第50卷第12期 中国科学技术大学学报 Vol. 50, No. 12
2020年12月 JOURNAL OF UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA Dec. 2020
Received: 2020-10-10; Revised: 2020-11-24 doi:10.3969/j.issn.0253-2778.2020.12.004

A novel linear iteration method for M-matrix algebraic Riccati equations

GUAN Jinrui[1*], FENG Yuehua[2]
1. Department of Mathematics, Taiyuan Normal University, Jinzhong 030619, China;
2. School of Mathematics, Physics and Statistics, Shanghai University of Engineering Science, Shanghai 201620 China
* Corresponding author. E-mail: guanjinrui2012@163.com

Abstract: Numerical solutions of the M-matrix algebraic Riccati equation (MARE) were studied, which has become a hot topic in recent years due to its broad applications. A novel linear iteration method for computing the minimal nonnegative solution of MARE was proposed, in which only matrix multiplications are needed at each iteration. Convergence of the new method was proved by choosing proper parameters for the MARE associated with a nonsingular M-matrix or an irreducible singular M-matrix. Theoretical analysis and numerical experiments show that the new method is feasible and is effective than some existing methods under certain conditions.

Keywords: algebraic Riccati equation; M-matrix; minimal nonnegative solution; Newton method; doubling algorithm
CLC number: O241.6 **Document code**: A
2010 Mathematics subject classifications: 65F30; 65F60; 15B99

1 Introduction

In this paper, we consider numerical solution of the matrix equation

$$XCX - XD - AX + B = 0 \tag{1}$$

where A, B, C, D are real matrices of sizes $m\times m$, $m\times n$, $n\times m$, $n\times n$ respectively and

$$K = \begin{pmatrix} D & -C \\ -B & A \end{pmatrix} \tag{2}$$

is an M-matrix. This class of nonlinear matrix equation is called M-matrix algebraic Riccati equation (MARE).

MARE arises from many areas of scientific computing and engineering applications, such as transport theory, Markov chains, applied statistics, control theory and so on[1-5,17]. In recent years, MARE has been extensively studied (see Refs. [6-10,18]).

In the following, we first give some notations and definitions which will be used in the sequel.

Denote $\mathbb{R}^{m\times n}$ to be the set of all real $m\times n$ matrices. Let $A\in\mathbb{R}^{m\times n}$, if $a_{ij}\geqslant 0$ ($a_{ij}>0$) for all i,j, then A is called a nonnegative (positive) matrix, denoted by $A\geqslant 0$ ($A>0$). Let $A=(a_{ij})$, $B=(b_{ij})\in\mathbb{R}^{m\times n}$, we write $A\geqslant B$ ($A>B$), if $a_{ij}\geqslant b_{ij}$ ($a_{ij}>b_{ij}$) for all i,j. A matrix $A\in\mathbb{R}^{n\times n}$ is called a Z-matrix, if $a_{ij}\leqslant 0$ for all $i\neq j$. A Z-matrix A is called an M-matrix if there exists a nonnegative matrix B such that $A=sI-B$ and $s\geqslant\rho(B)$, where $\rho(B)$ is the spectral radius of B. In particular, A is called a nonsingular M-matrix if $s>\rho(B)$ and singular M-matrix if $s=\rho(B)$.

The following are some basic results of M-matrix, which can be found in Ref. [11, Chapter 6].

Lemma 1.1 Let A be a Z-matrix. Then the following statements are equivalent:

① A is a nonsingular M-matrix;
② $A^{-1}\geqslant 0$;
③ $Av>0$ for some vectors $v>0$;
④ All eigenvalues of A have positive real part.

Lemma 1.2 Let A and B be Z-matrices. If A is a nonsingular M-matrix and $A\leqslant B$, then B is also a nonsingular M-matrix. In particular, for any nonnegative real number α, $B=\alpha I+A$ is a nonsingular M-matrix.

Lemma 1.3 Let A be a nonsingular M-matrix or an irreducible singular M-matrix. Let A be partitioned as

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

where A_{11} and A_{22} are square matrices. Then A_{11} and A_{22} are nonsingular M-matrices.

For the Eq. (1), the solution of practical interest is the minimal nonnegative solution. The following important result on the minimal nonnegative solution is from Refs. [2,5,12].

Citation: GUAN Jinrui, FENG Yuehua. A novel linear iteration method for M-matrix algebraic Riccati equations. J. Univ. Sci. Tech. China, 2020, 50(12): 1472-1477.

Lemma 1.4 If K is a nonsingular M-matrix or an irreducible singular M-matrix, then Eq. (1) has a minimal nonnegative solution S. If K is nonsingular, then $A-SC$ and $D-CS$ are also nonsingular M-matrices. If K is irreducible, then $S>0$ and $A-SC$ and $D-CS$ are also irreducible M-matrices.

In the past decades, many efficient numerical methods have been proposed for solving the MARE, such as fixed-point iteration methods, Newton method, ALI iteration method, SDA, and so on. For details see Refs. [2,5-7,9,10].

In this paper, we propose a novel linear iteration method for solving the MARE, in which at each iteration only matrix multiplications are needed. Hence, as compared with other methods, less CPU time are required for solving the MARE. Theoretical analysis and numerical experiments show that the new method is feasible and effective than some existing methods under suitable conditions.

The rest of the paper is organized as follows. In Section 2, we review some methods for solving the MARE. In Section 3, we propose the novel linear iteration method and give its convergence analysis. In Section 4, we use some numerical examples to show the feasibility and effectiveness of the new method. Conclusion is given in Section 5.

2 Some existing methods

In this section, we will review some efficient methods for solving the MARE.

In Refs. [2,7], the Newton method was proposed for solving the MARE as follows:

$$X_{k+1}(D - CX_k) + (A - X_kC)X_{k+1} = B + X_kCX_k,\ X_0 = 0 \quad (3)$$

Convergence analysis in Ref. [7] showed that the Newton method is quadratically convergent for the noncritical case, and is linearly convergent for the critical case. At each iteration, the overall cost of Newton method is one Sylvester matrix equation and three matrix multiplications. However, since solving a Sylvester matrix equation will costs $60n^3$ if $m=n$, the Newton method is very expensive.

In Refs. [2, 10], some fixed-point iteration methods were proposed for solving the MARE. The general form of the fixed-point iteration methods is as follows:

$$A_1X_{k+1} + X_{k+1}D_1 = X_kCX_k + X_kD_2 + A_2X_k + B,\ X_0 = 0 \quad (4)$$

where $A=A_1-A_2$, $D=D_1-D_2$ are given splittings of A and D. Some choices of splitting are given in the following:

$FP1$: $A_1 = \mathrm{diag}(A)$, $D_1 = \mathrm{diag}(D)$;

$FP2$: $A_1 = \mathrm{tril}(A)$, $D_1 = \mathrm{triu}(D)$;

$FP3$: $A_1 = A$, $D_1 = D$.

Though the fixed-point iteration methods are easy to implement, they may require a large number of iterations to converge. Convergence analysis in Ref. [2] showed that the fixed-point iteration methods have linear convergence rate for the noncritical case and have sublinear convergence rate for the critical case.

Ref. [6] proposed an alternately linearized implicit iteration method (ALI) for solving the MARE as follows:

$$\left.\begin{aligned} X_{k+1/2}(\alpha I + (D - CX_k)) &= (\alpha I - A)X_k + B,\\ (\alpha I + (A - X_{k+1/2}C))X_{k+1} &= X_{k+1/2}(\alpha I - D) + B \end{aligned}\right\} \quad (5)$$

where $X_0=0$ and $\alpha>0$ is a given parameter. In the ALI iteration method, only two linear matrix equations are needed to solve, which are easier than Sylvester matrix equations. At each iteration, the overall cost is two matrix inverses and six matrix multiplications. The ALI iteration method is linearly convergent for the noncritical case and is sub-linearly convergent for the critical case.

The doubling algorithm is one of the most efficient methods for solving the MARE. It was first proposed in Rer. [9]. Later, some extensions were proposed in Refs. [5,10,13,14,19], among which the ADDA in Ref. [10] is the fastest one. The iterations of the doubling algorithm are defined as follows:

$$\left.\begin{aligned} E_{k+1} &= E_k(I - G_kH_k)^{-1}E_k,\\ F_{k+1} &= F_k(I - H_kG_k)^{-1}F_k \end{aligned}\right\} \quad (6)$$

$$\left.\begin{aligned} G_{k+1} &= G_k + E_k(I - G_kH_k)^{-1}G_kF_k,\\ H_{k+1} &= H_k + F_k(I - H_kG_k)^{-1}H_kE_k \end{aligned}\right\} \quad (7)$$

where E_0, F_0, G_0, H_0 are some initial settings (see Refs. [9,10] for details). At each iteration, the overall cost of the doubling algorithm is two matrix inverses and ten matrix multiplications. Convergence analysis in Refs. [5, 10, 13] showed that the doubling algorithm is quadratically convergent for the noncritical case and is linearly convergent for the critical case.

For other efficient numerical methods, see Refs. [5,9,10,14-16] for an incomplete references.

3 A novel linear iteration method

In the following we will propose a novel linear iteration method for the MARE.

First, write Eq. (1) as the following form

$$AX + XD = B + XCX \quad (8)$$

By choosing $\alpha>0$, $\beta>0$, Eq. (8) can be written as

$$(\beta I + A)X(\alpha I + D) - (\alpha I - A)X(\beta I - D) = (\alpha + \beta)(B + XCX).$$

Making use of generalized Cayley transform and multiplying both sides by $(\beta I + A)^{-1}$, $(\alpha I + D)^{-1}$ respectively, we then can transform Eq. (8) into the following discrete form

$$X = (\beta I + A)^{-1}(\alpha I - A)X(\beta I - D)(\alpha I + D)^{-1} +$$

$$(\alpha+\beta)(\beta I+A)^{-1}B(\alpha I+D)^{-1}+(\alpha+\beta)(\beta I+A)^{-1}XCX(\alpha I+D)^{-1}.$$

Denote

$$U=(\beta I+A)^{-1}(\alpha I-A),\ V=(\beta I-D)(\alpha I+D)^{-1} \tag{9}$$

$$W=(\alpha+\beta)(\beta I+A)^{-1}B(\alpha I+D)^{-1}=\frac{1}{\alpha+\beta}(I+U)B(I+V) \tag{10}$$

then we can get the iteration as follows

$$X_{k+1}=UX_kV+W+\frac{1}{\alpha+\beta}(I+U)X_kCX_k(I+V).$$

Thus we have the following novel linear iteration method for computing the minimal nonnegative solution of Eq. (1).

Novel linear iteration (NLI) method:

① Set $X_0=0\in\mathbb{R}^{m\times n}$.

② Compute U,V,W as in (9)−(10), where $\alpha>0$, $\beta>0$ are two given parameters.

③ For $k=0,1,\cdots$, until $\{X_k\}$ converges, compute X_{k+1} from X_k by

$$X_{k+1}=UX_kV+W+\frac{1}{\alpha+\beta}(I+U)X_kCX_k(I+V) \tag{11}$$

At each iteration of the novel linear iteration method, only matrix multiplications are needed, which is much cheaper than the existing methods in Section 2. The overall cost is only six matrix multiplications. Hence less CPU time are required for solving the MARE.

In the following, we give convergence analysis of the novel linear iteration method.

Lemma 3.1 For the MARE (1), if K in Eq. (2) is a nonsingular M-matrix or an irreducible singular M-matrix and the parameters α, β satisfy

$$\alpha\geqslant\max\{a_{ii}\},\ \beta\geqslant\max\{d_{ii}\} \tag{12}$$

then U,V,W in (9)−(10) are well defined and satisfy

$$U\geqslant 0,\ V\geqslant 0,\ W\geqslant 0.$$

Proof Since K in Eq. (2) is a nonsingular M-matrix or an irreducible singular M-matrix, we know that A and D are nonsingular M-matrices by Lemma 1.3. Thus $\beta I+A$ and $\alpha I+D$ are nonsingular M-matrices by Lemma 1.2. When α and β satisfy Eq. (12), we have $\alpha I-A\geqslant 0$ and $\beta I-D\geqslant 0$. Hence

$$U=(\beta I+A)^{-1}(\alpha I-A)\geqslant 0,$$
$$V=(\beta I-D)(\alpha I+D)^{-1}\geqslant 0.$$

It is trivial that $W\geqslant 0$.

Lemma 3.2 Under the assumptions in Lemma 3.1, the sequence $\{X_k\}$ generated by Eq. (11) satisfy

$$0\leqslant X_k\leqslant X_{k+1},\ X_k\leqslant S,\ k=0,1,\cdots \tag{13}$$

Proof ① First, we prove by induction that $0\leqslant X_k\leqslant X_{k+1}$ holds true for $k=0,1,\cdots$.

When $k=0$, it is clear that $0=X_0\leqslant X_1=W$. Suppose that the assertion holds for $k=l$. It is clear from Eq. (11) that $X_{l+1}\geqslant 0$. On the other hand, from

$$X_{l+1}=W+UX_lV+\frac{1}{\alpha+\beta}(I+U)X_lCX_l(I+V),$$

$$X_l=W+UX_{l-1}V+\frac{1}{\alpha+\beta}(I+U)X_{l-1}CX_{l-1}(I+V),$$

we have

$$X_{l+1}-X_l=U(X_l-X_{l-1})V+\frac{1}{\alpha+\beta}(I+U)(X_lCX_l-X_{l-1}CX_{l-1})(I+V)=U(X_l-X_{l-1})V+\frac{1}{\alpha+\beta}(I+U)[(X_lC(X_l-X_{l-1})+(X_l-X_{l-1})CX_{l-1}](I+V)\geqslant 0.$$

Hence the assertions hold for $k=l+1$. Thus we have proved by induction that $0\leqslant X_k\leqslant X_{k+1}$ holds for all $k\geqslant 0$.

② Next, we prove by induction that $X_k\leqslant S$ holds true for all $k\geqslant 0$.

When $k=0$, it is clear that $0=X_0\leqslant S$. Suppose that the assertions hold for $k=l$. Then from

$$X_{l+1}=W+UX_lV+\frac{1}{\alpha+\beta}(I+U)X_lCX_l(I+V),$$

$$S=W+USV+\frac{1}{\alpha+\beta}(I+U)SCS(I+V),$$

we have

$$X_{l+1}-S=U(X_l-S)V+\frac{1}{\alpha+\beta}(I+U)(X_lCX_l-SCS)(I+V)=U(X_l-S)V+\frac{1}{\alpha+\beta}(I+U)[X_lC(X_l-S)+(X_l-S)CS](I+V)\leqslant 0.$$

Hence the assertions hold for $k=l+1$. Thus we have proved by induction that $X_k\leqslant S$ holds for all $k\geqslant 0$.

Using the lemmas above, we can prove the following convergence theorem of NLI method.

Theorem 3.1 For the MARE (1), if K in Eq. (2) is a nonsingular M-matrix or an irreducible singular M-matrix and the parameters α, β satisfy $\alpha\geqslant\max\{a_{ii}\}$, $\beta\geqslant\max\{d_{ii}\}$, then the sequence $\{X_k\}$ generated by the NLI method is well defined, monotonically increasing, and converges to the minimal nonnegative solution S of equation Eq. (1).

Proof We have shown in Lemma 3.2 that $\{X_k\}$ is nonnegative, monotonically increasing and bounded from above. Thus there is a nonnegative matrix S^* such that $\lim_{k\to\infty}X_k=S^*$. From Lemma 3.2, we have $S^*\leqslant S$. On the other hand, take the limit in the NLI method, we know S^* is a solution of the MARE (1), thus by Lemma 1.4 $S\leqslant S^*$. Hence $S=S^*$.

Note In the NLI method there have two parameters α and β, and how to choose the optimal values of them is very important. However, this is very difficult to discuss. Numerical examples show that when

$\alpha = \max\{a_{ii}\}$ and $\beta = \max\{d_{ii}\}$, the NLI method needs the least iteration numbers. So in the numerical experiments we just choose $\alpha = \max\{a_{ii}\}$ and $\beta = \max\{d_{ii}\}$.

4 Numerical experiments

In this section we use several examples to show the feasibility and effectiveness of the NLI method. We compare the NLI method with the fixed-point iteration method (FP3)[2], Newton method (Newton)[2,7], ALI iteration method (ALI)[6], ADDA[10], and present computational results in terms of the numbers of iterations (IT), CPU time (CPU) in seconds and the residue (RES), where

$$\mathrm{RES} = \frac{\| XCX - XD - AX + B \|_\infty}{\| XCX \|_\infty + \| XD \|_\infty + \| AX \|_\infty + \| B \|_\infty}.$$

In our implementations all iterations are performed in MATLAB (version R2012a) on a personal computer with 2 GHz CPU and 8 GB memory, and are terminated when the current iterate satisfies RES<10^{-6}. In the FP3, Newton, ALI, and NLI methods, the iterations are all started from $X_0 = 0$, while in the ADDA the initial settings are chosen according to Ref. [10].

Example 4.1 Consider the MARE (1) with

$$A = -10E_{n\times n} + 180.002\, I_n,\ B = 0.001\, E_{n\times m},$$
$$C = B^{\mathrm{T}},\ D = 0.018\, I_m,$$

where $E_{m\times n}$ is the $m\times n$ matrix with all ones and I_m is the identity matrix of size m with $m = 2$, $n = 18$. This example is from Ref. [5] where the corresponding M is an irreducible singular M-matrix. The numerical results are summarized in Tab. 1. For this example, the ALI iteration method can not converge in 9000 iterations, while the other four methods perform very well.

Example 4.2 Consider the MARE (1) with

$$A = D = \begin{pmatrix} 3 & -1 & & \\ & 3 & \ddots & \\ & & \ddots & -1 \\ -1 & & & 3 \end{pmatrix} \in \mathbb{R}^{n\times n},$$
$$B = I_n,\ C = \xi I_n,\ 0 < \xi \leqslant 2.$$

This example is from Ref. [6], where the corresponding K is a nonsingular M-matrix. For $n=500$ with different values of ξ, we report the numerical results in Tab. 2. For this example, all the four methods perform very well, while FP3 is a little expensive.

Example 4.3 Consider the MARE (1) with

$$A = \begin{pmatrix} 3 & -1 & & \\ & 3 & \ddots & \\ & & \ddots & -1 \\ -1 & & & 3 \end{pmatrix},$$
$$B = 2I,\ C = 10B,\ D = 10A.$$

This example is from Ref. [10], where the corresponding K is an irreducible singular M-matrix. For different sizes of n, the numerical results are summarized in Tab. 3. For this example, all the four methods perform very well, while FP3 is a little expensive.

Tab. 1 Numerical results of Example 4.1

Method	IT	CPU	RES
Newton	3	0.0022	7.4339e-8
ADDA	3	0.0012	8.2535e-9
FP3	8	0.0043	4.8065e-7
ALI	–	–	–
NLI	8	0.0007	4.8065e-7

Tab. 2 Numerical results of Example 4.2

ξ	Method	IT	CPU	RES
0.2	Newton	3	2.9530	1.2567e-13
	ADDA	3	2.4787	8.1793e-12
	FP3	4	3.6906	9.9510e-8
	ALI	5	1.8868	8.9388e-8
	NLI	5	1.6646	4.3011e-7
0.5	Newton	3	2.8326	2.0915e-11
	ADDA	3	2.3911	3.1459e-11
	FP3	5	4.4415	2.5827e-7
	ALI	5	1.9527	2.0268e-7
	NLI	6	1.8987	3.5791e-7
1	Newton	3	2.6065	3.9526e-9
	ADDA	3	2.3925	2.8378e-10
	FP3	7	6.2273	1.6131e-7
	ALI	5	1.8948	7.6678e-7
	NLI	7	2.0148	9.8902e-7
2	Newton	4	3.3157	1.0240e-12
	ADDA	3	2.4871	2.2541e-8
	FP3	10	8.8082	7.4006e-7
	ALI	7	2.4239	1.7466e-7
	NLI	11	2.9268	6.6354e-7

Example 4.4 Consider the MARE (1) for which

$$A = D = \mathrm{tridiag}(-I, T, -I) \in \mathbb{R}^{n\times n}$$

are block tridiagonal matrices,

$$C = \frac{1}{50}\mathrm{tridiag}(1,2,1) \in \mathbb{R}^{n\times n}$$

is a tridiagonal matrix, and

$$B = SD + AS - SCS$$

such that

$$S = \frac{1}{50} ee^{\mathrm{T}} \in \mathbb{R}^{n \times n}$$

is the minimal nonnegative solutionn of Eq. (1), where

Tab. 3 Numerical results of Example 4.3

n	Method	IT	CPU	RES
50	Newton	3	0.0386	3.6818e-8
	ADDA	3	0.0118	4.1856e-11
	FP3	8	0.0445	2.3094e-7
	ALI	10	0.0292	3.5113e-7
	NLI	8	0.0165	3.9584e-7
100	Newton	3	0.0835	3.6818e-8
	ADDA	3	0.0301	4.1856e-11
	FP3	8	0.1337	2.3094e-7
	ALI	10	0.0584	3.5113e-7
	NLI	8	0.0295	3.9584e-7
500	Newton	3	2.8493	9.3985e-8
	ADDA	3	1.6299	4.1856e-11
	FP3	8	7.6668	2.3094e-7
	ALI	10	2.7959	3.5113e-7
	NLI	8	1.6058	3.9584e-7
1000	Newton	3	21.7717	3.6818e-8
	ADDA	3	15.3004	4.1856e-11
	FP3	8	57.9162	2.3094e-7
	ALI	10	19.0729	3.5113e-7
	NLI	8	12.7018	3.9584e-7

Tab. 4 Numerical results of Example 4.4

m	Method	IT	CPU	RES
5	Newton	2	0.0135	1.3048e-8
	ADDA	3	0.0026	2.0662e-12
	FP3	3	0.0184	5.3544e-7
	ALI	5	0.0023	9.0345e-8
	NLI	5	0.0015	5.9335e-8
10	Newton	3	0.0726	2.5071e-11
	ADDA	4	0.0365	4.6985e-13
	FP3	5	0.0876	3.4722e-7
	ALI	9	0.0456	4.8599e-7
	NLI	9	0.0341	2.8401e-7
20	Newton	3	1.8199	6.9198e-7
	ADDA	4	0.7517	5.3607e-7
	FP3	13	5.0155	6.2371e-7
	ALI	20	1.4911	5.0121e-7
	NLI	19	1.2310	8.5316e-7

Tab. 5 Numerical results of Example 4.5

α	β	IT
4	4	10
5	5	11
6	6	12
8	8	14
10	10	17
20	20	31
50	50	73
4	10	11
4	20	12
4	50	12
10	4	11
20	4	12
50	4	12

$$T = \mathrm{tridiag}\left(-1, 4 + \frac{200}{(m+1)^2}, -1\right) \in \mathbb{R}^{m \times m},$$
$$e = (1, \cdots, 1)^{\mathrm{T}} \in \mathbb{R}^{n},$$

and $n = m^2$. This example is from Ref. [6]. For different sizes of m, the numerical results are summarized in Tab. 4.

For this example, all the four methods perform very well, while FP3 is a little expensive.

From the four examples, we can conclude that NLI is feasible. In addition, it is effective when the problems are far from critical.

Example 4.5 Consider the MARE (1) for which

$$A = \begin{pmatrix} 4 & -1 \\ -1 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$
$$C = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, D = \begin{pmatrix} 4 & -1 \\ -1 & 4 \end{pmatrix}.$$

In this example, we choose different values of parameters in the NLI method and show its numerical behaviour. For different values of α and β, the iteration numbers are summarized in Tab. 5.

From Tab. 5 we can conclude that, when choosing $\alpha = \max\{a_{ii}\} = 4$ and $\beta = \max\{d_{ii}\} = 4$, the NLI method needs the least iteration numbers.

5 Conclusions

We have proposed a novel linear iteration (NLI) method for computing the the minimal nonnegative solution of MARE. Convergence of the NLI method is proved for the MARE associated with a nonsingular M-matrix or an irreducible singular M-matrix. Numerical experiments have shown that the novel linear iteration method is feasible and needs less CPU time than some existing methods in some cases. However, for the critical case or near-critical case, the NLI method will be very slowly and in this case the doubling algorithms will be more preferable. Finally, how to design algorithms with small computation cost and fast convergence remains to be further studied.

Acknowledgements

The work is supported by National Natural Science Foundation of China (12001363, 12001395), Natural Science Foundation of Shanxi province, China (201901D211423).

Conflict of interest

The authors declare no conflict of interest.

Author information

GUAN Jinrui (corresponding author) is now an associate professor at Departement of Mathematics, Taiyuan Normal University, China. He received his Ph.D. degree in computational mathematics at Xiamen University in 2016. His research interests focus on matrix theory and numerical linear algebra with applications.

References

[1] Bean N G, O'Reilly M M, Taylor P G. Algorithms for return probabilities for stochastic fluid flows. Stochastic Models, 2005, 21: 149-184.
[2] Guo C H. Nonsymmetric algebraic Riccati equations and Wiener-Hopf factorization for Mmatrices. SIAM J. Matrix Anal. Appl., 2001, 23: 225-242.
[3] Juang J, Lin W W. Nonsymmetric algebraic Riccati equations and Hamilton-like matrices. SIAM J. Matrix Anal. Appl., 1999, 20: 228-243.
[4] Rogers L C G . Fluid models in queueing theory and Wiener-Hopf factorization of Markov chains. Ann. Appl. Probab., 1994, 4: 390-413.
[5] Bini D A , Iannazzo B, Meini B. Numerical solution of algebraic Riccati equations. Philadelphia: Society for Industrial and Applied Mathematics, 2012.
[6] Bai Z Z, Guo X X, Xu S F. Alternately linearized implicit iteration methods for the minimal nonnegative solutions of the nonsymmetric algebraic Riccati equations. Numer. Linear Algebra Appl., 2006, 13: 655-674.
[7] Guo C H, Higham N J. Iterative solution of a nonsymmtric algebraic Riccati equation. SIAM J. Matrix Anal. Appl., 2007, 29: 396-412.
[8] Guo C H, Lu D. On algebraic Riccati equations associated with regular singular M-matrices. Linear Algebra Appl., 2016, 493: 108-119.
[9] Guo X X, Lin W W, Xu S F. A structure-preserving doubling algorithm for nonsymmetric algebraic Riccati equation. Numer. Math., 2006, 103: 393-412.
[10] Wang W G, Wang W C, Li R C. Alternating-directional doubling algorithm for M-matrix algebraic Riccati equations. SIAM J. Matrix Anal. Appl., 2012, 33: 170-194.
[11] Berman A, Plemmons R J. Nonnegative Matrices in the Mathematical Sciences. New York: Academic Press, 1994.
[12] Guo C H. A note on the minimal nonnegative solution of a nonsymmtric algebraic Riccati equation. Linear Algebra Appl., 2002, 357: 299-302.
[13] Guo C H. On algebraic Riccati equations associated with M-matrices. Linear Algebra Appl., 2013, 439: 2800-2814.
[14] Xue J G, Li R C. Highly accurate doubling algorithms for M-matrix algebraic Riccati equations. Numer. Math., 2017, 135:733-767.
[15] Guan J R, Lu L Z. New alternately linearized implicit iteration for M-matrix algebraic Riccati equations. J. Math. Study, 2017, 50(1): 54-64.
[16] Guan J R. Modified alternately linearized implicit iteration method for M-matrix algebraic Riccati equations. Appl. Math. Comput., 2019, 347: 442-448.
[17] Bean N G, O'Reilly M M, Taylor P G. Algorithms for the Laplace-Stieltjes transforms of first return times for stochastic fluid flows. Methodol Comput Appl Probab, 2008, 10: 381-408.
[18] Guo C H. A new class of nonsymmtric algebraic Riccati equations. Linear Algebra Appl., 2007, 426: 636-649.
[19] Guo C H, Iannazzo B, Meini B. On the doubling algorithm for a (shift) algebraic Riccati equation. SIAM J. Matrix Anal. Appl., 2007, 29: 1083-1100.

M-矩阵代数 Riccati 方程的一类新的线性迭代法

关晋瑞[1*],冯月华[2]

1. 太原师范学院数学系，山西晋中 030619; 2. 上海工程技术大学数理与统计学院,上海 201620

摘要：研究了 M-矩阵代数 Riccati 方程的数值解法，这类方程由于广泛的应用成为近年来的研究热点. 提出了一种新的线性迭代法来计算方程的最小非负解，该方法在每步迭代中只需要矩阵乘法. 通过适当选取参数，证明了当系数矩阵为非奇异 M-矩阵或不可约奇异 M-矩阵时新方法的收敛性. 理论分析和数值实验表明，新方法是可行的，而且在一定情况下比现有的一些方法更加有效.

关键词：代数 Riccati 方程;M-矩阵;最小非负解;牛顿法;加倍算法