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Hadronic contribution from light by light processes in ($g-2$) of muon in nonlocal quark model

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Abstract: The hadronic corrections to the muon anomalous magnetic moment α_μ, due to the full gauge-invariant set of diagrams with dynamical quark loop and intermediate pseudoscalar and scalar states light-by-light scattering insertions, are calculated in the framework of the nonlocal chiral quark model. These diagrams correspond to all hadronic light-by-light scattering contributions to α_μ in the leading order of the $1/N_c$ expansion in quark model. The result of the quark loop contribution is $\alpha_\mu^{\mathrm{HLbL,Loop}} = (11.0 \pm 0.9) \cdot 10^{-10}$, and the total result is $\alpha_\mu^{\mathrm{HLbL,N\chi QM}} = (16.8 \pm 1.2) \cdot 10^{-10}$.

Key words: anomalous magnetic moment of muon; nonlocal model; light-by-light; chiral model**CLC number**: O572.3 **Document code**: A doi: 10.3969/j.issn.0253-2778.2016.06.002

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非局域夸克模型中强子 light-by-light 过程对缪子($g-2$)因子的贡献

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摘要:在非局域手征夸克模型框架下,计算了来源于全规范不变性动力学夸克圈图和中间赝标量介子和标量介子态圈图 light-by-lightt 散射过程对缪子反常磁矩 α_μ 的强子修正. 在夸克模型中,这些圈图对应于强子 light-by-light 散射过程对 α_μ 的最低价 $1/N_c$ 的贡献. 夸克圈图贡献的结果是 $\alpha_\mu^{\mathrm{HLbL,Loop}} = (11.0 \pm 0.9) \cdot 10^{-10}$,总贡献结果是 $\alpha_\mu^{\mathrm{HLbL,N\chi QM}} = (16.8 \pm 1.2) \cdot 10^{-10}$.

关键词:缪子反常磁矩; 非局域模型; light-by-light; 手征模型

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0 Introduction

The anomalous magnetic moment (AMM) of lepton and contribution of light by light (LbL) processes has a long history of investigation. After recent experiments on measurement of AMM of muon in Brookhaven National Laboratory (BNL) E821[1] the interest in this topic has returned. Two new experiments on measurement of AMM of muon are under construction in Fermilab[2] and J-PARC[3]. New precision data are demand more accurate calculations.

The most problematic part of the calculation AMM of muon is the segment associated with the strong interaction because most of this contribution is in nonperturbative low energy region. This contribution consist of hadron vacuum polarization (HVP) part (leading in α) and LbL scattering through the nonperturbative QCD vacuum (sub-leading in α). The HVP contribution can be extracted from the experimental data but the contribution of LbL scattering needs to be modeled.

What degrees of freedom (DoF) are relevant to modeling strong interaction at low energy: Mesons or quarks (and gluons)? This question is connected with the confinement problem and is one of the most important tasks in physics of strong interaction. One can separate two different approaches for description of LbL processes. In the first one the only mesonic DoF are used. The second one starts from quark Lagrangian and have mesonic DoF as a bound state.

1 Model

The LbL contribution to AMM of muon is in the low energy region where the perturbative methods of QCD are not applicable.

Nonlocal quark model NχQM is nonlinear realization of Nambu-Jona-Lasinio model. Nonlocality can be motivated by instanton liquid model. The model is formulated in terms of quark degrees of freedom and bound states corresponding to mesons. The circumscribing of the model is made in Refs. [4-5] and here we give a brief description of model properties that is needed for calculation of AMM.

1.1 Lagrangian

The Lagrangian of the $SU(3)$ nonlocal chiral quark model with the $SU(3)\times SU(3)$ symmetry has the form.

$$\mathscr{L} = \bar{q}(x)(i\hat{\partial} - m_c)q(x) + \frac{G}{2}[J_S^a(x)J_S^a(x) + J_{PS}^a(x)J_{PS}^a(x)] - \frac{H}{4}T_{abc}[J_S^a(x)J_S^b(x)J_S^c(x) - 3J_S^a(x)J_{PS}^b(x)J_{PS}^c(x)] \quad (1)$$

where $q(x)$ are the quark fields, $m_c(m_u = m_d \neq m_s)$ is the diagonal matrix of the quark current masses, G and H are the four-and six-quark coupling constants. The second line in the Lagrangian represents the Kobayashi-Maskawa-t′ Hooft determinant vertex with the structural constant

$$T_{abc} = \frac{1}{6}\epsilon_{ijk}\epsilon_{mnl}(\lambda_a)_{im}(\lambda_b)_{jn}(\lambda_c\alpha)_{kl} \quad (2)$$

where λ_a are the Gell – Mann matrices for a $=1,\cdots,8$ and $\lambda_0 = \sqrt{2/3}I$.

The nonlocal structure of the model is introduced via the nonlocal quark currents

$$J_M^a(x) = \int d^4x_1 d^4x_2 f(x_1) f(x_2)\bar{q}(x-x_2)\Gamma_M^a q(x+x_2) \quad (3)$$

where $M = S$ for the scalar and $M = PS$ for the pseudoscalar channels, $\Gamma_S^a = \lambda^a$, $\Gamma_{PS}^a = i\gamma^5\lambda^a$ and $f(x)$ is a form factor with the nonlocality parameter Λ reflecting the nonlocal properties of the QCD vacuum.

The model can be bosonized using the stationary phase approximation which leads to the system of gap equations for the dynamical quark masses $m_{d,i}$.

$$m_{d,i} + GS_i + \frac{H}{2}S_jS_k = 0 \quad (4)$$

with $i = u, d, s$ and $j, k \neq i$, and S_i is the quark loop integral

$$S_i = -8N_c\int\frac{d_E^4k}{(2\pi)^4}\frac{f^2(k^2)m_i(k^2)}{D_i(k^2)},$$

where $m_i(k^2) = m_{c,i} + m_{d,i}f^2(k^2)$, $D_i(k^2) = k^2 + m_i^2(k^2)$ is the dynamical quark propagator obtained by solving the Dyson-Schwinger equation, $f(k^2)$ is the nonlocal form factor in the momentum representation. For calculation we use two different form-factors: Gaussian form

$$f(p^2) = \exp(-\frac{p^2}{2\Lambda^2}) \quad (5)$$

monopole form

$$f(p^2) = (1 + \frac{p^2}{\Lambda^2})^{-1} \quad (6)$$

where Λ is the cutoff parameter. The model have five parameters which can be fitted on physical observables. In order to investigate the sensitivity of the model to the change in model parameters the dynamical mass of light quark varies between 200-350 MeV with corresponding refit of other parameters. This region corresponds to the more or less physical range of dynamical quark mass.

1.2 Meson propagator

The quark-meson vertex functions and the meson masses can be found from the solution of Bethe-Salpeter equation Fig. 1. For the separable interaction[5] the quark-antiquark scattering matrix in each (PS or S) channels becomes

$$\boldsymbol{T} = \dot{\boldsymbol{T}}(p^2)\delta^4(p_1 + p_2 - (p_3 + p_4))\prod_{i=1}^{4} f(p_i^2) \quad (7a)$$

$$\dot{\boldsymbol{T}}(p^2) = i\gamma_5\lambda_k(\frac{1}{-\boldsymbol{G}^{-1} + \prod(p^2)})_{ki} i\gamma_5\lambda_l \quad (7b)$$

where p_i are the momenta of external quark lines, $\boldsymbol{G}$ and $\prod(p^2)$ are the corresponding matrices of the four-quarkcoupling constants and the polarization operators of mesons($p = p_1 + p_2 = p_3 + p_4$). The meson masses can be found from the zeros of determinant $\det(\boldsymbol{G}^{-1} - \prod(-M^2)) = 0$. The $\dot{\boldsymbol{T}}$-matrix for the system of mesons in each neutral channelcan be expressed as

$$\dot{\boldsymbol{T}}_{ch}(p^2) = \sum_{M_{ch}} \frac{\bar{V}_{M_{ch}}(P^2) \otimes V_{M_{ch}}(P^2)}{-(P^2 + M_{ch}^2)}) \quad (8)$$

where M_M are the meson masses, and $V_M(P^2)$ are the vertex functions ($\bar{V}_M(p^2) = \gamma^0 V_M^+(P^2)\gamma^0$). The sum in Eq. (8) is over the full set of light mesons: ($M_{PS} = \pi^0, \eta, \eta'$) in the pseudoscalar channel and ($M_S = a_0(980), f_0(980), \sigma$) in the scalar one.

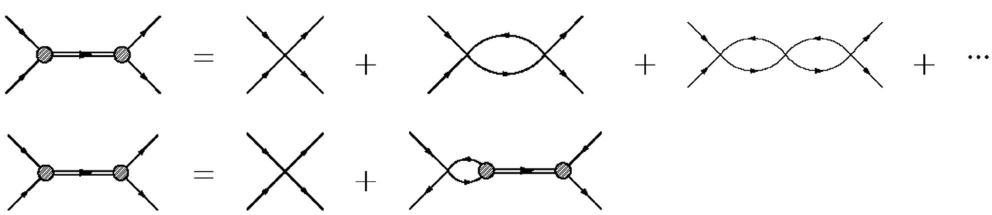

Fig. 1 The set of diagram with four-quarks interaction vertex can be represented by pure four-quarks vertex and sum of diagram that will be associated with meson exchange

Details about the vertex of interaction of mesons with quarks, meson propagator, etc. can be found in Refs. [4, 6-7].

1.3 Interaction with external photons

The next step for description LbL processes is to introduce in the nonlocal chiral Lagrangian (Eq. (1)) the gauge-invariant interaction with an external photon field $A_\mu(z)$ by Schwinger factor (Eq. (9)). In the result we obtain infinite series of vertexes quark – antiquark interactions with photons.

$$q(y) \to Q(x,y) = p\exp\{i\int_x^y dz^\mu A_\mu(z)\} q(y) \quad (9)$$

The scheme, based on the rules that the derivative of the contour integral does not depend on the path shape

$$\frac{\partial}{\partial y^\mu}\int_x^y dz^\nu F_\nu(z) = F_\mu(z),\ \delta^{(4)}(x-y)\int_x^y dz^\nu F_\nu(z) = 0,$$

was suggested in Ref. [8] and applied to nonlocal models in Ref. [9]. The actual form of the vertexes shown in Fig. 2 can be found in Ref. [10].

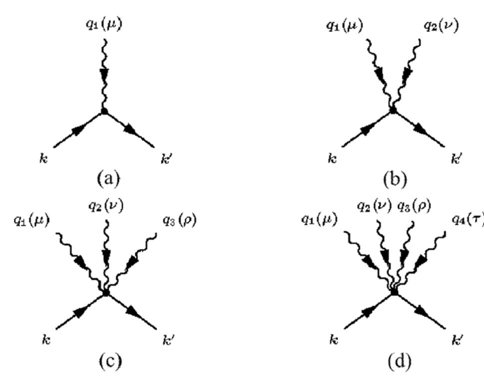

Fig. 2 The quark-photon vertex $\Gamma_\mu^{(1)}(q)$, $\Gamma_{\mu\nu}^{(2)}(q_1, q_2)$, $\Gamma_{\mu\nu\rho}^{(3)}(q_1, q_2, q_3)$, $\Gamma_{\mu\nu\rho\tau}^{(4)}(q_1, q_2, q_3, q_4)$

1.4 Box diagram

In effective quark model under consideration there are two different parts which correspond to contact contribution or contribution with the intermediate meson.

Using quark-antiquark interactions vertexes with one, two, three or four photons, see Fig. 3 (The numbers in front of the diagrams are the combinatoric factors), we can build five types of diagrams.

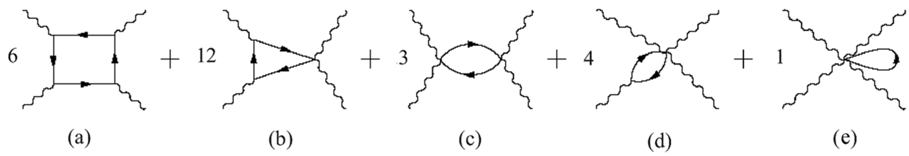

Fig. 3 The box diagram and the diagrams with nonlocal multiphoton interaction vertices that give the contributions to $\prod_{\mu\nu\lambda\rho}(q_1, q_2, q_3)$

The contribution of the box diagram with the local vertices, Fig. 3(a), is the dot (olive) line (Loc); the box diagram, Fig. 3(a), with the nonlocal parts of the vertices is the dash line (NL_1); the triangle, Fig. 3(b), and loop, Fig. 3(c), diagrams with the two-photon vertices is the dash-dot line (NL_2); the loop with the three-photon vertex, Fig. 3(d), is the dot-dot line (NL_3); the loop with the four-photon vertex, Fig. 3(e), is the dash-dot-dot line (NL_4); the sum of all contributions (Total) is the solid line. At zero all contributions are finite. Only the sum of all diagrams is gauge invariant and corresponds to contact (or quark loop) contribution.

2 LbL in AMM of muon

The contribution of light by light process to AMM of muon has the form:

$$a_\mu^{\mathrm{LbL}} = \frac{e^6}{48m_\mu}\int\frac{\mathrm{d}^4 q_1}{(2\pi)^4}\int\frac{\mathrm{d}^4 q_2}{(2\pi)^4}\times \frac{\prod_{\rho\mu\nu\lambda\sigma}(q_2, -q_3, q_1)\,T^{\rho\mu\nu\lambda\sigma}(q_1, q_2, p)}{q_1^2 q_2^2 q_3^2((p+q_1)^2-m_\mu^2)((p+q_2)^2-m_\mu^2)} \tag{10}$$

where the tensor $T^{\rho\mu\nu\lambda\sigma}$ is the Dirac trace

$$T^{\rho\mu\nu\lambda\sigma}(q_1, q_2, p) = \mathrm{Tr}((\hat{p}+m_\mu[\gamma^\rho,\gamma^\sigma](\hat{p}+m_\mu)\times \gamma^\mu(\hat{p}-\hat{q}_2+m_\mu)\gamma^\nu(\hat{p}+\hat{q}_1+m_\mu)\gamma^\lambda).$$

Taking the Dirac trace, the tensor $T^{\rho\mu\nu\lambda\sigma}$ becomes a polynomial in the momenta p, q_1, q_2.

After that, it is convenient to convert all momenta into the Euclidean space, and we will use the capital letters P, Q_1, Q_2 for the corresponding counterparts of the Minkowskian vectors p, q_1, q_2, e.g. $P^2=-p^2=-m_\mu{}^2$, $Q_1{}^2=-q_1{}^2$, $Q_2^2=-q_2^2$. Then Eq. (10) becomes

$$\left.\begin{aligned} a_\mu^{\mathrm{LbL}} &= \frac{e^6}{48m_\mu}\int\frac{\mathrm{d}_E^4 Q_2}{(2\pi)^4}\int\frac{\mathrm{d}_E^4 Q_2}{(2\pi)^4}\frac{1}{Q_1^2Q_2^2Q_3^2}\frac{T^{\rho\mu\nu\lambda\sigma}\prod_{\rho\mu\nu\lambda\sigma}}{D_1D_2} \\ D_1 &= (P+Q_1)^2=m_\mu^2=2(P\cdot Q_1)+Q_1^2,\ x \\ D_2 &= (P-Q_2)^2=m_\mu^2=2(P\cdot Q_2)+Q_2^2 \end{aligned}\right\} \tag{11}$$

Since the highest order of the power of the muon momentum P in $T^{\rho\mu\nu\lambda\sigma}$ is two * and $\prod_{\rho\mu\nu\lambda\sigma}$ is independent of P, the factors in the integrand of Eq. (11) can be rewritten as

$$\frac{T^{\rho\mu\nu\lambda\sigma}\prod_{\rho\mu\nu\lambda\sigma}}{D_1D_2} = \sum_{a=1}^{6}A_a\tilde{\prod}_a \tag{12}$$

with the coefficients

$$\left.\begin{aligned} A_1 &= \frac{1}{D_1},\ A_2=\frac{1}{D_2},\ A_3=\frac{(P\cdot Q_2)}{D_1} \\ A_4 &= \frac{(P\cdot Q_2)}{D_2},\ A_5=\frac{1}{D_1D_2},\ A_6=1 \end{aligned}\right\} \tag{13}$$

where all P-dependence is included in the A_a factors, while $\tilde{\prod}_a$ are P-independent.

Then, one can average over the direction of the muon momentum P (as was suggested in Ref. [11] for the pion-exchange contribution)

$$\int\frac{\mathrm{d}_E^4 Q_1}{(2\pi)^4}\int\frac{\mathrm{d}_E^4 Q_2}{(2\pi)^4}\frac{A_a}{Q_1^2Q_2^2Q_3^2}\cdots = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}Q_1\int_0^\infty \mathrm{d}Q_2\int_{-1}^{1}\mathrm{d}t\ \sqrt{1-t^2}\ \frac{Q_1Q_2}{Q_3}\langle A_a\rangle\cdots \tag{14}$$

where the radial variables of integration $Q_1\equiv|Q_1|$ and $Q_2\equiv|Q_2|$ and the angular variable $t=(Q_1\cdot Q_2)/(|Q_1|\,|Q_2|)$ are introduced. The averaged A_a factors was introduced in Ref. [11].

3 Density function

For investigation of the dependence of contribution from photon legs virtuality one can watch for "density function". This is the function which corresponds to the LbL contribution to AMM before integration over intermediate photons virtualities.

$$\rho^{\mathrm{LbL}}(Q_1,Q_2)=\frac{Q_1Q_2}{2\pi^2}\sum_{a=1}^{6}\int_{-1}^{1}\mathrm{d}t\,\frac{\sqrt{1-t^2}}{Q_3^2}\langle A_a\rangle\widetilde{\prod}_a \quad (15)$$

The volume under 3D-density function is full contribution to AMM of muon. In Fig. 4, this function is shown for the nonlocal model in leading $1/N_c$ order.

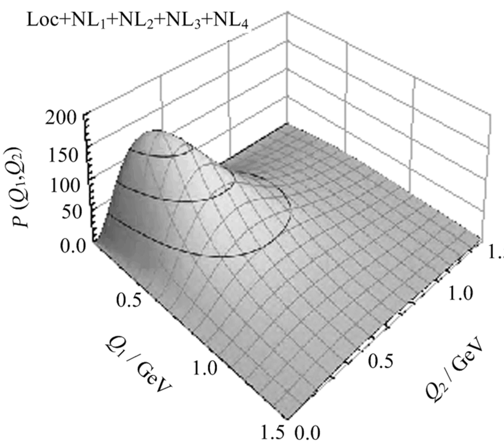

Fig. 4 The 3D density $\rho(Q_1,Q_2)$ defined in Eq. (15)

One can see that contribution is mainly localized in a range of virtualities of photons around 1 GeV, as shown in Fig. 5. Different curves correspond to the contributions of topologically different sets of diagrams drawn in Fig. 3.

In Fig. 5, the slice of $\rho^{\mathrm{HLbL}}(Q_1,Q_2)$ in the diagonal direction $Q_2=Q_1$ is presented together with the partial contributions from the diagrams of different topology. One can see, that the $\rho^{\mathrm{HLbL}}(0,0)=0$ is due to a nontrivial cancellation of different diagrams of Fig. 3. This important result is a consequence of gauge invariance and the spontaneous violation of the chiral symmetry, and represents the low energy theorem analogous to the theorem for the Adler function at zero momentum. Another interesting feature is that the large Q_1, Q_2 behavior is dominated by the box diagram with

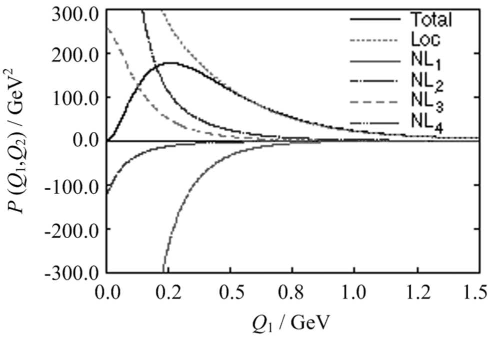

Fig. 5 The 2D slice of the density $\rho(Q_1,Q_2)$ at $Q_2=Q_1$

local vertices and quark propagators with momentum-independent masses in accordance with the perturbative theory. All these are very important characteristics of the NχQM, interpolating the well-known results of the chiral perturbative theory at low momenta and the operator product expansion at large momenta. Earlier, similar results were btained for the two-point [12] and three-point[13] correlators.

4 Conclusion

The contribution to AMM of muon from LbL process in NχQM corresponds to contributions from contact term and term with intermediate pseudoscalar and scalar channels. The contact term contribution is

$$a_\mu^{\mathrm{HLbL,Loop}}=(11.0\pm0.9)\cdot10^{-10} \quad (16)$$

and the total contribution is estimated as

$$a_\mu^{\mathrm{HLbL}}=16.8(1.25)\cdot10^{-10} \quad (17)$$

where the error bar is the band in the region of physical dynamical mass.

In Fig. 6, one can see that it is important, at least in the framework of quark model, to take into account not only diagrams with intermediate mesons but also the contact term (or quark loop) contribution. The solid curve is total contribution, the others dashed curves: contact terms in Fig. 3 and meson exchange Refs. [6-7], respectively.

In comparison with other model calculations, our results are quite close to the recent results obtained in Refs. [14-15]. The specific feature of our model and Dyson-Schwinger approach[14] is that due to nonlocal interaction kernel the quarks become dynamical ones

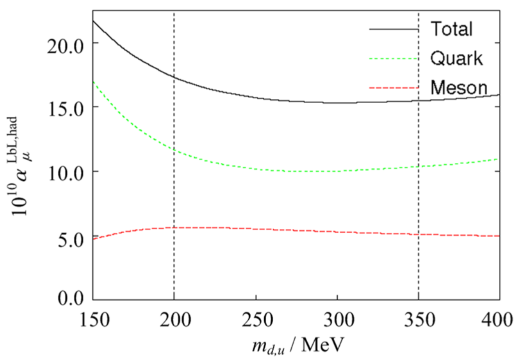

Fig. 6 The contribution of muon AMM from LbL depending on the dynamical mass of quark $m_{d,u}$ in zero order on $1/N_c$

with momentum-dependent mass. The predictions of the NχQM for the different contributions to the muon $g-2$ are in agreement with Ref. [14] within 10%.

The next step of calculations is to extend the quark model in order to estimate subleading in $1/N_c$ terms. This subleading contribution from diagrams with the meson loop has a negative sign[16].
To solve the puzzle of LbL contribution, we should to better understand the physics of strong interaction at a long distance. In principle one can do this with more accurate measurement of meson form factors.

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