increases and then decreases with an increase in the amount of SRD per unit, and there is an optimal level. However, when consumers have a low preference for social responsibility, the retailer will not implement donation because the reputation of socially responsible behavior is inadequate to obtain sufficient demand growth. Under these conditions, the retailer has no financial motivation to engage in social responsibility. These results are oriented toward maximizing the retailer’s profit. However, when the retailer makes decisions based on maximizing social welfare and paying attention to the impact of SRDs on corporate profits and social welfare, we present the following findings.

**Proposition 4.1.** \( \frac{\partial \pi_r^\Phi}{\partial d} > 0 \) and \( \frac{\partial \pi_s^\Phi}{\partial d} > 0 \) if \( 0 < d < d^\Phi \), then \( \frac{\partial \pi_r^\Phi}{\partial d} > 0 \) and \( \frac{\partial \pi_s^\Phi}{\partial d} < 0 \) if \( d^\Phi < d < d^\Phi^* \), when social welfare is maximized, the optimal amount of SRD per unit is

\begin{equation}
d^\Phi^* = \frac{7(a - c)(\beta - 1)}{32k - 7(\beta - 1)^2}.
\end{equation}
From Proposition 4.1, we know that the social welfare level first increases and then decreases with an increase in SRD. Moreover, there is a gap between firms’ donation decisions in pursuit of profit and social welfare maximization. To illustrate Proposition 4.1 better, we use Fig. 1 as a graphical representation of our results. Comprehensively considering the conditions of the models and previous literature, we set the parameter values as follows: \(a = 100, \ c = 10, \ \beta = 2, \ \ k = 10\). Under these parameter settings, we show the retailer’s profit and social welfare as SRD changes in Fig. 1. Under the condition of no subsidy, we notice an important phenomenon: When the retailer maximizes the profit, the firm’s optimal SRD decision does not make the social welfare optimal (i.e., \(d^* < d^*_w\)). Social welfare can only be maximized by encouraging profit-oriented retailers to increase their donations. Therefore, it is necessary for the government to implement subsidy policies to reduce the indirect cost of retailers’ SRD campaigns. Through subsidy policy, retailers are encouraged to increase donations for improving social welfare and optimizing the allocation of social resources. The related analysis will be extensively discussed in the next section.

### 4.2 Model I: Government provides indirect investment cost subsidy

In addition to the donation itself, it is expensive to prepare and maintain social responsibility activities, and this is also a major obstacle for firms to make donations. To enhance social welfare, the government needs to choose a subsidy for the indirect cost of SRD campaigns to encourage retailers to increase the amount of SRD per unit. In this section, we consider a three-stage game in which the government subsidy is provided. The decision variable of the supplier is the wholesale price, the decision variables of the retailer are the retail price and amount of SRD per unit, and the decision variable of the government is the subsidy rate. The decision sequence is expressed as follows.

**Step 1:** The government determines \(\tau\).

**Step 2:** The retailer determines \(d\).

**Step 3:** The supplier determines \(w\).

**Step 4:** The retailer determines \(p\).

We limit the government subsidy of indirect costs to \(\tau \in (0, 1)\) to ensure that the government subsidy does not destroy the existing market mechanism. In this case, the government subsidy is less than the retailer’s indirect costs. According to the notation described in the previous section, we obtain the following profit function:

\[
\pi^r = (p - w - d)(a - p + \beta d) - kd^2 + k\tau d^2, \tag{4}
\]

\[
\pi^s = (w - c) * (a - p + \beta d), \tag{5}
\]

\[
\pi^r + \pi^s + CS^r - \Phi. \tag{6}
\]

By solving the game model via backward induction, we can obtain the equilibrium solutions and establish the following results.

**Lemma 4.2.** Given the investment subsidy \(\tau\), in Model I, the equilibrium results are as follows:

\[
p^r(\tau) = \frac{a - (a + c)\beta + (12ak + 4ck)\tau - 1 + c^2}{(\beta - 1)^2 + 16k(\tau - 1)},
\]

\[
w^r(\tau) = \frac{8k(a + c)(\tau - 1) + c^2(\beta - 1)}{(\beta - 1)^2 + 16k(\tau - 1)},
\]

\[
d^r(\tau) = \frac{k(\tau - 1) - (a - c)^2}{(\beta - 1)^2 + 16k(\tau - 1)},
\]

\[
\pi^r(\tau) = \frac{k(\tau - 1) - (a - c)^2}{(\beta - 1)^2 + 16k(\tau - 1)},
\]

\[
\pi^s(\tau) = \frac{k(\tau - 1)(6k\tau - 1) - (\beta - 1)^2}{(\beta - 1)^2 + 16k(\tau - 1)},
\]

\[
\pi^r(\tau) = \frac{k(\tau - 1)(6k\tau - 1) - (\beta - 1)^2}{(\beta - 1)^2 + 16k(\tau - 1)}.
\]

The only optimal amount of SRD and optimal pricing decisions indicate that the retailer can obtain the optimal benefits when using SRD as a CSR tool. Focusing on the equilibrium solutions in Lemma 4.2, the conclusion is similar to that shown in Corollary 4.1. When consumers have a high degree of social responsibility preference (\(\beta > 1\)), the retailer takes the initiative to implement SRD campaigns (i.e., \(d^*(\tau) > 0\)). In the presence of government subsidy, members of the supply chain still have sole optimal management decisions. Therefore, it is theoretically feasible for governments to subsidize donations.

## 5 Equilibrium analysis

### 5.1 The impact of donation subsidy on the optimal decisions

Based on the optimal decisions in the two models, we investigate how the government subsidy affects the SRD decisions, pricing decisions, and profits of members in the supply chain. It should be noted that we only consider the parameter range that satisfies the feasibility of all models. For clarity, we summarize the impact of government investment subsidy as follows:

**Proposition 5.1.** In Model I, the more the investment subsidy, the higher the amount of SRD, retail price, and the profit of the retailer; the more the investment subsidy, the higher the wholesale price and profit of the supplier. That is, \(\frac{\partial d^r(\tau)}{\partial \tau} > 0, \frac{\partial p^r(\tau)}{\partial \tau} > 0, \frac{\partial w^r(\tau)}{\partial \tau} > 0, \frac{\partial \pi^r(\tau)}{\partial \tau} > 0, \frac{\partial \pi^s(\tau)}{\partial \tau} > 0\).

The results show that when firms choose to implement so-
cial responsibility activities, the government donation subsidy can encourage them to increase their amount of SRD, although this also leads to an increase in the product price. This is because the value of the government subsidy is always less than the indirect cost of donation. As the number of SRD increases, the demand for the final product increases, and the supplier also increases the wholesale price to capture a higher profit. The retailer’s donation expenditures and procurement costs increase. At this time, the retail enterprise usually raises the product price to compensate for losses\textsuperscript{30}. However, charging higher prices to obtain more consumer surplus violates the rights and interests of consumers with social responsibility, to some extent. Therefore, when the government encourages firms to use SRD as a tool of CSR, the subsidy level must be set carefully. A lower subsidy cannot incentivize firms, while excessive subsidy may cause retail prices to rise. The government’s goal is to maximize social welfare; therefore, we next explored the impact of government investment subsidy on corporate performance and social welfare.

**Proposition 5.2.** When the government provides a subsidy, social welfare increases first and then decreases with an increase of the subsidy rate, and the optimal subsidy rate is 
\[
\tau^* = \frac{5}{7}.
\]

The above-stated results indicate that government subsidy is an effective measure for improving social welfare. However, there is a ceiling for the subsidy. Given the optimal subsidy rate, each participant has a unique equilibrium solution. We summarize the optimal decision for each participant and the final equilibrium profit in Table 2. From Table 2, we find that the final donation can meet the requirements for maximizing social welfare (i.e., \(d^o = d_{sw}^o = \frac{7(a-c)(\beta-1)}{32k-7(\beta-1)^2}\)) under the optimal subsidy rate. Corollary 5.1 summarizes these results.

**Corollary 5.1.** In Model I, the optimal subsidy rate can improve the level of social welfare and eliminate the gap to maximize social welfare.

From these results, we can conclude that government subsidy has significant positive effects. The government subsidy for the indirect costs of SRD campaigns can improve the level of social welfare and bridge this gap (the retailer’s optimal choice cannot achieve the optimal level of social welfare). Therefore, the government should subsidize the social responsibility activities of SRD campaigns. The object of the subsidy is an indirect cost that cannot be ignored when social responsibility firms implement SRD campaigns. The existence of this government subsidy leads to the maximization of social welfare. In other words, a profit-oriented retailer can make decisions that maximize social welfare when the government sets the optimal subsidy rate. Government support allocates social resources better, which is also in line with the rule that the market sometimes requires government financial support.

Next, the focus of this research is to visually demonstrate the specific impact of government subsidy on the economic performance and social welfare of firms through a numerical example. Referring to the parameters in the previous literature\textsuperscript{30, 34}, we set the parameter values as follows: \(a = 100, c = 10, \beta = 2,\) and \(k = 10.\) Then, according to the range of government subsidy rates, \(\tau \in [0, 0.9],\) we draw the trend charts of the members’ profits in the supply chain and social welfare influenced by the government subsidy rate in Fig. 2.

As shown in Fig. 2, within the scope of the government subsidy rate, the profits of supply chain members increase with the government subsidy rate. However, the level of social welfare initially increases and then suddenly drops after exceeding a certain threshold. The main reason is that government subsidy can increase a retailer’s donations, and consumers are willing to buy socially responsible products at a higher price. This, in turn, leads to an increase in market demand and, ultimately, an increase in the profit of the dominant retailer. As the number of orders increases, the supplier takes the opportunity to increase the wholesale price to obtain higher profits. The final result is that an increased subsidy will lead to higher commodity prices and better firm performance, but encroach on consumer surplus. As the government makes decisions from the perspective of maximizing social welfare, it is impossible to increase the subsidy indefinitely. Social welfare increases as the government subsidy rate increases and then decreases outside this range. These results are consistent with those of Proposition 5.2.

Compared to the situation without government subsidy, the level of SRD, retail price, and wholesale price are higher under government subsidy.

**Proposition 5.3.** As for the optimal decision-making, \(d^o > d^*, w^o > w^*, p^o > p^*\).

Proposition 5.3 reveals that government subsidy can effectively optimize the level of SRD. The optimal decisions of firms with government subsidy are better than those of firms without government subsidy. Because an increase in donation will stimulate consumers’ intention to purchase, demand will increase. The supplier then raises the wholesale price to obtain higher yields. Retail prices are largely influenced by wholesale prices; thus, retailers set higher retail prices to compensate for purchasing costs. Consumers with a stronger sense of social responsibility are more willing to pay for products that have CSR attributes\textsuperscript{30}. For example, the percentage of the European population willing to buy CSR products increased by 40% over three years\textsuperscript{34}. In this case,

<table>
<thead>
<tr>
<th>(d^o)</th>
<th>(\frac{7(a-c)(\beta-1)}{32k-7(\beta-1)^2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w^o)</td>
<td>(\frac{16ak+c(16k-7(\beta-1)^2)}{32k-7(\beta-1)^2})</td>
</tr>
<tr>
<td>(p^o)</td>
<td>(\frac{8ck + 7(\beta-1)/2 + a(24k + 7(\beta-1))}{32k-7(\beta-1)^2})</td>
</tr>
</tbody>
</table>

Table 2. The equilibrium results of Model I.
the existence of government subsidy helps to optimize the distribution of social resources.

**Proposition 5.4.** As for the performance of profits and social welfare, $\pi_r^{g} > \pi_r^{ng}$, $\pi_d^{g} > \pi_d^{ng}$, $SW^{g} > SW^{ng}$.

Proposition 5.4 elaborates the relationship between the subsidy, corporate profits, and social welfare. Compared with the absence of subsidy, government subsidy can increase the retailer’s donation, which in turn leads to increased market demand, ultimately increasing the profit of the dominant retailer. As the number of orders increases, suppliers seize the opportunity to raise the wholesale price and gain higher profits. In brief, compared with the situation without subsidy, the existence of government subsidy can effectively improve CSR, corporate profits, and overall social welfare. In addition, our analysis also explains an interesting phenomenon where the supplier does not need to undertake donation activities; the government does not subsidize it, but its profit increases with the increase in subsidy. Notably, this phenomenon is actually a common form of hitchhiking. For supply firms that do not actively undertake social responsibilities, the government may regulate their behavior through tax policy.

**5.2 The impact of the cost parameter and consumer CSR preference**

Because the donation amount of supply chain members is limited by their corresponding implementation costs, we analyze the impact of the cost parameter on the decision variables by using mathematical methods.

**Proposition 5.5.** By increasing the cost of SRD campaigns, regardless of whether there is a government subsidy, the optimal amount of SRD, the overall level of firms’ profits, and social welfare will decline.

Referring to the previous requirements, by taking the following parameter values: $a = 100$, $c = 10$, $\beta = 2$, we can obtain the changes in the impact of the cost parameter on the level of SRD, the firms’ profits, and social welfare, as shown in Fig. 3.

According to Fig. 3, the amount of SRD per unit, firms’ profits, and the level of social welfare decrease with the cost factor of corporate donation, regardless of the existence of government subsidy. In other words, they are negatively related to the corporate donation cost parameter ($k$). This phenomenon can be explained as follows: First, an increase in indirect costs reduces the level of SRDs by retailers. Furthermore, consumers will reduce the purchase of products with lower CSR attributes, which leads to less demand for this product in the market. Therefore, the retailer reduces the retail price to stimulate consumers to purchase, resulting in a decline in the retailer’s total profit. The supplier’s profits will also be reduced because of the reduction in the number of retailer orders. Simultaneously, consumer surplus decreases with the market demand, and the overall level of social welfare declines. These results are consistent with Proposition 5.5. Hence, when firms implement a donation policy, they must improve their efficiency and minimize process costs.

As the donation and pricing decisions of supply chain members are influenced by consumers’ preferences for social responsibility, we also use mathematical methods to analyze the influence of the sensitivity parameters of consumer demand on optimal decisions. Through the numerical example, we draw the influence trend of the consumer CSR preference factor on SRD, supply chain members’ profits, and social welfare in Fig. 4. The specific parameter settings are the same as those described above.

**Proposition 5.6.** As the consumer’s preference for CSR increases, regardless of whether there is a government subsidy.
or not, the optimal amount of SRD, the level of corporate profits and social welfare will rise.

The results of the numerical analysis are consistent with the conclusions of Proposition 5.6. Fig. 4 shows a comparison of

Fig. 3. The impact of the cost parameter on SRD, the firms' profits, and social welfare.

Fig. 4. The impact of consumer CSR preference on SRD, the corporate profits, and social welfare.
corporate donation, retailer profit, supplier profit, and social welfare in two different situations. The profits of supply chain members and the resulting social welfare level with government subsidy are higher than those without government subsidy. The retailer’s CSR behavior has a positive impact on consumer demand. When the retailer increases its social donation level, the increased demand can compensate for the retailer’s donation expenses and even make the retailer more profitable. For the supplier, as its order increases, its profit increases. In addition, we can see that with the improvement in consumers’ social preferences, the increments in corporate profits and social welfare (due to the existence of government subsidy) are more significant. Therefore, governments should enhance citizens’ social responsibility awareness before implementing subsidy policies. Only when consumers pay more attention to social responsibility can the implementation of government subsidy policies significantly optimize the allocation of social resources.

6 Extensions

In this section, we expand the basic content of this study from two aspects: Uncertainty of market size and centralized decision-making. In subsequent work, we discuss each extension in detail.

6.1 Equilibrium result under uncertainty

Based on the assumptions of the basic model, we extend the research content to explore equilibrium results under uncertain market sizes.

The rapid economic development of the market economy leads to a great deal of uncertainty. In recent years, with the acceleration of product iterations, the emergence of alternative products has become bound to meet the market demand for existing products. In real firm’s activities, this effect leads to uncertain market demand. Therefore, we assume that the market size is $a$. Excluding the market occupied by competitors’ alternative products, the market potential of this firm is $\lambda a$, where $\lambda$ represents the level of market encroachment by alternative products and $\lambda = 0$ indicates that the market is completely encroached by alternative products. Thus, the demand function is expressed as $q = \lambda a - p + \beta d$. A larger $\lambda$ value indicates a larger market size. We employ numerical analysis to study the impact of market size uncertainty on the final decision variables and equilibrium results. The results are depicted in Fig. 5.

As can be seen from Fig. 5, the decision variables increase with the market size, and the retail price and wholesale price are significantly affected, while the decision result of donation is less affected. Regarding the equilibrium result, the profits of firms and the government increase with the market size, and the influence of market size changes more significantly with an increase in market size. Therefore, firms and governments should adjust their strategies in time for a larger market size with greater potential changes to ensure that the decision results conform to market demand. When the market size is small, the decision results change minimally; therefore, a relatively conservative strategy can be adopted. Additionally, through verification, the previous conclusion still holds true when the market size changes.

6.2 Equilibrium results under centralized decision-making

In this section, we explore the equilibrium results of government subsidy under centralized decision-making. In a centralized channel, there is a single sales channel in which both suppliers and retailers are willing to cooperate and want to implement joint decisions. The pricing and donation decisions are made by a single decision maker. Therefore, the total profit function of this channel is: $\pi^c = (p - c - d)\cdot (a - p + \beta d) - kd' + k\beta d'$. The government’s objectives can be expressed as follows:

$$\pi^g_c = \pi + CS' - \Phi.$$  \hspace{1cm} (7)

By solving the game model, we can obtain the following results.

**Proposition 6.1.** In centralized decision-making, social welfare increases first and then decreases with the increase in government subsidy, and the optimal subsidy rate is $\tau^* = \frac{1}{3}$, the optimal social welfare is $\pi^g_{\tau^*} = \frac{3(a - c)k}{8k - 3(1 + \beta)}$.

From Proposition 6.1, in the case of centralized decision-making, the impact trend of the government’s subsidy on social welfare is the same as that under decentralized decision-making in the supply chain. However, the government’s optimal subsidy rate under centralized decision-making is lower.

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Fig. 5. The impact of market size on decision variables and equilibrium results.
than that under decentralized decision making. Under centralized decision-making, social welfare is better than under decentralized decision-making. Therefore, enterprises in the supply chain should strengthen cooperation to achieve higher performance, reduce double marginalization, and improve social welfare. In addition, the results show that a centralized supply chain has a low demand for government financial support when it achieves optimal social welfare, which helps to reduce the financial pressure of the government. In other words, it is necessary for the government to actively adjust policies, encourage and guide firms to strengthen cooperation, and integrate production and retail. Thus, the resources can be allocated more efficiently.

7 Conclusions

The influence of corporate donation behavior on firms and society has received considerable attention from academia and practitioners. Although many scholars have explored the factors affecting the effectiveness of corporate SRD campaigns through empirical investigation, few have provided support for pricing and donation design from the operational management perspective. Through the established models, this study defines management decision-making whereby firms can successfully implement SRD campaigns. In addition, from the perspective of social performance, through two Stackelberg game models, we analyzed the profits of all members and social welfare, and further discussed the gap between profit-oriented and welfare-oriented retailers. This study examined the impact of government subsidy on profit-oriented firms’ decision-making and found that government subsidy could address the gap in the social performance contribution between the two different retailers. Finally, we explored two important extensions in terms of the uncertain size of market demand and the increasing centralization of supply chain channels.

7.1 Theoretical contributions

This study provides three important theoretical contributions. First, compared with previous studies that only regarded social responsibility as a general concern of consumer surplus or a type of expenditure cost, this study reflects the real practice of CSR and contributes to the implementation of CSR activities. Based on the premise of maximizing corporate profits, this study explores how to maximize social welfare under government subsidy and draws conclusions and actionable suggestions that are more in line with corporate motives. We have further broadened the scope of research on CSR in supply chain management. Second, the previous literature on corporate donation behavior mainly focuses on output donation, believing that product donation is beneficial to firms. However, this study focuses on the form of donation related to consumer purchasing behavior. In addition, the literature on SRD behavior mainly focuses on empirical research, rarely using the method of operations management to provide support for firms to implement strategic decisions. This study also fills this research gap. Last but not least, the indirect costs in the process of SRD campaigns cannot be ignored, and this hinders profit-oriented firms from increasing their contribution to society. This study considers this neglected factor in previous studies. Using a three-stage Stackelberg game model to obtain and analyze optimal management decisions, we recognize that government subsidy can effectively improve business performance and social welfare by offsetting the indirect cost of donations. Our study contributes to the literature on government subsidy.

7.2 Managerial implications

In addition to the theoretical contributions emphasized above, this study also provides managerial contributions to the effectiveness of the firm and government decision-making. Our findings will help firms succeed in implementing SRD campaigns and help the government provide support in optimizing social welfare. The specific management implications are as follows:

First, when consumers have high social responsibility preferences, a retailer’s SRD is conducive toward improving the overall level of corporate profits and social welfare. Therefore, it is necessary to promote consumers’ awareness of social responsibility through relevant advertising and publicity. Moreover, as the supply chain leader, retailers should actively use CSR activities to help improve the overall performance of the supply chain, promote the sustainable development of relevant parties, and establish a good corporate image. Second, we find that supply chain performance and the resulting social welfare under government subsidy are higher than those under non-government subsidy. To maximize social welfare, the government needs to set incentives to stimulate retailers to offer higher social donations. In actual business operations, the government plays a key role in the social responsibility of the entire supply chain. Therefore, when society needs donation behaviors (for example, in the situation of the current COVID-19 pandemic, corporate donation is of great significance), the government should provide subsidy to reduce the corresponding indirect costs incurred in corporate donation activities, such as promotion of the activity and construction of supporting facilities, to increase the enthusiasm of firms. Notably, we find that, once the subsidy exceeds a certain range, the level of social welfare declines. Therefore, from the perspective of social welfare maximization, the government should not increase the level of subsidy indefinitely. Third, an interesting discovery is that the profit of the supplier increases with government subsidy under the government subsidy mechanism, and the profit is higher than that under the non-government subsidy mechanism. However, the supplier, as a follower in the supply chain, does not clearly bear CSR. This free-riding behavior is conducive to maintaining a long-term relationship with the retailer. To create more opportunities to cooperate with retailers and improve market competitiveness, suppliers should actively undertake social responsibility. The government can formulate corresponding punishment measures to avoid this free-riding behavior. Fourth, the larger the market size, the more significant the positive effect of the subsidy. It is necessary for the government to implement subsidy policies for larger markets. Meanwhile, compared with decentralized decision-making, centralized decision-making can improve corporate profits and social welfare and reduce the financial burden of the govern-
ment. Therefore, firms in the supply chain should strengthen cooperation, and the government should also encourage firms to form centralized production and sales chains.

7.3 Limitations and future research

Although our study provides significant management insights into government subsidy strategies and corporate SRD decisions, there are still some limitations. First, we consider only a three-level supply chain. The analysis of more complex supply chain structures is a promising research direction because there may be upstream (downstream) competition among suppliers (retailers). Second, environmental issues are currently receiving more attention, and the supply chain structure is gradually forming a closed loop, which is also an interesting phenomenon that is worth considering.

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Conflicts of interest

The authors declare that they have no conflict of interest.

Biographies

Dongxu Du is a graduate student at the School of Management, University of Science and Technology of China. His research interests focus on corporate social responsibility management.

Manman Wang is currently a postdoctoral researcher of operation management at the School of Management, University of Science and Technology of China (USTC). She received her Ph.D. degree from USTC in 2021. Her research interests include sustainable operations and the reverse supply chain management.

References

Appendix

Proof of Lemma 4.1. We solve this game using backward induction. First, the retailer determines the optimal retail price satisfying the first-order condition $\partial p^r / \partial p^r = a + d - 2p + w + \beta d = 0$. We can obtain $p^r(w) = (a + d + w + \beta d)$. Substituting $p^r(w)$ into Eq. (2), and according to the first-order condition of the supplier’s optimal wholesale price, $\partial w^s / \partial w = (a + c - 2w + d - \beta d) = 0$, so $w^s(d) = (1/2)(a - d + \beta d)$. Then, we obtain the following results: $p^r(d) = (1/4)[(a + c + d + 2\beta d)]$ and $w^s(d) = (1/2)[(a + c + d(-1 + \beta))]$. Therefore, substituting $p^r(d)$ and $w^s(d)$ into Eq. (1) and solving $\partial w^s(d)/\partial d = 0$, we obtain $d^s = (a - c)(\beta - 1)/16k(\beta - 1)^2$. Substituting $d^s$ into $p^r(d)$ and $w^s(d)$, it gives $p^r = a(12k/\beta(1 - \beta))$ and $w^s = 8ak + c(8k(-1 + \beta^2))$. Similarly, substituting $d^s$, $p^r$, and $w^s$, into Eqs. (1) and (2), we obtain $\pi^r$ and $\pi^s$.

Proof of Corollary 4.1. Because $d^s = (a - c)(\beta - 1)/16k(\beta - 1)^2$, $a - c > 0$, and $16k - (\beta - 1)^2 > 0$, so $d^s < 0$ occurs at $\beta > 1$. If $\beta < 1$, the retailer will not implement donations, that is, $d^s < 0$.

Proof of Proposition 4.1. From the process of Proof of Lemma 4.1, by simultaneously substituting $d^s$, $p^r$, and $w^s$ into $7(a - c)^2 + d^s(7(\beta - 1)^2 - 32k + 144d(a - c)(\beta - 1))$, we can obtain $w^s(d) = (32k - 7(\beta - 1)^2)/32k - 7(\beta - 1)^2$ and $\partial^2 w^s(d) / \partial d^2 = -(32k - 7(\beta - 1)^2)/32k - 7(\beta - 1)^2 < 0$. Therefore, $\partial^2 w^s(d) / \partial d^2 > 0$ occurs at $0 < d < d^s$, and $\partial^2 w^s(d) / \partial d^2 < 0$ occurs at $d > d^s$. Thus, Proposition 4.1 is proven.

Proof of Lemma 4.2. To solve the three-stage game of government participation, the first phase is to solve the game between the retailer and supplier. For the retailer’s optimal retail price decision, the first-order condition is $\partial p^r / \partial p^r = 0$. Therefore, we can obtain $p^r(w) = (1/2)(a + d + w + \beta d)$. By substituting $p^r(w)$ into Eq. (5), and according to the first-order condition of the supplier’s optimal wholesale price, $\partial w^s / \partial w = (a + c - 2w + d - \beta d) = 0$, we obtain $w^s(d) = (1/2)(a + c + d(-1 + \beta))$. Then, substituting $p^r(w)$ and $w^s(d)$ into Eq. (4) gives the following result, $\pi^r(\tau) = (1/16)[(a^2 + 2ad(\beta - 1) + d^2(\beta - 1)^2 + 16k(\beta - 1))]$. Then, we get $\partial^2 \pi^r(\tau) / \partial d^2 = 1/(8k + c)(\beta - 1)^2 + 16k(\beta - 1)^3 < 0$. We can obtain optimal donation decisions for retailer $d^s(\tau) = (a - c)(\beta - 1)/16k(\beta - 1)^2$. According to $d^s(\tau)$, so we can get $w^s(\tau) = 8k(a + c)(\beta - 1)^2 + c(\beta - 1)^2$ and $p^r(\tau) = a - c + \beta + 16k(\beta - 1) + 4ck(\tau)(\beta - 1)$. Similarly, by substituting $d^s(\tau)$, $w^s(\tau)$, and $p^r(\tau)$ into Eqs. (3) and (4), we can obtain $\pi^s(\tau)$, $\pi^r(\tau)$, and $\pi^s(\tau)$ in Lemma 4.2.

Proof of Proposition 4.2. From Lemma 4.2, we can get $\partial^2 p^r(\tau) / \partial \tau^2 = (4k(a - c)(\beta - 1)(1 + 3\beta))/((\beta - 1)^2 + 16k(\beta - 1)^3)$. Because $\beta > 1$ and $a - c > 0$, so $\partial^2 p^r(\tau) / \partial \tau^2 > 0$. For $d^s(\tau), \partial d^s(\tau) / \partial \tau = 16k(\beta - 1)(\beta - 1)^2 + 16k(\beta - 1)^3$, noting that $(a - c)(\beta - 1) > 0$, so $\partial d^s(\tau) / \partial s > 0$. For $w^s(\tau), \partial w^s(\tau) / \partial \tau = 8k(a - c)(\beta - 1)/(\beta - 1)^2 + 16k(\beta - 1)^3$, and noting that $(a - c)(\beta - 1) > 0$, so $\partial w^s(\tau) / \partial s > 0$. For $\pi^r(\tau)$ and $\pi^s(\tau)$, proofs are omitted for brevity.

Proof of Proposition 5.2. From Lemma 4.2, the government’s objective function can be expressed as $\pi^s(\tau) = (1/16)[(a^2 + 2ad(\beta - 1) + d^2(\beta - 1)^2 + 16k(\beta - 1))]/(8k + c)(\beta - 1)^2 + 16k(\beta - 1)^3$. Therefore, we can get $\partial \pi^s(\tau) / \partial \tau = (4k(a - c)(\beta - 1)(1 + 3\beta))/((\beta - 1)^2 + 16k(\beta - 1)^3)$. Because $\beta > 1$ and $a - c > 0$, so $\partial \pi^s(\tau) / \partial \tau > 0$. For $d^s(\tau), \partial d^s(\tau) / \partial \tau = 16k(\beta - 1)(\beta - 1)^2 + 16k(\beta - 1)^3$, noting that $(a - c)(\beta - 1) > 0$, so $\partial d^s(\tau) / \partial s > 0$. For $w^s(\tau), \partial w^s(\tau) / \partial \tau = 8k(a - c)(\beta - 1)/(\beta - 1)^2 + 16k(\beta - 1)^3$, and noting that $(a - c)(\beta - 1) > 0$, so $\partial w^s(\tau) / \partial s > 0$. For $\pi^r(\tau)$ and $\pi^s(\tau)$, proofs are omitted for brevity.
\[
\frac{k(a - c)^2 (56k(\tau - 1)^2 - (\beta - 1)^2)}{((\beta - 1)^2 + 16k(\tau - 1))}. \]
From \( \pi_c^\circ(\tau) \), we can get \( \frac{\partial \pi_c^\circ(\tau)}{\partial \tau} = 16a'k'(\beta - 1)(\tau - 5) \). And we find \( \frac{\partial \pi_c^\circ(\tau)}{\partial \tau} > 0 \) at \( \tau < \frac{5}{7} \). And \( \frac{\partial \pi_c^\circ(\tau)}{\partial \tau} < 0 \) at \( \tau > \frac{5}{7} \). Thus, when the optimal subsidy rate is \( \tau = \frac{5}{7} \), the overall welfare of society is optimal. Thus, Proposition 5.2 is proven.

**Proof of Proposition 5.3.** From Lemma 4.1 and Table 2, we can get \( d^\tau - d^\nu = \frac{80k(a - c)(\beta - 1)}{(32k - 7(\beta - 1)^2)(16(\beta - 1)^2)} \), \( w^\nu - w^\nu' = \frac{20k(a - c)(3\beta' - 1 - 2\beta)}{(32k - 7(\beta - 1)^2)(16(\beta - 1)^2)} \). Because \( (32k - 7(\beta - 1)^2)(16(\beta - 1)^2) > 0 \) and \( \beta > 1 \), so \( d^\tau - d^\nu > 0 \), \( w^\nu - w^\nu' > 0 \) and \( p^\nu - p^\nu' > 0 \).

**Proof of Proposition 5.4.** The proof is the same as that of Proposition 5.3; here, it is omitted for brevity.

**Proof of Proposition 5.5.** When the government does not subsidize corporate donations, By taking the first derivative, we can get \( \frac{\partial d^\tau}{\partial k} = \frac{16(a - c)(\beta - 1)}{(16k - (\beta - 1)^2)} < 0 \), \( \frac{\partial \pi_c^\circ(\tau)}{\partial k} = \frac{-k(a - c)(\beta - 1)^2}{(16k - (\beta - 1)^2)} < 0 \), \( \frac{\partial \pi_c^\circ(\tau)}{\partial k} = \frac{-32k(a - c)(\beta - 1)^2(16k - (\beta - 1)^2)}{(16k - (\beta - 1)^2)} < 0 \). At this point, the optimal amount of SRD, the overall level of corporate profits, and social welfare will decline as the cost of SRD campaigns increases. Similarly, under government subsidy we can get \( \frac{\partial d^\nu}{\partial k} < 0 \), \( \frac{\partial \pi_c^\circ(\tau)}{\partial k} < 0 \), \( \frac{\partial \pi_c^\circ(\tau)}{\partial k} < 0 \), \( \frac{\partial S^W}{\partial k} < 0 \). Therefore, we can get the conclusion in the proposition.

**Proof of Proposition 5.6.** The proof is the similar to that of Proposition 5.5; here, it is omitted for brevity.

**Proof of Proposition 6.1.** The first step is to establish the concave condition of the centralized channel profit function. The concavity condition can be tested by using Hessian matrix. Hessian matrix of \( \pi^\circ \) is:

\[
H^\circ(p,d) = \begin{pmatrix}
\frac{\partial^2 \pi^\circ}{\partial p^2} & \frac{\partial^2 \pi^\circ}{\partial \pi_c^\circ \partial p} \\
\frac{\partial^2 \pi^\circ}{\partial \pi_c^\circ \partial d} & \frac{\partial^2 \pi^\circ}{\partial d^2}
\end{pmatrix} = \begin{pmatrix}
-2 & 1 + \beta \\
1 + \beta & 2(-k - \beta + k\tau)
\end{pmatrix}.
\]
If \( |H^\circ(p,d)| = 4k(1 - \tau) - (\beta - 1)^2 > 0 \), \( \pi^\circ \) is concave with respect to \( d \). By solving \( \frac{\partial \pi_c^\circ}{\partial p} = 0 \) and \( \frac{\partial \pi_c^\circ}{\partial d} = 0 \), we can obtain the \( p^\circ = \frac{a(1 - \beta + 2k(\tau - 1)) + c((\beta - 1)\beta + 2k(\tau - 1))}{4k(1 - \tau) - (\beta - 1)^2} \) and \( d^\circ = \frac{(a - c)(\beta - 1)}{4k(1 - \tau) - (\beta - 1)^2} \). Substituting \( p^\circ \) and \( d^\circ \) into Eq. (7), we can obtain \( \pi_c^\circ = \frac{3(a - c)^2 k}{8k - 3(1 + \beta)^2} \). From \( \pi_c^\circ \), we can get \( \frac{\partial \pi_c^\circ}{\partial \tau} = \frac{4(a - c)^2 k((\beta - 1)^2(1 - 3\tau))}{4k(1 - \tau) - (\beta - 1)^2} \). Because \( 4k(1 - \tau) - (\beta - 1)^2 > 0 \), we can find \( \frac{\partial \pi_c^\circ(\tau)}{\partial \tau} > 0 \) at \( \tau < \frac{1}{3} \). And \( \frac{\partial \pi_c^\circ(\tau)}{\partial \tau} < 0 \) at \( \tau > \frac{1}{3} \). Thus, when the government subsidy rate is \( \tau^* = \frac{1}{3} \), the overall welfare of society is optimal, that is, \( \pi_c^\circ = \frac{3(a - c)^2 k}{8k - 3(1 + \beta)^2} \). Thus, Proposition 6.1 is proven.