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Linear inviscid damping for monotonic shear flows under the two dimensional β-plane equation

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Abstract: The linear asymptotic stability of a class of strictly monotonic shear flows was established under the two dimensional β-plane equation in an infinite periodic channel of period 2π, $\mathbb{T}\times\mathbb{R}$.

Key words: β-plane; linear asymptotic stability; inviscid damping

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二维β平面方程关于单调剪切流的无黏性衰减

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摘要:研究了在$\mathbb{T}\times\mathbb{R}$区域上二维$\beta$平面方程关于一类严格单调的剪切流的线性渐近稳定性,其中$\mathbb{T}$的周期为2π.

关键词:β-平面方程;线性渐近稳定;无黏性衰减

0 Introduction

In this article, we study the asymptotic stability of monotonic shear flows $(U(y), 0)$ under the linearized 2D β-plane equation in an infinite periodic channel of periodic 2π, $\mathbb{T}\times\mathbb{R}$.

The β-plane equation is a classical equation in geophysical fluid physics which describes the motion of perfect liquid under the influence of Coriolis effect. The β-plane equation in the momentum formulation is

$$\left.\begin{aligned}&\partial_t u+u\cdot\nabla u+(-fu^2,fu)^{\mathrm{T}}+\nabla p=0,\\&\nabla\cdot u=0\end{aligned}\right\}\quad(1)$$

where $(t,x,y)\in\mathbb{R}\times\mathbb{T}\times\mathbb{R}$, $u=(u^x, u^y)$, and $p:(t,x,y)\in\mathbb{R}\times\mathbb{T}\times\mathbb{R}\to\mathbb{R}$ are the velocity and the pressure of the fluid, respectively. Here $f:\mathbb{T}\times\mathbb{R}\to\mathbb{R}$ is the Coriolis force. The strength of the Coriolis force is usually assumed to be linear on the latitude, i.e., $f=f_0+\beta(y-y_0)$. More details about how to derive β-plane equation and physical background can be found in Refs.[1-3].

The 2D β-plane equation in the vorticity

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formulation is

$$\left.\begin{aligned}&\partial_t\omega+u\cdot\nabla\omega=\beta\partial_x\phi,\\&u=\nabla^{\perp}(\triangle)^{-1}\omega,\omega(t=0)=\omega_{\text{in}}\end{aligned}\right\}\quad(2)$$

Here $\nabla^{\perp}=(-\partial_y,\partial_x)$, $\phi=\triangle^{-1}\omega$, and β is a constant.

The study of linear stability of special solutions is an important and active topic in fluid mechanics. It began in the nineteenth century with the works of Stokes, Helmholtz, Reynolds, Rayleigh, Kelvin. Rayleigh[4] studied the linear stability and instability of planar inviscid shear flows using what is now referred to as normal mode methods. In fact Rayleigh's argument showed whether the operator's spectral instability or stability depends on whether or not an unstable eigenvalue exists. Meanwhile, Kelvin[5] constructed exact solutions to the linearized problem around the Couette flow. Physically, the inviscid damping can be understood as the transfer of enstrophy to small scales (high frequencies in frequency space) which yields the decay of the velocity by the Biot-Savart law. The transfer to small scales by mixing is now considered as a fundamental mechanism intimately connected with the stability of coherent structures and the theory of 2D turbulence[6-7]. Recently there has been great progress on fluid asymptotic stability at the linear level. Tung[2] studied shear flows $U(t,y)$ which satisfy $U'(t,y)=U'(t)$ for the viscous β-plane system. Lin and Zeng[9] used the explicit solution to establish inviscid damping in a finite periodic channel. Zillinger[9] applied the ghost weight method to get inviscid damping in both finite and infinite periodic channel for monotone shear flows. And recently Wei et al.[10-11] applied spectral methods to establish inviscid damping in the finite periodic channel for monotone shear flows. The nonlinear asymptotic stability problem for Couette flows under the two dimensional Euler equation in $\mathbb{R}\times\mathbb{T}$ was first proved by Ref.[12] and their methods were applied to many other models see for instance Refs.[12-14]. Bedrossian and Masmoudi[12] pointed out it would be a very interesting question to prove the decay of the β-plane model at the nonlinear level. In this paper, we consider the linear behavior first, which might give some clues to understanding the nonlinear problem raised by Ref.[12].

1 Main results

Linearizing the vorticity equation (2) around the Couette flow yields the following equation:

$$\left.\begin{aligned}&\partial_t\omega+y\partial_x\omega=\beta\partial_x\phi,\\&\omega=\triangle\phi,u=\nabla^{\perp}\phi,\omega(t=0)=\omega_{\text{in}}\end{aligned}\right\}\quad(3)$$

For the Couette flow, we have

Theorem 1.1 Let $\omega(t)$ be a solution to Eq. (3) with initial data $\omega_{\text{in}}\in H^s$ for $s=3$ satisfying $\int\omega_{\text{in}}\,\mathrm{d}x\,\mathrm{d}y=0$. Then we have

$$\|P_{\neq0}u^x(t)\|_2+\langle t\rangle\|u^y(t)\|_2\lesssim\frac{\|w_{\text{in}}\|_{H^s}}{\langle t\rangle},$$

where all implicit constants are independent of t and $P_{\neq0}f=f-\frac{1}{2\pi}\int f(x,y)\mathrm{d}x$.

For general monotonic shear flows, the linearized equation for (2) is

$$\left.\begin{aligned}&\omega_t+U(y)\partial_x\omega-U''(y)\partial_x\psi=\beta\partial_x\psi,\\&\omega=\triangle\psi,u=\nabla^{\perp}\psi,\\&\omega(t=0)=\omega_{\text{in}}\end{aligned}\right\}\quad(4)$$

The main result of this article is

Theorem 1.2 Let $\omega(t)$ be a solution of Eq. (4) with initial data $\omega_{\text{in}}\in H^s$ for $s=3$. If there exists a universal constant $\epsilon_0\leqslant\frac{1}{4}\mathrm{e}^{-\pi(8|\beta|+1)}$ such that $\|U'(y)-1\|_{H^6}\leqslant\epsilon_0$, then we have inviscid damping

$$\|P_{\neq0}u^x(t)\|_2+\langle t\rangle\|u^y(t)\|_2\lesssim\frac{1}{\langle t\rangle}\|w_{\text{in}}\|_{H^s}\quad(5)$$

where all implicit constants are independent of t and $P_{\neq0}f=f-\frac{1}{2\pi}\int f(x,y)\mathrm{d}x$.

Theorem 1.1 is proved by explicitly solving (3) via Fourier method. With the ghost weight method as Refs.[9,15], we will prove Theorem 1.2 by regarding the $\beta\partial_z\phi$ term as a perturbation for linearized Euler equation. The key is to construct a suitable weight which appropriately compensates the loss of derivatives in the energy argument.

Notation 1.1 We use the notation $f \lesssim g$ when there exists a constant $C>0$ independent of the parameters of interest such that $f \leqslant Cg$. $f \approx g$ means there exists some universal constant $C>1$ such that $\frac{1}{C} f \leqslant g \leqslant Cf$. The vector norm $|(k, \eta)| = |k| + |\eta|$ is used. For a given scalar or vector v in $\mathbb{R}^n$, we denote

$$\langle v \rangle = (1 + |v|^2)^{1/2}.$$

The x (or z) average of a function is denoted by: $\langle f \rangle = \frac{1}{2\pi}\int f(x,y)\mathrm{d}x = f_0$. The nonzero frequency projection is denoted by $P_{\neq 0} f = f - f_0$.

2 Proof of Theorem 1.1

By the change of variables

$$f(t,z,y) = w(t,z+ty,y) = w(t,x,y)$$

and $\phi(t,z,y) = \psi(t,z+ty,y) = \psi(t,x,y)$, (3) can be rewritten as

$$\left.\begin{aligned} \partial_t f &= \beta \partial_z \phi, \\ \triangle_L \phi &= f \end{aligned}\right\} \tag{6}$$

where $\triangle_L \phi = \partial_{zz}\phi + (\partial_y - t\partial_z)^2 \phi$. Using Fourier transformation we obtain f and ϕ in frequency space as follows:

$$\hat{f}(t,k,\eta) = \hat{\omega}_{\text{in}}(k,\eta)\exp\left[\beta \int_0^t \frac{ik}{k^2 + |\eta - \tau k|^2}\mathrm{d}\tau\right] \tag{7}$$

$$\hat{\phi}(t,k,\eta) = -\frac{\hat{f}(t,k,\eta)}{k^2 + |\eta - tk|^2} \tag{8}$$

From (7)～(8) and the bound $\frac{1}{k^2 + |\eta - tk|^2} \lesssim \frac{\langle \eta \rangle^2}{\langle kt \rangle^2}$ which holds for any non-zero integer k, we have

$$\| \phi_{\neq 0} \|_{H^{s-2}} = \left(\sum_{k \neq 0} \int \frac{|\hat{f}(t,k,\eta)|^2}{k^2 + |\eta - tk|^2} \mathrm{d}\eta \right)^{1/2} \lesssim \frac{1}{\langle t \rangle^2} \| w_{\text{in}} \|_{H^s}.$$

Then Theorem 1.1 follows from

$$u^x = -\partial_y \psi(t,x,y) = -\partial_y(\phi(t,x-ty,y)) = ((\partial_y - t\partial_x)\phi)(t,x-ty,y),$$

$$u^y(t,x,y) = \partial_x \psi(t,x,y) = \partial_x((\phi(t,x-ty,y)) = (\partial_x \phi)(x,x-ty,y).$$

3 Proof of Theorem 1.2

Now we turn to the proof of Theorem 1.2. We will use the ghost weight method introduced in Ref.[16].

Before we prove Theorem 1.2, we give two lemmas.

Lemma 3.1 For $\| g^2 - 1 \|_{H^{s+3}} + \| b \|_{H^{s+3}}$ sufficiently small, there holds

$$\left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{\triangle_L \phi}{\omega} \right\|_{H^s} \leqslant 4 \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{f}{\omega} \right\|_{H^s}.$$

Proof The key ingredient is the identity:

$$\triangle_L \phi = f + (1-g^2)(\partial_v - t\partial_z)^2 \phi - b(\partial_v - t\partial_z)\phi.$$

Direct calculations yield

$$\begin{aligned} \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{\triangle_L \phi}{\omega} \right\|_{H^s} \leqslant & \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{1}{\omega} (1-g^2)(\partial_v - t\partial_z)^2 \phi) \right\|_{H^s} + \\ & \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{1}{\omega} f \right\|_{H^s} + \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{1}{\omega} (b(\partial_v - t\partial_z)\phi) \right\|_{H^s} = \\ & L_1 + \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{1}{\omega} f \right\|_{H^s} + L_2. \end{aligned}$$

L_1 can be estimated as follows

$$\begin{aligned} L_1 = & \left\| \int_\xi \sqrt{\frac{\partial_t \omega}{\omega}}(t,k,\eta) \frac{\langle k,\eta \rangle^s}{\omega(t,k,\eta)} \widehat{(1-g^2)}(t,\eta-\xi)(\xi - tk)^2 \hat{\phi}(t,k,\xi)\mathrm{d}\xi \right\|_{L^2_{k,\eta}} \leqslant \\ & \mathrm{e}^{\pi(8|\beta|+1)} \left\| \int_\xi \langle \eta - \xi \rangle^{s+1} \widehat{(1-g^2)}(t,\eta-\xi)(\xi - tk)^2 \sqrt{\frac{\partial_t \omega}{\omega}}(t,k,\xi) \frac{\langle k,\xi \rangle^s}{\omega(t,k,\xi)} \hat{\phi}(t,k,\xi)\mathrm{d}\xi \right\|_{L^2_{k,\eta}}. \end{aligned}$$

Here we use

$$\sqrt{\frac{\partial_t \omega}{\omega}}(t,k,\eta) \leqslant \langle \eta - \xi \rangle \sqrt{\frac{\partial_t \omega}{\omega}}(t,k,\xi),$$

$$\frac{1}{\omega(t,k,\eta)} \leqslant e^{(8|\beta|+1)\pi} \frac{1}{\omega(t,k,\xi)}.$$

Then Young' inequality gives

$$L_1 \leqslant e^{\pi(8|\beta|+1)} \| 1-g^2 \|_{H^{s+3}} \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{\triangle_L \phi}{\omega} \right\|_{H^s}.$$

For L_2 we can get a similar estimate

$$L_2 \leqslant e^{\pi(8|\beta|+1)} \| b \|_{H^{s+3}} \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{\triangle_L \phi}{\omega} \right\|_{H^s}.$$

Therefore we get

$$\left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{\triangle_L \phi}{\omega} \right\|_{H^s} \leqslant e^{\pi(8|\beta|+1)} \| 1-g^2 \|_{H^{s+3}} \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{\triangle_L \phi}{\omega} \right\|_{H^s} + e^{\pi(8|\beta|+1)} \| b \|_{H^{s+3}} \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{\triangle_L \phi}{\omega} \right\|_{H^s} + \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{1}{\omega} f \right\|_{H^s}.$$

Hence we conclude that

$$\left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{\triangle_L \phi}{\omega} \right\|_{H^s} \leqslant \frac{1}{1-e^{\pi(8|\beta|+1)} \| 1-g^2 \|_{H^{s+3}} - e^{\pi(8|\beta|+1)} \| b \|_{H^{s+3}}} \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{1}{\omega} f \right\|_{H^s}.$$

Lemma 3.2 For $\| g^2-1 \|_{H^{s+3}} + \| b \|_{H^{s+3}}$ sufficiently small, there holds

$$\| \phi \|_{H^{s-2}} \lesssim \frac{1}{\langle t \rangle^2} \| f \|_{H^s}.$$

Proof Since

$$\frac{1}{k^2+|\eta-tk|^2} \lesssim \frac{\langle \eta^2 \rangle}{\langle t \rangle^2},\ k \neq 0.$$

Hence we deduce

$$\| \phi \|_{H^{s-2}} \lesssim \frac{1}{\langle t \rangle^2} \| \triangle_L \phi \|_{H^s}.$$

Combining Lemma 3.1, we obtain Lemma 3.2.

Proof of Theorem 1.2

Step one Change of variables

It is natural to introduce the following change of variables:

$$z = x - tU(y) \tag{9}$$

$$v = U(y) \tag{10}$$

By choosing ϵ_0 small enough, $\| U'-1 \|_{H^6} \leqslant \epsilon_0$ implies U is strictly monotone. Hence by the implicit theorem, this coordinate transform is always invertible and we can always solve x, y in terms of (z,v): $x = x(t,z,v)$, $y = y(v)$. Define the new variables

$$f(t,z,v) = w(t,z+tU(y),y) = w(t,x,y),$$
$$\phi(t,z,v) = \psi(t,z+tU(y),y) = \psi(t,x,y),$$

then (4) becomes

$$\left.\begin{aligned} &\partial_t f - b(v)\partial_z \phi = \beta\, \partial_z \phi, \\ &\triangle_t \phi = f \end{aligned}\right\} \tag{11}$$

where $\triangle_t \phi = \partial_{zz}\phi + (g(v))^2(\partial_v - t\partial_z)^2\phi + b(v)(\partial_v - t\partial_z)\phi$, $b(v) = U''(U^{-1}(v))$ and $g(v) = U'(U^{-1}(v))$.

Step two Toy model

As shown in Refs.[9,15], we consider the toy model for (11)

$$\left.\begin{aligned} &\partial_t g = (\beta - b)\partial_z \varphi, \\ &\partial_{zz}\varphi + (\partial_v - t\partial_z)^2 \varphi = \beta g \end{aligned}\right\} \tag{12}$$

for some $b, \beta \in \mathbb{R}$. The solution to (12) satisfies

$$\partial_t |\hat{g}|(t,k,\eta) \leqslant \frac{|\beta - b||k|}{k^2+|\eta-tk|^2} |\hat{g}|(t,k,\eta) \leqslant \frac{|b|}{1+\left|\frac{\eta}{k}-t\right|^2} |\hat{g}|(t,k,\eta).$$

Back to (11), in order to absorb the increase, we introduce a multiplier ω:

$$\left.\begin{aligned} &\partial_t \omega(t,k,\eta) = (8|\beta|+1) \frac{1}{1+\left|\frac{\eta}{k}-t\right|^2} \omega(t,k,\eta), \\ &\omega(0,k,\eta) = 1, k \neq 0 \end{aligned}\right\} \tag{13}$$

It is easy to see $e^{-\pi(8|\beta|+1)} \leqslant \omega^{-1} \leqslant 1$. Therefore we immediately get

$$\| f(t) \|_{H^s} \lesssim_\beta \left\| \frac{1}{\omega(t,\nabla)} f(t) \right\|_{H^s} \leqslant \| f(t) \|_{H^s} \tag{14}$$

Step three Variable coefficient estimates

For the solution to (11), direct calculations show

$$\frac{1}{2} \frac{d}{dt} \| \omega^{-1}(t,\nabla) f \|_{H^s}^2 = -\sum \int \frac{\partial_t \omega}{\omega^3} \langle k,\eta \rangle^{2s} |\hat{f}|^2 d\eta + (\omega^{-1} f, \omega^{-1}\partial_t f)_{H^s} = CK_\omega + L \tag{15}$$

where the CK_ω term is

$$CK_\omega = -\left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{1}{\omega} f \right\|_{H^s}^2.$$

For the L term we aim to get

$$L \leqslant \frac{2}{3} \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{1}{\omega} f \right\|_{H^s}^2 \tag{16}$$

If (16) holds, then we can absorb he CK_ω term and thus obtain

$$\| f \|_{H^s} \lesssim_\beta \left\| \frac{1}{\omega} f \right\|_{H^s} \leqslant \| \omega_{\text{in}} \|_{H^s}.$$

In order to prove (16), the key point is that one can approximate $\triangle_t \phi$ with $\triangle_L \phi$ in a rather specific manner, i.e.,

$$| L | = | (\omega^{-1}(t,\nabla) f, \omega^{-1}(\beta \partial_z \phi - b(v) \partial_z \phi_{H^s} | \leqslant \frac{1}{8} \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{\triangle_L \phi}{\omega} \right\|_{H^s} + \frac{1}{8|\beta|+1} \| b \|_{H^{s+3}} \left\| \sqrt{\frac{\partial_t \omega}{\omega}} \frac{\triangle_L \phi}{\omega} \right\|_{H^s} \tag{17}$$

where $\triangle_L \phi = \partial_{zz} \phi + (\partial_v - t\partial_z)^2 \phi$.

Combining Lemma 3.1 and (17), we see for $\| g^2 - 1 \|_{H^{s+3}} + \| b \|_{H^{s+3}}$ sufficiently small,

$$\| f \|_{H^s} \approx \| \omega(t,\nabla)^{-1} f \|_{H^s} \leqslant \| \omega_{\text{in}} \|_{H^s}.$$

Step four Final conclusion

By

$$u^x(t,x,y) = -U'(y)((\partial_v - t\partial_z)\phi)(t, x - tU(y), U(y)),$$

$$u^y(t,x,y) = (\partial_z \phi)(t, x - tU(y), U(y)),$$

and the assumption $\| U'(y) - 1 \|_{H^6} \leqslant \epsilon_0$, Theorem 1.2 is implied by Lemma 3.2:

$$\| u^x \|_{L^2} \lesssim \langle t \rangle \| \nabla \phi(t) \|_{H^1} \lesssim \langle t \rangle^{-1} \| w_{\text{in}} \|_{H^3},$$

$$\| u^y \|_{L^2} \lesssim \| \nabla \phi(t) \|_{H^1} \lesssim \langle t \rangle^{-2} \| w_{\text{in}} \|_{H^3}.$$

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