

Design and analysis of a universal model reference adaptive controller based on extended state observer

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Abstract: A universal model reference adaptive control (MRAC) method based on extended state observer (ESO) was presented, which was used to directly estimate states and uncertainties and then compensate the uncertainties. Benefiting from the ESO, we only need to know the order of a system without distinction between linear and nonlinear system, time-varying and time-invariant system, the internal (parameter or structure) uncertainty and the external (disturbance) uncertainty etc. Furthermore, the reference model which meets desired performance index and has the same order with the system can be chosen arbitrarily largely independently of the system structure. Therefore, the controller design process is greatly simplified. After proposing the control architecture, the control law was designed and a strict stability analysis was given. The features mentioned above in addition to strong robustness, small control moment and high steady state accuracy are demonstrated by the simulation results.

Key words: model reference adaptive control; extended state observer

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基于扩张状态观测器的通用模型参考自适应控制设计与分析

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摘要: 提出一种基于扩张状态观测器(ESO)的任意参考模型自适应控制方法, 该方法利用 ESO 直接估计系统状态, 且对不确定性进行估计并补偿。基于 ESO 的模型参考自适应控制(MRAC), 仅需要知道被控对象的阶次, 而不必区分线性、非线性, 时变、时不变, 内部(参数、结构)不确定性、外部(扰动)不确定性等。此外, 当参考模型与被控对象同阶并在满足期望性能指标的前提下, 对被控对象模型依赖程度很低, 简化了设计过程。首先, 提出了基于 ESO 的 MRAC 控制器结构, 并设计了控制律, 进而给出了稳定性分析, 最后进行了仿真验证。仿真结果表明, 该方法具有鲁棒性好、控制量少、稳态精度高等优点。

关键词: 模型参考自适应控制; 扩张状态观测器

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0 Introduction

For a system with internal (parameter or structure) uncertainty or external (disturbance) uncertainty is difficult to control, adaptive control theory had been proposed. So far, the major adaptive control methods, which are more mature and widely used in application, are model reference adaptive control and self-tuning regulators. The main concept of MRAC is to promote the output of the system to track a reference model that has been constituted in advance, thereby accommodating the performance index of the system.

State feedback MRAC needs to obtain all state information accurately, which is usually not practical for many practical systems. Aiming at this problem, the following methods are usually used; adaptive laws based on transfer function^[1]; designing an adaptive observer to estimate the unknown parameters and states of the system^[2]. In addition, the output feedback adaptive control is also an important research direction, see e. g. Refs. [3-6] and the references therein. Note that there are often external disturbance and measurement noise to a system, in addition to un-modeled dynamics. However, adaptive laws designed based on stability theory can only guarantee the stability of the ideal system nominally. Adaptive control system is a non-linear system, hence the impact of a simplified model, external disturbance and noise should be taken into account, otherwise the system stability cannot be guaranteed. This gave rise to the robustness problem of the model reference adaptive control systems^[7]. In order to minimize the negative influence of internal or external system uncertainties, since the 1980s there have been lots of work done on robust adaptive control^[8-11].

Recently, a novel L1 adaptive controller was developed, which ensures uniformly bounded transient response for system's both signals, input and output, simultaneously in Refs. [12-13]. This new architecture has a low-pass filter in the

feedback loop and relies on the small-gain theorem for the proof of asymptotic stability. In Ref. [14], the methodology was extended to systems with unknown time-varying parameters and bounded disturbances in the presence of unknown high-frequency gain; In Ref. [15], the paper extended the approach to the un-modeled actuator dynamics system; In Ref. [16], the extending approach was utilized in a class of system with unknown nonlinearities; In Ref. [17], the paper extended the methodology to a class of systems in the presence of un-modeled dynamics; In Ref. [18], the methodology was extended to time-varying reference systems in the presence of un-modeled dynamics.

It seems that several important aspects are worthy of taking into consideration in these papers. Firstly, the L_1 adaptive controller needs to adjust its architecture, adaptation law and control law according to different system types. In other words, it is much too dependent on model rather than universals. As we all know, proportional-integral-derivative (PID) control law is widely used in application for its simple architecture and less dependence on models. Secondly, as mentioned above, several methods such as neural networks have been used to improve the robustness of adaptive control. However, as presented in Refs. [19-20], the design of neural networks is usually complicated and there still remains several difficult problems to solve. Finally, much effort has been done in the design of full-state feedback adaptive controllers. What's more, the problem of design and analysis of output feedback adaptive control becomes more complicated due to the absence of a damping factor introduced by the controller^[21].

In this paper, we propose a new model reference adaptive controller based on extended state observer (ESO-MRAC). Robustness and adaptability of ESO is stronger than an ordinary observer/adaptive observer, and with better effect of estimating the disturbance^[22-24]. The presented

controller possesses the following characteristics:

(I) It is simple and universal, requiring only the order of the system, without any distinction between linear and nonlinear, time-varying and time-invariant, internal (parameter or structure) uncertainty and external (disturbance) uncertainty, etc.

(II) The reference model which meets desired performance index and has the same order with the system can be chosen arbitrarily independently of the system structure.

(III) Since the uncertainties are compensated by ESO, the controller has strong robustness, small control moment and high steady state accuracy.

1 Mathematical model formulation of ESO

Consider a generally non-linear time-varying dynamic system with single-input single-output described by

$$\left. \begin{aligned} \dot{x}^{(n)} &= f(x(t), \dot{x}(t), \dots, x^{(n-1)}(t), \omega(t)) + b_0 r(t) \\ y &= x \end{aligned} \right\} \quad (1)$$

where $\omega(t)$ is the external disturbance and b_0 is a given constant, r is the control input, y is the regulated output. Here, $f(x(t), \dot{x}(t), \dots, x^{(n-1)}(t), \omega(t))$ simply denoted as $f(\cdot)$, represents the unknown non-linear time-varying dynamics of the plant. That is, for this plant, only the order and the parameter b_0 are given. To this end, assuming $f(\cdot)$ is differentiable and letting $h = \dot{f}(\cdot)$, $x_{n+1}(t) = f(\cdot)$. Treat the $x_{n+1}(t)$ as an extended state of system (1), then system (1) can be written in an augmented state form as:

$$\left. \begin{aligned} x(t) &= x_1(t), \dot{x}(t) = x_2(t), \dots, x^{(n-1)}(t) = x_n(t) \\ x^{(n)}(t) &= x_{n+1}(t) + b_0 r \\ x^{(n+1)}(t) &= h(x, \omega) \\ y &= x_1(t) \end{aligned} \right\} \quad (2)$$

where $\mathbf{x} = [x_1, x_2, \dots, x_{n+1}]^T \in R^{n+1}$, $u \in R$ and $y \in R$ are the state, input and output of the system, respectively. Any state observer of system (2), will estimate the derivatives of y and $f(\cdot)$ since

the latter is now a state in the extended state model. Such observers are known as ESO. A particular ESO is given as follows:

$$\left. \begin{aligned} e &= x_1 - \hat{x}_1 \\ \dot{\hat{x}}_1 &= \hat{x}_2 + l_1 g_1(e) \\ \dot{\hat{x}}_2 &= \hat{x}_3 + l_2 g_2(e) \\ &\vdots \\ \dot{\hat{x}}_n &= \hat{x}_{n+1} + l_n g_n(e) + b_0 r \\ \dot{\hat{x}}_{n+1} &= l_{n+1} g_{n+1}(e) \end{aligned} \right\} \quad (3)$$

where $\hat{\mathbf{x}} = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_{n+1}]^T \in R^{n+1}$, g_1, g_2, \dots, g_{n+1} are proper functions to be chosen, l_1, l_2, \dots, l_{n+1} are the observer gain parameters to be chosen. This ESO is able to estimate $x(t), \dot{x}(t), \dots, x^{(n)}(t)$ as well as $x_{n+1}(t)$, namely $\hat{x}_1(t) \rightarrow x_1(t), \dots, \hat{x}_n(t) \rightarrow x_n(t), \hat{x}_{n+1}(t) \rightarrow x_{n+1}(t) = f(\cdot)$. We adopt control signal as $r = r_0 - \hat{x}_{n+1}(t)/b_0$, then system (1) can be rewritten as:

$$\left. \begin{aligned} \dot{x}^{(n)} &= b_0 r_0 \\ y &= x \end{aligned} \right\} \quad (4)$$

In general, g_1, g_2, \dots, g_{n+1} are defined as $g_i(e, \alpha_i, \delta) = \text{fal}(e, \alpha_i, \delta)$, and the observer is called non-linear extended state observer (NLESO). The nonlinear function $\text{fal}(\cdot)$ is described by

$$\text{fal}(e, \alpha_i, \delta) = \begin{cases} e/\delta^{1-\alpha_i}, & |e| \leq \delta \\ |e|^{\alpha_i} \text{sgn}(e), & |e| > \delta \end{cases} \quad (5)$$

where δ is a positive constant, and when $\alpha < 1$, the function $\text{fal}(\cdot)$ has the following property: small error and big gain; big error and small gain.

For convenience, we can obtain linear extended state observer (LESO) by adopting g_1, g_2, \dots, g_{n+1} as simple linear functions. However, LESO's efficiency is much lower. LESO is described by

$$\left. \begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + l_1(x_1 - \hat{x}_1) \\ &\vdots \\ \dot{\hat{x}}_{n-1} &= \hat{x}_n + l_{n-1}(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_n &= \hat{x}_{n+1} + l_n(x_1 - \hat{x}_1) + bu \\ \dot{\hat{x}}_{n+1} &= l_{n+1}(x_1 - \hat{x}_1) \end{aligned} \right\} \quad (6)$$

Gao^[25] presented a simple method to select the LESO gain parameters based on bandwidth, as follows:

$$[l_1 \quad l_2 \quad \cdots \quad l_{n+1}] = [\omega_0 \alpha_1 \quad \omega_0^2 \alpha_2 \quad \cdots \quad \omega_0^{n+1} \alpha_{n+1}] \quad (7)$$

where $\omega_0 > 0$. Here, $\alpha_i (i = 1, 2, \dots, n + 1)$ are selected such that the characteristic polynomial $s^{n+1} + \alpha_1 s^n + \cdots + \alpha_n s + \alpha_{n+1}$ is Hurwitz. For simplicity, let $s^{n+1} + \alpha_1 s^n + \cdots + \alpha_n s + \alpha_{n+1} = (s + 1)^{n+1}$, where $\alpha_i = \frac{(n+1)!}{i! (n+1-i)!}$, $i = 1, 2, \dots, n+1$. Then the characteristic polynomial is equal to $(s + \omega_0)^n$, and ω_0 becomes the only tuning parameter of the observer.

2 ESO-MRAC

2.1 Control law design of ESO-MRAC

Fig.1 shows the closed-loop ESO-MRAC architecture, where $i = 1, 2, \dots, (n - 1)$, and assuming $[r_1, r_2, \dots, r_n, r_{n+1}] = [r, \dot{r}, \dots, r^{(n-1)}, r^{(n)}]$. The dotted line means that when the state is measurable, we can either make use of the real state information directly or the estimated state information from the extended state observer. Given that the extended state observer parameters are set reasonably, the estimated state can track the real state well, and we assume that $\hat{x}_i = x_i$ here. Consider the non-linear system (1), and its reference model is described by

$$r^{(n)} = f(\dot{r}, \ddot{r}, \dots, r^{(n-1)}) + b_0 r(t) \quad (8)$$

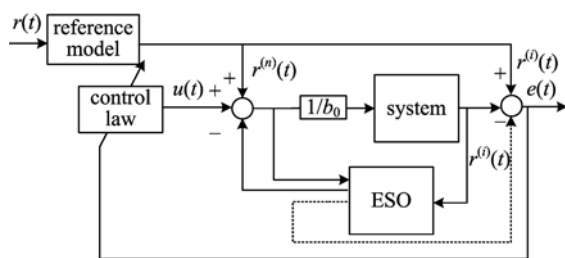


Fig.1 Closed-loop ESO-MRAC architecture

Assuming

$$e^{(i)} = x^i - r^i, \quad i = 1, 2, \dots, n \quad (9)$$

$$u(t) = - \sum k_i e^{(i)}, \quad i = 1, 2, \dots, (n-1) \quad (10)$$

Then we obtain the input of the object

$$r(t) = (r^{(n)}(t) + u(t) - \hat{x}_{n+1}(t))/b_0 \quad (11)$$

Notice that

$$x^{(n)}(t) = x_{n+1}(t) + b_0 r(t) \quad (12)$$

Substituting Eq. (11) into Eq. (12) gives

$$x^{(n)}(t) = r^{(n)}(t) + u(t) - \hat{x}_{n+1}(t) + x_{n+1}(t) \quad (13)$$

and then one has

$$\sum k_i e^{(i)} = - \hat{x}_{n+1}(t) + x_{n+1}(t), \quad i = 1, 2, \dots, n \quad (14)$$

According to the assumption

$$\hat{x}_{n+1}(t) = x_{n+1}(t) \quad (15)$$

We have

$$\sum k_i e^{(i)} = 0, \quad i = 1, 2, \dots, n \quad (16)$$

Let $k_i, i = 1, 2, \dots, n$ be some constants. For this high order constant coefficient homogeneous linear differential equation, its characteristics equation is

$$\sum k_i r^{(i)} = 0, \quad i = 1, 2, \dots, n \quad (17)$$

Let the real part of equation root be negative enough, that is $\text{real}(r_i) < 0$. Besides

$$\sum k_i r^{(i)} = \prod_{i=1}^n (r - r_i) = 0 \quad (18)$$

We can get $k_i, i = 1, 2, \dots, n$ by equation expansion. For convenience, we consider that the characteristics equation has single negative root, then the tracking error

$$\lim_{t \rightarrow \infty} e(t) = \lim_{t \rightarrow \infty} \sum C_i e^{r_i t} = 0, \quad i = 1, 2, \dots, n \quad (19)$$

In other cases, $\lim_{t \rightarrow \infty} e(t) = 0 (i = 1, 2, \dots, n)$ is also attainable as long as $\text{real}(r_i) < 0$ is guaranteed, according to the solution structure of n -order constant coefficient homogeneous linear differential equation.

The deduction above is based on the assumption $x_i = \hat{x}_i$. As a matter of fact, it is difficult to guarantee that $x_i = \hat{x}_i$ is strictly equal. We give a strict stability analysis to make sure the system is stable and the tracking error is bounded.

2.2 Stability analysis

In Refs. [23, 26], LESO was studied observation error bounded under a class of disturbance respectively from different point of view, and in Ref. [27] the convergence of NLESO is proved. Combining these results, we present here the system stability analysis. It follows by

two steps: Firstly, we prove that LESO state estimation error is bounded; secondly, we prove that the closed-loop ESO-MRAC system is stable and the tracking error is bounded under the bounded error of state estimation both by LESO and ESO.

2.2.1 Convergence of the LESO

Let $\tilde{x}_i = x_i - \hat{x}_i$ ($i = 1, 2, \dots, n + 1$), which denote the state estimation error. From Eqs. (2), (5) and (6), the observer estimation error can be shown as:

$$\left. \begin{aligned} \dot{\tilde{x}}_1 &= \tilde{x}_2 - \omega_0 \alpha_1 \tilde{x}_1 \\ &\vdots \\ \dot{\tilde{x}}_{n-1} &= \tilde{x}_n - \omega_0^{n-1} \alpha_{n-1} \tilde{x}_1 \\ \dot{\tilde{x}}_n &= \tilde{x}_{n+1} - \omega_0^n \alpha_n \tilde{x}_1 \\ \dot{\tilde{x}}_{n+1} &= h(x, \omega) - \omega_0^{n+1} \alpha_{n+1} \tilde{x}_1 \end{aligned} \right\} \quad (20)$$

Now let $\epsilon_i = \frac{\tilde{x}_i}{\omega_0^{i-1}}$ ($i = 1, 2, \dots, n + 1$), then system (20) can be rewritten as:

$$\dot{\epsilon} = \omega_0 \mathbf{A} \epsilon + \mathbf{B} \frac{h(x, \omega)}{\omega_0^n} \quad (21)$$

where

$$\mathbf{A} = \begin{bmatrix} -\alpha_1 & 1 & 0 & \cdots & 0 \\ -\alpha_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\alpha_n & 0 & \cdots & 0 & 1 \\ -\alpha_{n+1} & 0 & \cdots & 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{(n+1) \times 1}.$$

Here, α_i ($i = 1, 2, \dots, n + 1$) are selected such that \mathbf{A} is Hurwitz.

Theorem 2.1 Assuming $h(x, \omega)$ is bounded, there exist a constant $\sigma_i > 0$ and a finite $T_1 > 0$ such that $|\tilde{x}_i(t)| \leq \sigma_i, i = 1, 2, \dots, n + 1, \forall t \geq T_1 > 0$ and $\omega_0 > 0$. Furthermore, $\sigma_i = o\left(\frac{1}{\omega_0^k}\right)$, for some positive integer k .

Proof Solving Eq. (21), it follows that

$$\epsilon(t) = e^{\omega_0 A t} \epsilon(0) + \int_0^t e^{\omega_0 A(t-\tau)} \mathbf{B} \frac{h(x(\tau), \omega)}{\omega_0^n} d\tau \quad (22)$$

Let

$$p(t) = \int_0^t e^{\omega_0 A(t-\tau)} \mathbf{B} \frac{h(x(\tau), \omega)}{\omega_0^n} d\tau \quad (23)$$

since $h(x(\tau), \omega)$ is bounded, that is, $h(x(\tau), \omega) \leq \delta$,

where δ is a positive constant, for $i = 1, 2, \dots, n + 1$, we have

$$|p_i(t)| \leq \frac{\delta}{\omega_0^{n+1}} [|(\mathbf{A}^{-1} \mathbf{B})_i| + |(\mathbf{A}^{-1} e^{\omega_0 A t} \mathbf{B})_i|] \quad (24)$$

For \mathbf{A} and \mathbf{B} defined in Eq. (21),

$$\mathbf{A}^{-1} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -\frac{1}{\alpha_{n+1}} \\ 1 & 0 & \cdots & 0 & -\frac{\alpha_1}{\alpha_{n+1}} \\ 0 & 1 & \cdots & 0 & -\frac{\alpha_2}{\alpha_{n+1}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -\frac{\alpha_n}{\alpha_{n+1}} \end{bmatrix},$$

$$|(\mathbf{A}^{-1} \mathbf{B})_i| \leq v \quad (25)$$

where $v = \max_{i=2, \dots, n+1} \left\{ \frac{1}{\alpha_{n+1}}, \frac{\alpha_{i-1}}{\alpha_{n+1}} \right\}$. Since \mathbf{A} is Hurwitz, there exists a finite time $T_1 > 0$ such that

$$[e^{\omega_0 A t}]_{ij} \leq \frac{1}{\omega_0^{n+1}} \quad (26)$$

for all $t \geq T_1, i = 1, 2, \dots, n + 1$. Hence

$$[e^{\omega_0 A t} \mathbf{B}]_i \leq \frac{1}{\omega_0^{n+1}} \quad (27)$$

$$|(\mathbf{A}^{-1} e^{\omega_0 A t} \mathbf{B})_i| \leq \frac{\mu}{\omega_0^{n+1}} \quad (28)$$

for all $t \geq T_1, i = 1, 2, \dots, n + 1$, where

$$\mu = \max_{i=2, \dots, n+1} \left\{ \frac{1}{\alpha_{n+1}}, 1 + \frac{\alpha_{i-1}}{\alpha_{n+1}} \right\}.$$

From Eqs. (24), (25) and (28), we obtain

$$|p_i(t)| \leq \frac{\delta v}{\omega_0^{n+1}} + \frac{\delta \mu}{\omega_0^{2n+2}} \quad (29)$$

for all $t \geq T_1, i = 1, 2, \dots, n + 1$. Let

$$\epsilon_{\text{sum}}(0) = |\epsilon_1(0)| + |\epsilon_2(0)| + \cdots + |\epsilon_{n+1}(0)|.$$

It follows that

$$|[e^{\omega_0 A t} \epsilon(0)]_i| \leq \frac{\epsilon_{\text{sum}}(0)}{\omega_0^{n+1}} \quad (30)$$

for all $t \geq T_1, i = 1, 2, \dots, n + 1$. From Eq. (22), one has

$$|\epsilon_i(t)| \leq |[e^{\omega_0 A t} \epsilon(0)]_i| + |p_i(t)| \quad (31)$$

Let $\tilde{x}_{\text{sum}}(0) = |\tilde{x}_1(0)| + |\tilde{x}_2(0)| + \cdots + |\tilde{x}_{n+1}(0)|$.

According to $\epsilon_i = \frac{\tilde{x}_i}{\omega_0^{i-1}}$ and Eqs. (29) ~ (31),

we have

$$|\tilde{x}_i(t)| \leq \left| \frac{\tilde{x}_{\text{sum}}(0)}{\omega_0^{n+1}} \right| + \frac{\delta v}{\omega_0^{n-i+2}} + \frac{\delta \mu}{\omega_0^{2n-i+3}} = \sigma_i \quad (32)$$

for all $t \geq T_1, i=1, 2, \dots, n+1$. Q. E. D.

As shown in Eq. (32), the estimation error of the system LESO (6) is bounded and its upper bound monotonously decreases with the observer bandwidth.

2.2.2 Stability analysis of MRAC based on LESO

Theorem 2.2 Assuming $h(x, \omega)$ is bounded, there exist a constant $\rho_i > 0$ and a finite time $T_2 > 0$ such that $|e_i(t)| \leq \rho_i, i=1, 2, \dots, n+1, \forall t \geq T_2 > 0$, and $\omega_c > 0$. Furthermore, $\rho_i = o\left(\frac{1}{\omega_c^j}\right)$ for some positive integer j .

Proof From Eqs. (10) and (14), we obtain

$$(f - \hat{x}_{n+1}) + k_i(r_i - \hat{x}_i) + r_{n+1} - x_{n+1} = 0 \quad (33)$$

Let $e_i = r_i - x_i (i=1, 2, \dots, n)$, we have

$$\left. \begin{aligned} \dot{e}_1 &= \dot{r}_1 - \dot{x}_1 = r_2 - x_2 = e_2 \\ &\vdots \\ \dot{e}_{n-1} &= \dot{r}_{n-1} - \dot{x}_{n-1} = r_n - x_n = e_n \\ \dot{e}_n &= \dot{r}_n - \dot{x}_n = r_{n+1} - x_{n+1} \\ &\quad - \sum_{i=1}^n k_i(e_i + \tilde{x}_i) - \tilde{x}_{n+1} \end{aligned} \right\} \quad (34)$$

Let $e = [e_1, e_2, \dots, e_n]^T \in R^n, \tilde{x} = [\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_{n+1}]^T \in R^{n+1}$, then

$$\dot{e}(t) = A_e e(t) + A_{\tilde{x}} \tilde{x}(t) \quad (35)$$

where

$$A_e = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -k_1 & -k_2 & -k_3 & \dots & -k_n \end{bmatrix},$$

$$A_{\tilde{x}} = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 \\ -k_1 & -k_2 & \dots & -k_n & -1 \end{bmatrix}.$$

Solving system (34), we have

$$e(t) = e^{A_e t} e(0) + \int_0^t e^{A_e(t-\tau)} A_{\tilde{x}} \tilde{x}(\tau) d\tau \quad (36)$$

According to Eq. (34) and Theorem 2.1, one has

$$\left. \begin{aligned} [A_{\tilde{x}} \tilde{x}(\tau)]_{i=1, \dots, n-1} &= 0 \\ |[A_{\tilde{x}} \tilde{x}(\tau)]_n| &\leq k_n \sigma_n = \gamma, \forall t \geq T_1 \end{aligned} \right\} \quad (37)$$

where $k_s = 1 + \sum_{i=1}^n k_i$. Let $k_i = \frac{n!}{(i-1)!(n+1-i)!}$ $\omega_c^{n+1-i} (i=1, 2, \dots, n)$, such that A_e is Hurwitz.

Define $\psi = [0 \ 0 \ \dots \ 0 \ \gamma]^T$. Let

$$\varphi(t) = \int_0^t e^{A_e(t-\tau)} A_{\tilde{x}} \tilde{x}(\tau) d\tau.$$

It follows that

$$\varphi(t) = \int_0^t e^{A_e(t-\tau)} A_{\tilde{x}} \tilde{x}(\tau) d\tau,$$

we have

$$|\varphi_i(t)| \leq |(A_e^{-1} \psi)_i| + |(A_e^{-1} e^{A_e t} \psi)_i| \quad (38)$$

and

$$\left. \begin{aligned} |(A_e^{-1} \psi)_1| &= \frac{\gamma}{k_1} = \frac{\gamma}{\omega_c^n} \\ |(A_e^{-1} \psi)_i|_{i=2, \dots, n} &= 0 \end{aligned} \right\} \quad (39)$$

Since A_e is Hurwitz, there exists a finite time $T_2 > 0$ such that

$$|[e^{A_e t}]_{ij}| \leq \frac{1}{\omega_c^{n+1}} \quad (40)$$

for all $t \geq T_2 (i, j=1, 2, \dots, n)$. Note that T_2 depends on A_e . Let $T_3 = \max\{T_1, T_2\}$. It follows that

$$|[e^{A_e t} \psi]_i| \leq \frac{\gamma}{\omega_c^{n+1}} \quad (41)$$

for all $t \geq T_3 (i, j=1, 2, \dots, n)$, and

$$|(A_e^{-1} e^{A_e t} \psi)_i| \leq \begin{cases} \left| \frac{1 + \sum_{i=1}^n k_i}{\omega_c^n} \frac{\gamma}{\omega_c^{n+1}} \right|_{i=1} \\ \left| \frac{\gamma}{\omega_c^{n+1}} \right|_{i=2, \dots, n} \end{cases} \quad (42)$$

for all $t \geq T_3$. From Eqs. (38), (39) and (42), we obtain:

$$|\varphi_i(t)| \leq \begin{cases} \left| \frac{\gamma}{\omega_c^n} + \frac{1 + \sum_{i=2}^n k_i}{\omega_c^n} \frac{\gamma}{\omega_c^{n+1}} \right|_{i=1} \\ \left| \frac{\gamma}{\omega_c^{n+1}} \right|_{i=2, \dots, n} \end{cases} \quad (43)$$

for all $t \geq T_3$. Let $e_{\text{sum}}(0) = |e_1(0)| + |e_2(0)| + \dots + |e_n(0)|$. It follows that

$$|[e^{A_e t} e(0)]_i| \leq \frac{e_s(0)}{\omega_c^{n+1}} \quad (44)$$

for all $t \geq T_3 (i, j=1, 2, \dots, n)$. From Eq. (36), one has

$$|e_i(t)| \leq |e^{A_s t} e(0)|_i + |\varphi_i(t)| \quad (45)$$

According to Eqs. (39), (43)~(45), we have

$$|e_i(t)| \leq \begin{cases} \left| \frac{e_s(0)}{\omega_c^{n+1}} + \frac{k_s \sigma_i}{\omega_c^n} + \frac{(1 + \sum_{i=2}^n k_i) k_s \sigma_i}{\omega_c^{2n+1}} \right|_{i=1} \leq \rho \\ \left| \frac{e_s(0) + k_s \sigma_i}{\omega_c^{n+1}} \right|_{i=2, \dots, n} \end{cases} \quad (46)$$

for all $t \geq T_3$ ($i, j=1, 2, \dots, n$), where

$$\rho_i =$$

$$\max \left\{ \frac{e_s(0)}{\omega_c^{n+1}} + \frac{k_s \sigma_i}{\omega_c^n} + \frac{(1 + \sum_{i=2}^n k_i) k_s \sigma_i}{\omega_c^{2n+1}}, \frac{e_s(0) + k_s \sigma_i}{\omega_c^{n+1}} \right\}.$$

Q. E. D.

As shown in Eq. (46), the tracking error and its up to $(n-1)$ th order derivatives of LESO-MRAC are bounded and their upper bounds monotonously decrease with the controller bandwidth.

Furthermore, on the basis of the conclusion drawn in Ref. [27] that the estimation error of the NLESO is bounded, the tracking error and its up to $(n-1)$ th order derivatives of NLESO-MRAC are also bounded and their upper bounds monotonously decrease with the controller bandwidth according to Eq. (46). In a word, the closed-loop system is stable and boundedly convergent as long as LESO or NLESO is bounded convergent.

3 Simulations

In this section, two examples are used to illustrate the practical performance by simulation.

Example 3.1 Consider a second order linear system with uncertainty described by

$$\left. \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -50x_2 - 50x_1 + \text{sign}(\sin(t)) + 3r \\ y &= x_1 \end{aligned} \right\} \quad (47)$$

Its second order reference model is described by

$$\left. \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -3x_2 - 3x_1 + 3r \\ y &= x_1 \end{aligned} \right\} \quad (48)$$

NLESO gain parameters: $l_1 = 40$, $l_2 = 400$, $l_3 = 800$, control laws gain parameters: $k_1 = 80$, $k_2 = 250$. For a second order system, k_1 and k_2 have a clear physical meaning, and they correspond to the differential and proportion in traditional PID. To illustrate the effectiveness of the presented method, we compare it with state feedback adaptive control without compensation, output feedback adaptive control (no differential, $k_d=0$) and PID (for models (48) and (47) with fixed parameters, that are $k_p=2$, $k_i=1$, $k_d=2$) by step response. Besides, the parameters remain the same in the whole process. The simulation results are shown in Figs. 2~7.

As shown in Fig. 2, the simulation results show that ESO-MRAC can well track the output of the object reference model and has the following advantages:

(I) As shown in Fig. 2, the PID with the fixed parameters is suitable for model (48), however, its performance become very poor when the system has a total disturbance of $-47x_2 - 47x_1 + \text{sign}(\sin(t))$, which is not a problem for ESO-MRAC.

(II) As shown in Figs. 3 and 4, the ESO can estimate system states well, and in Fig. 5, the ESO-MRAC is better than output feedback adaptive control. In this case, the output feedback adaptive control is equal to proportional control, and the ESO-MRAC is actually proportional-derivative control. Therefore, the ESO-MRAC is superior to the output feedback adaptive control.

(III) As shown in Fig. 6, the steady-state accuracy can be improved by compensation based on ESO. The ESO not only inhibits constant disturbance, but also eliminates almost any form of disturbance. Although the traditional error integral feedback can also inhibit constant disturbance, it often makes the closed-loop system slow in operation and easy to oscillate. At the same time, it also makes the control moment easy to saturate, which brings about other negative effects. Neural networks can also estimate and

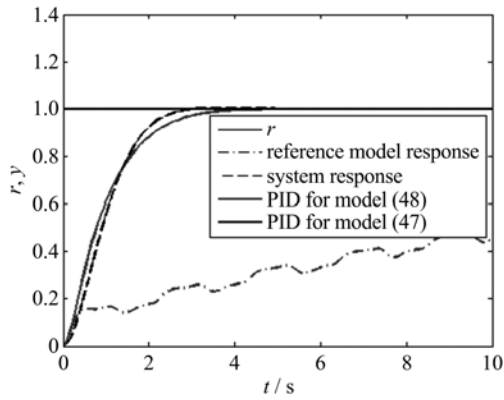


Fig. 2 Diagram of step response

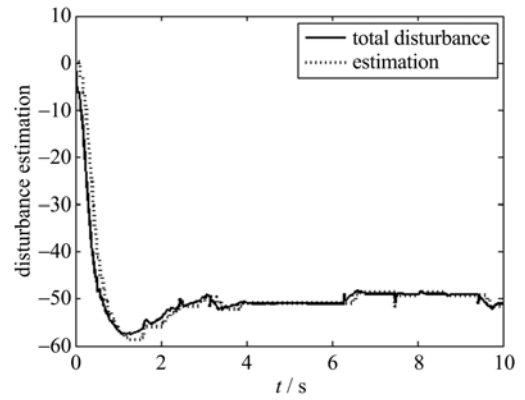


Fig. 3 The total disturbance and its estimation

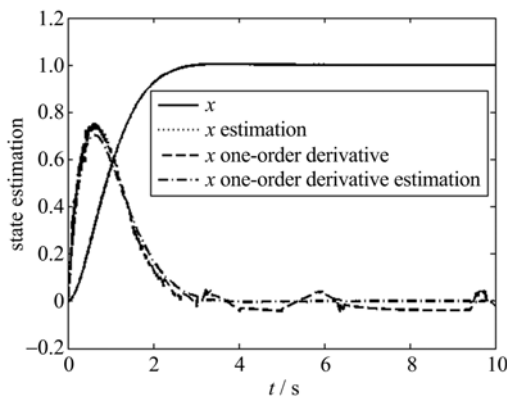


Fig. 4 System states and their estimation

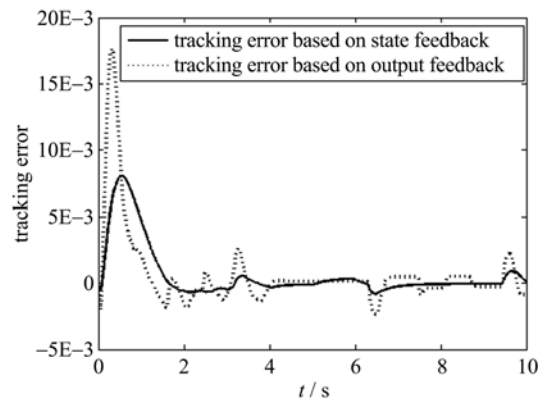


Fig. 5 Tracking error of adaptive control respectively based on state feedback and output feedback

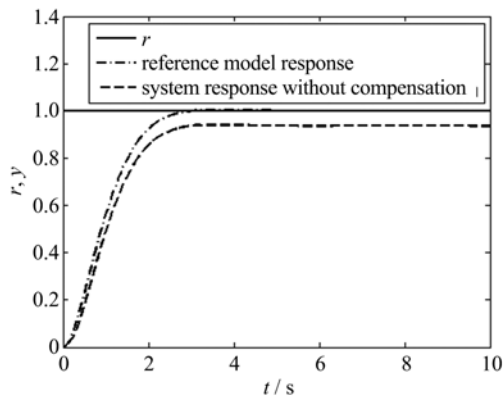


Fig. 6 Diagram of step response without compensation

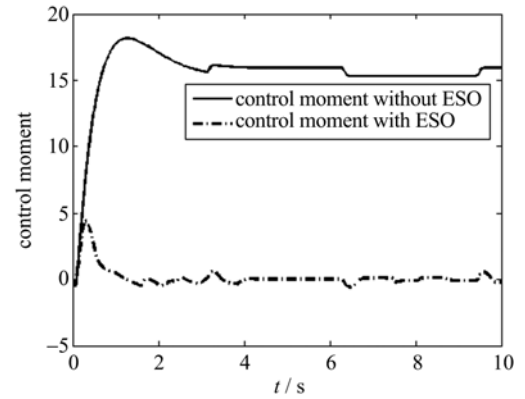


Fig. 7 Control moment with/without ESO

compensate disturbance, but it is complex to design and cannot estimate system states.

(IV) As shown in Fig. 7, due to disturbance estimation and compensation by the extended state, the control moment is greatly weakened, which is of great value in engineering applications.

Example 3.2 Consider a second order

nonlinear system with uncertainty described by

$$\left. \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -100x_2x_1 - 50x_1 + 200x_1x_1 + \\ &\quad 200x_2x_2 + 10\text{sign}(\sin(3t)) + 3r \\ y &= x_1 \end{aligned} \right\} \quad (49)$$

The reference model is described by Eq. (48) just as before, and controller parameters remain

the same. The simulation results are shown in Figs. 8 and 9.

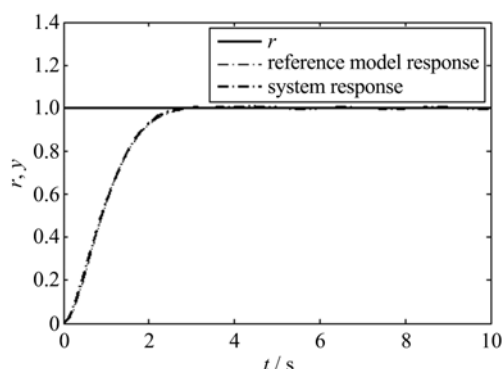


Fig. 8 Diagram of step response of ESO-MRAC

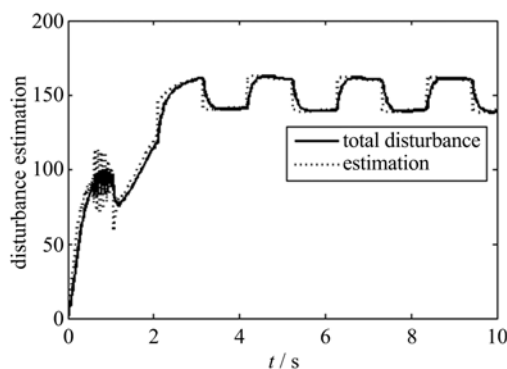


Fig. 9 The total disturbance and its estimation

As shown in Figs. 8 and 9, the reference model is the same for both nonlinear system (49) and linear system (47). Although the parameters remain the same, system can still track the reference model well. This means that the ESO-MRAC is suitable for systems with the same order, hence it is easier to choose a reference model, thus providing a great convenience for high order systems, especially higher order nonlinear systems to select a reference model. In addition, due to linearization with dynamic compensation, the closed-loop system works in a linear state, which avoids the robustness problem for a nonlinear system and the robustness mainly depends on estimating and compensating the total disturbance by ESO.

4 Conclusion

In this paper, a novel model reference adaptive

control method based on extended state observer (ESO-MRAC) has been presented. Closed-loop ESO-MRAC architecture, control law and stability analysis have been given. Compared with conventional adaptive controller, extended state observer was introduced first to estimate and compensate the uncertainties, which brings about a series of advantages. The proposed scheme is simple, universal and easier for choosing a reference model. Furthermore, it has strong robustness, as well as small control moment and high steady state accuracy. The features mentioned above have been demonstrated by simulations.

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References

- [1] Monopoli R V. Model reference adaptive control with an augmented error signal[J]. IEEE Transactions on Automatic Control, 1974, 19: 474-484.
- [2] Narendra K S, Valavani L S. Direct and indirect adaptive control [J]. Automatica, 1979, 15 (6): 653-664.
- [3] Wang C, Lin Y. Decentralised adaptive dynamic surface control for a class of interconnected non-linear systems[J]. IET Control Theory Applications, 2012, 6(9): 1 172-1 181.
- [4] Kamali M, Askari J. Output-feedback model reference adaptive control of linear continuous state delayed systems in the presence of actuator failures [C]// Proceedings of 8th IEEE International Conference on Control and Automation. Xiamen, China: IEEE Press, 2010: 2 126-2 131.
- [5] Hovakimyan N, Yang B J, Calise A J. Adaptive output feedback control methodology applicable to non-minimum phase nonlinear systems [J]. Automatica, 2006, 42 (4): 513-522.
- [6] Kim N, Calise A J. Neural network based adaptive output feedback augmentation of existing controllers [J]. Aerospace Science and Technology, 2008, 12(3): 248-255.
- [7] Rohrs C E, Valavani L, Athans M, et al. Robustness of adaptive control algorithms in the presence of unmodeled dynamics [C]// Proceedings of 21st IEEE

- Conference on Decision and Control. Orlando, USA; IEEE Press, 1982: 3-11.
- [8] Blažič S, Matko D, Škrjanc I. Adaptive law with a new leakage term [J]. IET Control Theory & Applications, 2010, 4(9): 1 533-1 542.
- [9] Huh S H, Bien Z. Robust sliding mode control of a robot manipulator based on variable structure-model reference adaptive control approach[J]. IET Control Theory & Applications, 2007, 1(5): 1 355-1 363.
- [10] Yan L, Hsu L, Costa R R, et al. A variable structure model reference robust control without a prior knowledge of high frequency gain sign [J]. Automatica, 2008, 44(4): 1 036-1 044.
- [11] Calise A J, Hovakimyan N, Idan M. Adaptive output feedback control of nonlinear systems using neural networks[J]. Automatica, 2001, 37 (8): 1 201-1 211.
- [12] Cao C Y, Hovakimyan N. Design and analysis of a novel L_1 adaptive controller[C]// American Control Conference. Minnesota, USA; IEEE Press, 2006: 3 397-3 402.
- [13] Cao C Y, Hovakimyan N. Design and analysis of a novel L_1 adaptive controller[C]// American Control Conference. Minnesota, USA; IEEE Press, 2006: 3 403-3 408.
- [14] Cao C Y, Hovakimyan N. Guaranteed transient performance with L_1 adaptive controller for systems with time-varying parameters and bounded disturbances [C]// American Control Conference. New York, USA; IEEE Press, 2007: 3 925-3 930.
- [15] Cao C Y, Hovakimyan N. L_1 adaptive controller for systems in the presence of unmodelled actuator dynamics[C]// Proceedings of 46th IEEE Conference on Decision and Control. New Orleans, USA; IEEE Press, 2007: 891-896.
- [16] Cao C Y, Hovakimyan N. L_1 adaptive controller for a class of systems with unknown nonlinearities [C]// American Control Conference. Baltimore, USA; IEEE Press, 2008: 4 093-4 098.
- [17] Cao C Y, Hovakimyan N. L_1 adaptive controller for nonlinear systems in the presence of unmodelled dynamics [C]// American Control Conference. Baltimore, USA; IEEE Press, 2008: 4 099-4 104.
- [18] Kharisov E, Hovakimyan N, Wang J, et al. L_1 adaptive controller for time-varying reference systems in the presence of unmodeled nonlinear dynamics[C]// American Control Conference. Baltimore, USA; IEEE Press, 2010: 886-891.
- [19] Hayakawa T, Haddad M M, Hovakimyan N. Neural network adaptive control for a class of nonlinear uncertain dynamical systems with asymptotic stability guarantees [J]. IEEE Transactions on Neural Networks, 2008, 19(1): 80-89.
- [20] Campa G, Fravolini M L, Mammarella M, et al. Bounding set calculation for neural network-based output feedback adaptive control systems[J]. Neural Computing and Applications, 2011, 20(3): 373-387.
- [21] Moreno-Valenzuela J, Santibañez V, Orozco-Manríquez E, et al. Theory and experiments of global adaptive output feedback tracking control of manipulators[J]. IET Control Theory & Applications, 2010, 4(9): 1 639-1 654.
- [22] Huang Y, Han J Q. A new synthesis method for uncertain systems the self-stable region approach[J]. International Journal of System Science, 1999, 30(1): 33-38.
- [23] Yang X X, Huang Y. Capabilities of extended state observer for estimating uncertainties[C]// American Control Conference. St. Louis, USA; IEEE Press, 2009: 3 700-3 705.
- [24] 韩京清. 一类不确定对象的扩张状态观测器[J]. 控制与决策, 1995, 10(1): 85-88.
- [25] Gao Z Q. Scaling and bandwidth-parameterization based controller tuning [C]// American Control Conference. Denver, USA; IEEE Press, 2003: 4 989-4 996.
- [26] Yoo D, Yau S S T, Gao Z Q. On convergence of the linear extended state observer[C]// Proceedings of IEEE International Conference on Control Applications. Texas, USA; IEEE Press, 2006: 1 645-1 650.
- [27] Guo B Z, Zhao Z L. On the convergence of an extended state observer for nonlinear systems with uncertainty [J]. Systems & Control Letters, 2011, 60 (6): 420-430.