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An applicable multiple-player's quantum market game

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Abstract: The quantum game of the classical Bertrand's duopoly model was generalized to N-player case. In a quantized game, the more entanglement is involved, the higher maximal profits it will be. It monotonously increases until the optimal collusive profit, which is restricted, and cannot be achieved in its classical game. With partial information entanglement between two adjacent firms, the generalizing evolutionary N-player Bertrand's model not only solved the Bertrand paradox, but also achieved a practical result.

Key words: quantum game; quantum entanglement; Bertrand model

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可扩展的多人市场模型的量子博弈

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摘要:将古典经济学模型中的Bertrand双寡头模型推广到了N人的情况.在量子化的博弈中,随着所加纠缠的增大,收益也增大.并且,收益值将不断增长直至达到最优的"合作收益"值.而这个"合作收益"的值在经典博弈的非合作情况中是无法达到的.我们的量子化方法是,在相邻两公司间加入部分信息纠缠.推广到N人的Bertrand模型不但解决了Bertrand模型中的悖论,而且得到一个较为可行的结论.

关键词:量子博弈;量子纠缠;Bertrand模型

0 Introduction

Quantum game, as an application of quantum computation, is very different from classical game, and attracts broad interest. In initial papers[1-2], we see the quantum characters, such as the quantum entanglement, break bounds of classical rules and primarily cause the improvement of results[3-10,15-16].

In our previous work, we discussed the quantum market game of Cournot's duopoly[11-12]. That is about two homogeneous-product firms, based upon the premise that each firm holds outputs as variables. Warranted with this

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assumption, each oligopolistic firm maximizes its profit by considering the outputs of the other constant. In Cournot's duopoly, the price in the market depends on the outputs. It is considered that the market requirement quantity adjusts according to the price of the market. As the market requirement is determined by factual outputs of the producers in the oligopoly, there will be a price variable dominating the whole competition, and the output of the firms should be adjusted to follow it. Another important oligopoly model is Bertrand's duopoly model[13-14]. It assumes the price as its initial variable, which accords to the situation shown above. Comparing to the Cournot's duopoly, we could get more practical results through Bertrand's duopoly model. Therefore the Bertrand's duopoly is more factual and more applicable.

We compared quantum market game to classical market game mainly on the profits of Nash equilibrium based upon the same duopoly model. The reason is for maximizing each firm's profits, the non-cooperative competition ultimately forms a Nash equilibrium situation. However, based on non-cooperative competition, Nash equilibrium yields a maximized but not optimal profit result, which is less than that of cooperative competition. Analogously, for market competition, when participating firms increase in number, it is difficult to collaborate for higher profits. In this case the final situation will also be a Nash equilibrium. We can regard it as multiple-player game. Being a little different from duopoly, it forms this Nash equilibrium more spontaneously, and with less adscititious assumption. However, it still has the same dilemma. The classical non-cooperative market game, unlike the cooperative one, cannot achieve the optimal profit result since it encounters an upper limit.

In the present study, firstly, we give a quantized Bertrand's duopoly to see what can be gained from this quantizing. Then, for generalized discussion, we consider a multiply-player market, and the corresponding classical game is evolved from Bertrand's duopoly model with the same symmetric rule. Correspondingly, a theoretical multiply-player quantizing structure is given. Like Ref. [11], we use the entire unitary operator to entangle different firm's information, and change the initial state before gaming. The entanglement degree γ is also used to represent the intercommunion degree.

1 The quantum game of Bertrand's model

In a duopoly market competition, we suppose firm 1 chooses p_1 as the price of its products, and firm 2 p_2. Due to practical supply and demand relation, the production quantity q_i can be determined by each firm's variables p_i and p_j. Expressions are as follows:

$$q_i(p_i, p_j) = a - p_i + bp_j. \tag{1}$$

All of these are based on the condition that $a > 0$, $0 < b < 1$. We use i, j to distinguish different firms ($i, j = 1, 2$). In this equation, a is a parameter determined by the actual market conditions, and b indicates the negative interaction since the productions are homogeneous. Suppose that the same unit cost of production is c, the profits will be quantity multiplied by profit per unit production:

$$\mathrm{U}_i(p_i, p_j) = (a - p_i + bp_j)(p_i - c). \tag{2}$$

At the unique Nash equilibrium, each firm can get its maximal profit p_1^*, p_2^* by setting $p_1 = p_1^* = \frac{a+c}{2-b}$, and $p_2 = p_2^* = \frac{a+c}{2-b}$. Then, the profits of the the two firms are

$$\mathrm{U}^{\mathrm{C}}(p_1^*, p_2^*) = \mathrm{U}_1(p_1^*, p_2^*) = \mathrm{U}_2(p_1^*, p_2^*) = \frac{(a - c + bc)^2}{(2-b)^2}. \tag{3}$$

The superscript "C" means "classical" condition[9].

In the quantized Nash equilibrium of Bertrand's duopoly, we suppose the symmetrical profits are

$$U_1^Q(x_1^*, x_2^*, \gamma) = U_2^Q(x_1^*, x_2^*, \gamma) = U^Q(\gamma).$$

The superscript "Q" means it is for "Quantized" condition. Then the profits of the two firms are

$$U^Q(\gamma) = (a - c + bc)^2 \cdot \frac{(e^{2\gamma}+1)[(1-b)e^{2\gamma}+1+b]}{4[(1-b)e^{2\gamma}+1]^2} > 0, \quad (4)$$

which is monotonously increasing for

$$\frac{\partial U^Q(\gamma)}{\partial \gamma} > 0. \quad (5)$$

The profit $U^Q(\gamma)$ reaches its minimum when $\gamma \to -\infty$, and reaches its maximum when $\gamma \to \infty$. Profits of equilibrium are monotonously increasing when γ increases. That is the strongpoint of this model. Therefore we can change the results. For example, to improve the profit $U^Q(\gamma)$, we can control the parameter γ, which is involved in the overall operation $J(\gamma)$ and $J(\gamma)^\dagger$. And the extent of increase is mainly determined by parameter b. The dominating trends of profit $U^Q(\gamma)$ when parameter γ and b increases are given in Fig. 1. For pithiness, we consider this $\frac{U^Q(\gamma)}{(a-c+bc)^2}$ part.

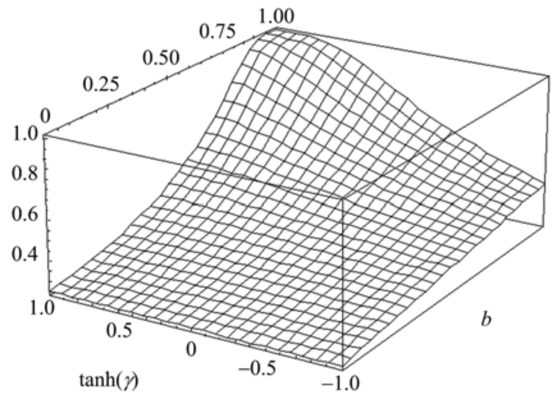

Fig. 1 The profit of quantized Bertrand's Duopoly $U^Q(\gamma)$, with respect to $\tanh(\gamma)$, which monotonously maps $\gamma \in (-\infty, \infty)$ into $\tanh(\gamma) \in (-1, 1)$, and the parameter b, using $\tanh\left(\frac{U^Q(\gamma)}{(a+bc-c)^2}\right)$ as z-axis

For the information is symmetric in this Bertrand's duopoly, we generalize the quantizing structure from two-player to N-player, using the same symmetrical model. Then we use it to quantizing an evolutionary Bertrand's duopoly which is different from Ref. [10], and give the comparing conclusion at last.

1.1 A generalized N-player game structure

We assume that there are symmetrical N firms in the market. The sales of firm j are affected by the other N−1 firms for they all sell the homogenous products. We use parameter j to denote firm 0, firm 1 to firm N−1, respectively. Suppose that the influence of the other firms are faint, the total effect of the other N−1 firms is less than the firm j itself. Then, we give the symmetrical function of q_j:

$$q_j(p_0, p_1, \cdots, p_{N-1}) = a - p_j + \frac{b}{N-1} \cdot (p_0 + p_1 + \cdots + p_{j-1} + p_{j+1} + \cdots + p_{N-1}). \quad (6)$$

Analogously, the profit is

$$U_j(p_0, p_1, \cdots, p_{N-1}) = \Big[a - p_j + \frac{b}{N-1} \cdot (p_0 + p_1 + \cdots + p_{j-1} + p_{j+1} + \cdots + p_{N-1})\Big](p_j - c). \quad (7)$$

There are N firms and N equations of profit U_j. At the Nash equilibrium,

$$p^* = p_0^* = p_1^* = \cdots = p_{N-1}^* = \frac{a+c}{2-b},$$

and the maximal profits of firm j are

$$U^{(n)C} = U_j(p_0^*, p_1^*, \cdots, p_{N-1}^*) = \frac{(a-c+bc)^2}{(2-b)^2}. \quad (8)$$

It is positive on the condition $a > 0$ and $0 < b < 1$. We can see $U^{(n)C}$ is independent of the number of players N. But if we consider a Collusive Profit $G_\omega(p_0, p_1, \cdots, p_{N-1})$, which indicates the whole profit of all the firms, there is

$$G_\omega(p_0, p_1, \cdots, p_{N-1}) = U_0(p_0, p_1, \cdots, p_{N-1}) + U_1(p_0, p_1, \cdots, p_{N-1}) + \cdots + U_{N-1}(p_0, p_1, \cdots, p_{N-1}). \quad (9)$$

It is a symmetrical situation, and the prices of each firm are equal at the maximal profit situation: $p_0 = p_1 = \cdots = p_{N-1}$. We denote it by price p_ω^*. For the maximal Collusive Profit,

$$p_\omega^* = \frac{a+c-bc}{2(1-b)},$$

and the individual profits are

$$U_\omega^{(n)} = U_j(p_\omega^*, p_\omega^*, \cdots, p_\omega^*) = \frac{(a-c+bc)^2}{4(1-b)}. \quad (10)$$

All of that has been considered above are under the condition $0<b<1$, and there will always be

$$\frac{(a-c+bc)^2}{4(1-b)} > \frac{(a-c+bc)^2}{(2-b)^2}.$$

It means that at the point of the maximal Collusive Profit, the individual profits of participants are always better than those at the Nash equilibrium of the non-cooperation game:

$$U_{co}^{(n)} > U^{(n)C}. \qquad (11)$$

In other words, the maximal profit of the non-cooperation game is not the optimal one. This result is obvious. The difference coming from that "collusive profit" is based on cooperative competition, while "Nash equilibrium" is the maximum of noncooperation — For most actual conditions, Nash equilibrium describes the realistic competition state rightly. When there are N firms in the market, the game participants prefer to depart from the cooperation, and trend to achieve its maximal profits selfishly. This will cut down the opponents' profits.

1.2 The N-player quantizing structure

We use the quantizing structure as shown in Fig. 2, which is an expanding N-player's version of Ref. [11]. Quantum entanglement is involved as partial information entanglement between two adjacent firms. This will lead to a quantizing result different from Ref. [10]. And we will discuss the the superiority of this model later. Consider using N single-mode electromagnetic fields as N parts to implement this quantum structure, of which the quadrature amplitudes have a continuous set of eigenstates, to represent the continuous-variable in the Bertrand competition. In this figure, a series of operations are listed in turn.

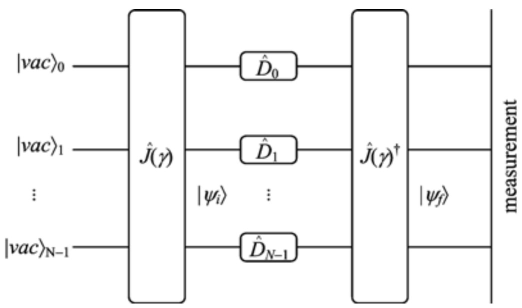

Fig. 2 The quantizing structure of N-player

This structure starts from the state $|vac\rangle_0 \otimes |vac\rangle_1 \otimes \cdots \otimes |vac\rangle_{N-1}$, which is a tensor product of N vacuum states. They represent N different parts: firm 0, firm 1, …, and firm $N-1$, respectively. Consequently, there is an exoteric unitary operation $\hat{J}(\gamma)$, implemented on every part. In electromagnetic fields, $\hat{J}(\gamma)$ is symmetric with respect to the interaction of different abutting electromagnetic fields. For conciseness, we choose $\hat{J}(\gamma)$ in the following form to describe this tangly action:

$$\hat{J}(\gamma) = \exp\{i\gamma(\hat{X}_0\hat{P}_1 + \hat{X}_1\hat{P}_2 + \hat{X}_2\hat{P}_3 + \cdots + \hat{X}_{N-1}\hat{P}_0)\}, \qquad (12)$$

where the "position" functor $\hat{X}_j = (\hat{a}_j^\dagger + \hat{a}_j)/\sqrt{2}$ is an observable variable, which can be measured finally. Correspondingly, the "momentum" functor $\hat{P}_j$ is $\hat{P}_j = i(\hat{a}_j^\dagger - \hat{a}_j)/\sqrt{2}$. $\hat{a}_j^\dagger(\hat{a}_j)$ is the creation (annihilation) functor, related to the electromagnetic fields. In this $\hat{J}(\gamma)$, the "position" functor of firm j is only entangled with the "momentum" functor of its abutting firm $J+1$, and we overlook the interaction between the "position" functor and the "momentum" functor of firm j itself. After this operation $\hat{J}(\gamma)$, we get an entangled initial state $|\psi_i\rangle$:

$$|\psi_i\rangle = \hat{J}(\gamma)(|vac\rangle_0 |vac\rangle_1 \cdots |vac\rangle_{N-1}), \qquad (13)$$

which can also be regarded as the beginning for this market competition. After this, firms undergo an operation $\hat{D}_j$ on their state, respectively, which is used to express different information of private strategy. These operations hold privately. With the strategy form $\hat{D}_j(x_j) = \exp(-ix_j\hat{P}_j)$, different firms choose different strategies from the strategy set:

$$S_j = \{\hat{D}_j(x_j) = \exp(-ix_j\hat{P}_j) \mid x_j \in (-\infty,\infty)\}, \qquad (14)$$

in which, $\hat{P}_j$ is also the "momentum" operators, yet x_j (not the "position" functor) a strategy parameter, which is classical and continuously changed. Subsequently, we use another unitary operation, denoted by functor $\hat{J}(\gamma)^{+}$, to meet with the operation of the first step. Finally, the whole state comes to

$$|\psi_f\rangle = \hat{J}(\gamma)^{+}(\hat{D}_0 \otimes \hat{D}_1 \otimes \cdots \otimes \hat{D}_{N-1}) \cdot \hat{J}(\gamma)\{|vac\rangle_0 |vac\rangle_1 \cdots |vac\rangle_{N-1}. \quad (15)$$

Through detailed calculation, we get a formal result for N-player model. For convenience, we use an expression $F(\gamma)_h$ instead of complicated progression expression:

$$F(\gamma)_h = \sum_{m=0}^{\infty} \frac{\gamma^{h+mN}}{(h+mN)!},$$

as $h=0,1,\cdots,N-1$, respectively. Therefore, the final state is

$$|\psi_f\rangle = \exp\{-i[\sum_{k=0}^{N-1} x_k F(\gamma)_{(-k)\bmod N}]\hat{P}_0\} |vac\rangle_0 \otimes \exp\{-i[\sum_{k=0}^{N-1} x_k F(\gamma)_{(1-k)\bmod N}]\hat{P}_1\} |vac\rangle_1 \otimes \cdots \otimes \exp\{-i[\sum_{k=0}^{N-1} x_k F(\gamma)_{(j-k)\bmod N}]\hat{P}_j\} |vac\rangle_j \otimes \cdots \otimes \exp\{-i[\sum_{k=0}^{N-1} x_k F(\gamma)_{(N-1-k)\bmod N}]\hat{P}_{N-1}\} |vac\rangle_{N-1}. \quad (16)$$

After measurement, we denote measured results with the price variable p_j of quantized market competition for $j=0,1,2,\cdots,N-1$:

$$p_j(x_0, x_1, \cdots, x_{N-1}) = \sum_{k=0}^{N-1} x_k F(\gamma)_{(j-k)\bmod N}. \quad (17)$$

1.3 The quantized N-player game

Using the quantizing structure we introduced, the price p_j can be expressed with the entanglement degree γ. Then, in the quantized N-player competition, the quantized profit $U_j^{(n)Q}$ is

$$U_j^{(n)Q}(x_0, x_1, \cdots, x_{N-1}) = \left[a - p_j + \frac{b}{N-1}(p_0 + p_1 + \cdots + p_{j-1} + p_{j+1} + \cdots + p_{N-1})\right](p_j - c) = \left[a - \frac{N-1+b}{N-1}\sum_{k=0}^{N-1} x_k F(\gamma)_{(j-k)\bmod N} + \frac{b}{N-1}e^{\gamma}\sum_{k=0}^{N-1} x_k\right](\sum_{k=0}^{N-1} x_k F(\gamma)_{(j-k)\bmod N} - c). \quad (18)$$

At the Nash equilibrium, there is

$$x^* = x_0 = x_1 = \cdots = x_{N-1},$$

while the formal value of x^* is

$$x^* = \frac{a(N-1)F(\gamma)_0 + c[(N-1+b)F(\gamma)_0 - be^{\gamma}]}{(2N-2+2b-Nb)e^{\gamma}F(\gamma)_0 - be^{2\gamma}}.$$

Then, the profit of firm j is

$$U_j^{(n)Q}(\gamma) = (a-c+bc)^2 \cdot \frac{(N-1)F(\gamma)_0[(b-1+N)F(\gamma)_0 - be^{\gamma}]}{[be^{\gamma} + (bN-2N+2-2b)F(\gamma)_0]^2} > 0, \quad (19)$$

which is the same for these N firms when $\gamma=0$. The competition is the same as that of the corresponding classical N-player game. There are

$$U_j^{(n)Q}(\gamma)|_{\gamma=0} = \frac{(a-c+bc)^2}{(2-b)^2}. \quad (20)$$

When $\gamma \to \infty$, $F(\gamma)_0$ tends to a non-progression expression:

$$F(\gamma)_0 \to \frac{e^{\gamma}}{N}. \quad (21)$$

In this condition, the profit $U_j^{(n)Q}(\gamma)$ of firm j will be

$$U_j^{(n)Q}(\gamma)|_{\gamma\to\infty} = \frac{(a-c+bc)^2}{4(1-b)}, \quad (22)$$

which is equal to the classical optimal Collusive Profit $U_{\infty}^{(n)}$ in the cooperative situation.

When the positive entanglement $\gamma \to \infty$, quantized competition reaches the optimal profit, which cannot be reached in classical Nash equilibrium. Hence, the Bertrand Paradox is solved. Similar to preterit quantum games, we improved profits by quantizing the model, and achieved the optimal profit results. In our particular quantizing structure, what has been quantized is the initial state by a unitary tangly operation. Since this operation is totally public before game, we can regard the entanglement degree γ as the intercommunion among different firms, which is determined by external restriction. The strategies x_j are still classical. Based on an applicable N-player market competition, we give one quantum algorithm to improve the optimal profit from the one in the non-cooperative market game to a cooperative case. Further, we find there is no parameter N in the profit. $U_j^{(n)Q}(\gamma)|_{\gamma\to\infty}$, meaning that with the maximal positive entanglement, profit remains unchanged whatever N is.

Our quantizing model is with partial information entanglement between two adjacent firms, while the quantizing model in Ref.[10] is complete information entanglement existing between each two firms. And this leads to different results. Let's focus on the special case: $\gamma\to\infty$. According to Eq.(5) in Ref.[10], the price of each firm is the same at this time, no matter which of the price strategies each firm takes. This price is $\gamma p_i=\frac{1}{N}\sum_{i=1}^{N}x_i$. When $N\to\infty$, $p_i\to 0$ and $U_i<0$. This is not rational as the price p_i should be constraint by the cost of production c at least ($p_i>c$). In contrast, we used a different quantizing N-player Bertrand game, and the problem didn't exist in such results. In $\gamma\to\infty$ of our results, no matter how much N is the profits of each company are kept to

$$U_j^Q(\gamma)|_{\gamma\to\infty}=\frac{(a-c+bc)^2}{4(1-b)}.$$

This profit result is the same as the full cooperation in classical game, which is more like a win-win competition result.

2 Conclusion

We generalized a quantized Bertrand's duopoly quantum game to the N-player case. For the maximal positive entanglement $\gamma\to\infty$, the profit in quantized Bertrand's duopoly is always better than its classical counterpoint. As the original Bertrand's duopoly is practical and the N-player model is general, it is an applicable optimized algorithm for market competition. In another point of view, the quantized equilibrium at $\gamma\to\infty$ can be improved to a maximal collusive profit, which is just the same as the classical cooperative situation. Different from Ref.[10], this is much more like a win-win competition result. This generalizing model not only solves the Bertrand paradox, but also achieves a practical result.

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