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A digital control approach for unstable-first-order-plus-dead-time systems with P and PI controllers

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Abstract: Many dynamical systems can be modeled by unstable-first-order-plus-dead-time (UFOPDT) transfer functions. However, analysis and synthesis of UFOPDT systems are much more challenging due to the general difficulties of infinite dimensionality and the instability of the plant. Considering the control of such systems, explicit tuning formulae were derived for proportional (P) and proportional-integral (PI) controllers, based on the digitized open loop systems. Stability range was also discussed for the feedback systems with delays. Compared with existing results, the presented method significantly improved the accuracy and sufficiency, and simplified the tuning process. Numerical example about an isothermal chemical reactor control problem was given to illustrate this algorithm, and several relevant methods were also compared.

Key words: time-delay systems; digital control; stability; Nyquist criterion

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一种基于数字控制的一阶时滞不稳定系统 P 和 PI 控制器设计方法

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摘要：在实际应用中，许多动力系统都可以以一阶时滞不稳定传递函数进行建模．但是，无穷维及系统不稳定等因素的存在使得这类系统的控制分析与综合具有很大挑战．考虑一阶时滞不稳定系统的控制问题，基于数字化的开环控制系统，对比例(P)控制器和比例积分(PI)控制器分别提出了明确的优化公式．同时，针对具有时滞的反馈系统，讨论了控制器的稳定范围．与现有的结果相比较，该数字控制方法简化了控制器的优化过程，且证明了稳定范围的充分性及精确性．最后，通过一个数值实例及与已有结果作比较验证该算法的可靠性及有效性．

关键词：时滞系统；数字控制；稳定性；Nyquist 判据

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0 Introduction

Time delays are ubiquitous in the dynamical modeling of many important industry processes[1]. The control of time delay systems has been widely studied in the past several decades[2-3] and still being further investigated. Among the various research results on time delay systems, a problem of particular interest is the analysis and control of unstable-first-order-plus-dead-time (UFOPDT) processes that represent many important industry applications such as chemical process control[4]. Unlike the classical controller tunings for stable systems without delays, the analysis and synthesis of UFOPDT systems are much more challenging due to the general difficulties of infinite dimensionality and the instability of the plant. When a controller (assuming the general form of PID) is applied, the transfer function of the feedback systems is in the form of quasi-polynomials[5]. Low order quasi-polynomials can be treated analytically, while high order quasi-polynomials are more complex. It should be noticed that for stable systems with delays, a frequency domain stability criterion, called Tsypkin criterion, is very useful to determine the stability of the feedback systems. Consider open loop transfer function $H_o(s)=\frac{P(s)}{Q(s)}e^{-s\tau}$. If $Q(s)$ is a stable polynomial, then the closed loop system $H_c(s)=\frac{P(s)e^{-s\tau}}{Q(s)+P(s)e^{-s\tau}}$ is asymptotically stable if $|Q(j\omega)|>|P(j\omega)|$ holds for all ω.

However, Tsypkin criterion cannot be applied to UFOPDT because the open loop system is unstable. Research on the control of UFOPDT has been very active in the past two decades, where different design and tuning approaches have been provided[6-12]. In particular, Ref. [6] developed a Ziegler-Nichols type control tuning method for unstable process, Bahavarnia et al. have discussed the nonfragility. Refs. [7] and [8] provided a more practical controller design formula for similar problems based on numerical approximations of the solutions of transcendental equations derived from Nyquist stability criterion. Ref. [9] made a low order numerical approximation for the arctan functions used in the stability conditions, with which a simplified tuning method was generated explicitly. A very recent result from Ref. [10] discussed the same problem with a first order Padé expansion of the time delay block of the original plant, thus the UFOPDT system is treated as a second order transfer function, instead of an infinite dimensional system. More analytical results are given in Ref. [11], but the algorithms are computationally complicated. In Ref. [12], a modified Smith-predictor structure was proposed for the UFOPDT system. These results provide some useful tuning methods for the controller design of UFOPDT systems, as well as practical tools of analyzing the gain/phase margin of the feedback systems.

In spite of the abundant research results on this topic, there are still important open problems yet to be solved. In particular, the existing results are either too complicated for practical use, or with approximations of the time delay blocks or transcendental equations, such that the necessity and sufficiency of the stability conditions are no longer precise for the original systems. Meanwhile the results on the analysis/comparison of the accuracies of different approximation methods are rare in the references. In this paper, we consider P and PI control of UFOPDT systems from a digital control perspective. Because the digitized system is finitely dimensional and thus can be analyzed based on Routh-Hurwitz criterion for polynomials. Therefore we can obtain the stability range more easily. More improtantly, we can also prove that the stability range for the digitized system is a sufficient condition for UFOPDT systems. Therefore, we can get the more accurate stability range of P and PI controller for UFOPDT systems applying the digital control method. A numerical example proves that the stability range obtained by the digital control method can certainly stablize the original UFOPDT system. The results are also compared with existing results.

1 Proportional control of UFOPDT systems

Consider the UFOPDT transfer function of the form:

$$G_p(s)=\frac{k_p e^{-hs}}{\tau s-1} \tag{1}$$

And the open loop transfer function with a proportion controller k_c is written as:

$$T_c^p(s)=\frac{k_p k_c e^{-hs}}{\tau s-1} \tag{2}$$

where the phase $\phi_c^p(\omega)$ and magnitude $A_c^p(\omega)$ can be derived as:

$$\phi_c^p(\omega)=\arctan(\omega t)-\omega h-\pi \tag{3}$$

$$A_c^p(\omega)=\frac{k_p k_c}{\sqrt{1+\tau^2\omega^2}} \tag{4}$$

Under Nyquist criterion, the Nyquist plot should encircle (−1, 0) exactly once in the anticlockwise direction. Note the fact that

$$\frac{d\phi_c^p(\omega)}{d\omega}=\frac{\tau}{1+\omega^2\tau^2}-h \tag{5}$$

It is straightforward that $\tau>h$ is a necessary condition for stability. Otherwise, there is no anticlockwise encircling. We refer to Fig. 1 for an illustrative example.

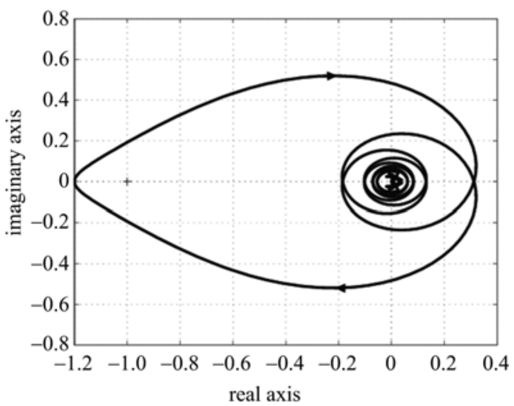

Fig. 1 The case of $h>\tau$, which cannot be stabilized by P controllers

We assume $\varepsilon := h/\tau < 1$ and consider the first crossover frequency, $\omega=\omega_0$, $\omega_0>0$:

$$\arctan(\omega_0\tau)=\omega_0 h$$

$$\Rightarrow \cos(\omega_0 h)^{-1}=\tau\omega_0/\sin(\omega_0 h)\approx 1/\varepsilon \tag{6}$$

Recall Eq. (4) and Nyquist stability criterion, we should have

$$1<k_p k_c<\sqrt{1+\tau^2\omega_0^2} \tag{7}$$

We further observe from Eq. (6) that

$$\sqrt{1+\tau^2\omega_0^2}=\sqrt{1+\tan(\omega_0 h)^2}=(\cos(\omega_0 h))^{-1}\approx 1/\varepsilon.$$

Thus the stability range of the P controller k_c is

$$k_{\min}=\frac{1}{k_p}<k_c<k_{\max}=1/k_p\varepsilon \tag{8}$$

It is worth noting that the stability condition of Eq. (8) is the same as that in Ref. [6], while we are not using the optimal phase margin method therein.

Now that we will revisit the same problem from the digital systems perspective. Assume the sample period is $\Delta T=h/M$, where $M=1,2,3\cdots$. The open loop transfer Eq. (2) can be digitized as:

$$T_d^p(z)=\frac{k_c k_p\varepsilon/M}{z(z-(1+\varepsilon/M))} \tag{9}$$

We further simplify the system by letting $M=1$, thus

$$T_d^p(z)=\frac{k_c k_p\varepsilon}{z(z-(1+\varepsilon))} \tag{10}$$

and have the following result:

Theorem 1.1 A proportional controller k_c can stabilize Eq. (10) if

$$k_c\in\left(\frac{1}{k_p},\frac{1}{k_p\varepsilon}\right) \tag{11}$$

Meanwhile, Eq. (11) is also a sufficient condition for k_c to stabilize the original UFOPDT Eq. (1).

Proof Define $z=(v+1)/(v-1)$ in the characteristic equation for the closed loop transfer function of Eq. (10), we have

$$(k_c k_p\varepsilon-\varepsilon)v^2+2(1-k_c k_p\varepsilon)v+2+\varepsilon+k_c k_p\varepsilon=0 \tag{12}$$

Recall Routh-Hurwitz criterion, the stability of the closed loop system is guaranteed if

$$k_c k_p(\varepsilon-\varepsilon)>0,\ \text{and}\ 1-k_c k_p\varepsilon>0 \tag{13}$$

which proves the first statement of the theorem. The second statement of the theorem follows from the fact that Eq. (10) is a digitized transfer function of Eq. (2).

Remark 1.1 The stability range of Theorem 1.1 is equivalent to Eq. (8) derived from s-domain analysis. However, the digital system based analysis proves sufficient of the condition, which is not explicit from other methods.

2 Proportional-Integral control of UFOPDT systems

In this section, we will study the stabilization problem of UFOPDT systems (1) using PI controllers defined as:

$$C_{pi}(s) = \frac{k_c(sT_i + 1)}{sT_i} \tag{14}$$

The open loop transfer function is:

$$T_c^{pi}(s) = C_{pi}(s) * G_p(s) = \frac{k_p k_c(sT_i + 1)e^{-hs}}{sT_i(\tau s - 1)} \tag{15}$$

where the phase $\phi_c^{pi}(\omega)$ and magnitude $A_c^{pi}(\omega)$ can be derived as:

$$\phi_c^{pi}(\omega) = \arctan(\omega\tau) + \arctan(\omega T_i) - \omega h - \frac{3\pi}{2} \tag{16}$$

$$A_c^{pi}(\omega) = \frac{k_c k_p}{T_i\omega}\sqrt{\frac{1 + (T_i\omega)^2}{1 + (\tau\omega)^2}} \tag{17}$$

Taking the derivative of $\phi_c^{pi}(\omega)$, we have

$$\frac{d\phi_c^{pi}(\omega)}{d\omega} = \frac{\tau}{1 + (\tau\omega)^2} + \frac{T_i}{1 + (T_i\omega)^2} - h \tag{18}$$

Similar to Section 1, we need to examine the crossover frequencies at ω-domain and determine the direction of encircling based on Nyquist criterion. As discussed in Ref. [8], the upper bound of $\varepsilon = h/\tau$ can be estimated by looking at the extreme case where $\phi_c^{pi}(\omega) = n\pi$ and $d\phi_c^{pi}(\omega)/d\omega = 0$. It has been further shown in Refs. [6, 8] that $\varepsilon < 0.6$.

Due to the general difficulties of explicit solutions for the transcendental equations associated with $\phi_c^{pi}(\omega)$ and $A_c^{pi}(\omega)$, widely deployed methods are to use different approximations (see Refs. [6,8-12], to cite just a few contributions). As a result, accuracy and complexity have been the primary issues for these methods[8,10]. In what follows, we will follow the same fashion of digital analysis as Section 1 to study the PI controller design problem. Again, we pick up the sample rate of $\Delta T = h/M$ (where $M=1$ to further simplify the analysis). Now that the open loop transfer function (15) is written as:

$$T_d^{pi}(z) = \frac{k_c k_p \varepsilon(z - \gamma)}{z(z-1)(z-(1+\varepsilon))} \tag{19}$$

where $\gamma = 1 - \Delta T/T_i$.

To determine T_i, we refer to the analysis of Ref. [8], where

$$T_i \approx 5/\omega_{co} \tag{20}$$

where ω_{co} is the crossover frequency of $e^{-hs}/(\tau s - 1)$, which can be approximated by

$$\omega_{co} \approx \omega_c \approx 0.2/(\tau - h) \tag{21}$$

Therefore

$$\beta := \Delta T/T_i = h/25(\tau - h) = \varepsilon/25(1-\varepsilon) \tag{22}$$

$$\gamma := 1 - \beta = (25 - 26\varepsilon)/25(1-\varepsilon) \tag{23}$$

We are now in the position to state the following theorem:

Theorem 2.1 Assuming the PI controller is in the form of Eq. (14), where T_i is determined by Eqs. (20) and (21). We further assume that $0 < \varepsilon < 0.577$. We have:

(Ⅰ) A necessary and sufficient condition for the stability of the closed loop digital systems with respect to Eq. (19) is:

$$k_c \in \left(\frac{1+\varepsilon-2\beta-\varepsilon\beta-\sqrt{\xi}}{2k_p\varepsilon(\beta-1)^2}, \frac{1+\varepsilon-2\beta-\varepsilon\beta+\sqrt{\xi}}{2k_p\varepsilon(\beta-1)^2}\right) \tag{24}$$

where

$$\xi = (\varepsilon(\beta-1) + 2(\beta-1) + 1)^2 - 4\varepsilon(\beta-1)^2 \tag{25}$$

and β defined by Eq. (23);

(Ⅱ) (Ⅰ) is also a sufficient condition for the PI controller to stabilize the UFOPDT system (1).

Proof The characteristic equation for the closed loop systems of Eq. (19) is:

$$z(z-1)(z-(1+\varepsilon)) + k_c k_p \varepsilon(z - \gamma) = 0 \tag{26}$$

Replacing z with $(v+1)/(v-1)$ in Eq. (26), we have

$$a_3 v^3 + a_2 v^2 + a_1 v + a_0 = 0 \tag{27}$$

where

$$\left.\begin{aligned} a_3 &= k_c k_p \varepsilon - k_c k_p \varepsilon\gamma = k_c k_p \varepsilon\beta \\ a_2 &= 3k_c k_p \varepsilon\gamma - 2\varepsilon - k_c k_p \varepsilon = \\ &\quad 2k_c k_p \varepsilon - 2\varepsilon - 3k_c k_p \varepsilon\beta \\ a_1 &= 4 - k_c k_p \varepsilon - 3k_c k_p \varepsilon\gamma = 4 - 4k_c k_p \varepsilon + 3k_c k_p \varepsilon\beta \\ a_0 &= 4 + 2\varepsilon + k_c k_p \varepsilon + k_c k_p \varepsilon\gamma \end{aligned}\right\} \tag{28}$$

Recall Routh-Hurwitz criterion. The necessary and sufficient condition for the closed loop stability is: $a_i > 0$, $i=0,1,2,3$; and $a_1 a_2 > a_0 a_3$. Observe the fact that $\gamma, \beta, \varepsilon > 0$. It is straightforward that $a_3 > 0$ and $a_0 > 0$. Meanwhile

$$a_2 > 0 \Leftrightarrow k_c k_p > \frac{2}{(2-3\beta)} \tag{29}$$

and

$$a_1 > 0 \Leftrightarrow k_c k_p < \frac{4}{\varepsilon(4-3\beta)} \tag{30}$$

which can be written as:

$$\frac{2}{2-3\beta} < k_p k_c < \frac{4}{\varepsilon(4-3\beta)} \tag{31}$$

From Eq. (28), we further have

$$\begin{aligned} & a_1 a_2 > a_0 a_3 \\ & \Leftrightarrow \varepsilon(\beta^2 - 2\beta + 1)(k_c k_p)^2 + \\ & \quad (\varepsilon\beta + 2\beta - \varepsilon - 1) k_c k_p + 1 < 0 \end{aligned} \tag{32}$$

We claim that $\xi > 0$ when $0 < \varepsilon < 0.577$, which can be analytically validated. A numerical plot is also provided to verify this result (see Fig. 2).

For $\xi > 0$, the explicit solution of the inequality in Eq. (32) can be written as:

$$k_c k_p \in \left(\frac{1+\varepsilon-2\beta-\varepsilon\beta-\sqrt{\xi}}{2\varepsilon(\beta-1)^2}, \frac{1+\varepsilon-2\beta-\varepsilon\beta+\sqrt{\xi}}{2\varepsilon(\beta-1)^2}\right) \tag{33}$$

Therefore the necessary and sufficient condition for the closed loop stability of the digital system (19) is to satisfy.

Eqs. (31) and (33) simultaneously. The proof of (Ⅰ) is concluded by further observing the fact that

$$\frac{4}{\varepsilon(4-3\beta)} - \frac{1+\varepsilon-2\beta-\varepsilon\beta+\sqrt{\xi}}{2\varepsilon(\beta-1)^2} > 0 \tag{34}$$

which is the upper line of Fig. 3; and

$$\frac{1+\varepsilon-2\beta-\varepsilon\beta-\sqrt{\xi}}{2\varepsilon(\beta-1)^2} - \frac{2}{(2-3\beta)} > 0 \tag{35}$$

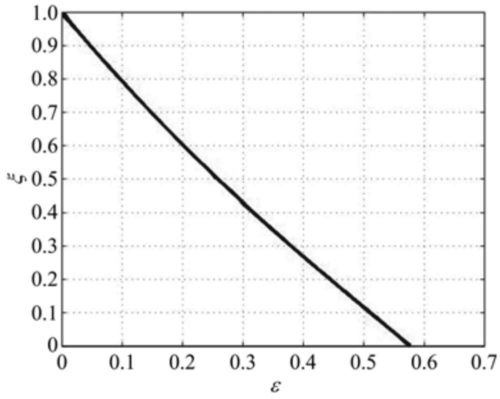

Fig. 2 Verification of $\xi > 0$ for any $0 < \varepsilon < 0.577$

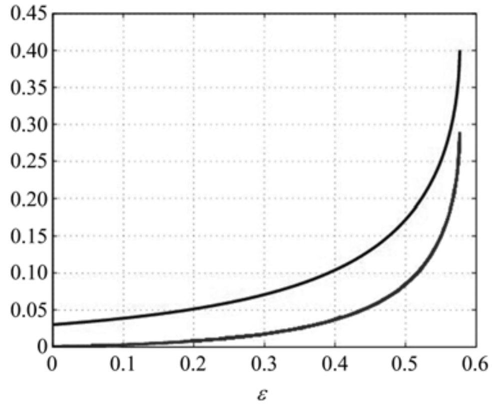

Fig. 3 Numerical verification of Eqs. (34) and (35)

which is the lower line of Fig. 3.

Part (Ⅱ) of the Theorem 2.1, follows from the fact that Eq. (19) is a digitized transfer function of Eq. (15). Therefore, the closed loop stability with respect to Eq. (19) is a sufficient condition of that of Eq. (15).

3 Numerical example

The effectiveness of our proposed method is evaluated by a representative numerical example, which is an isothermal chemical reactor control problem widely discussed in the references of chemical engineering.

The mathematical model equation of the reactor is written as:

$$\frac{\mathrm{d}C}{\mathrm{d}t} = \frac{Q}{V}(C_f - C) - \frac{k_1 C}{(k_2 C + 1)^2} \tag{36}$$

where Q is the inlet flow rate and C_f is the inlet concentration. The values of the operating parameters are given as $Q = 0.033\,33$ L/s, $V = 1$ L,

$k_1 = 10$ L/s, and $k_2 = 10$ L/mol. For the nominal value of $C_f = 3.288$ mol/L, the steady-state solution of the model equation gives two stable steady states at C = 1.767 3, C = 0.014 2 mol/L, and one unstable steady state at C = 1.316 mol/L. Feed concentration is considered as the manipulated variable. Linearization of the manipulated variable around this operating condition C = 1.316 gives the unstable transfer function model as 3.433/(103.1s − 1), and a time delay of 20 s is considered.

Therefore, the linearized model can be written as a UFOPDT system:

$$G_p(s) = \frac{3.433}{103.1s - 1} e^{-20\,s}.$$

We have $\tau = 103.1$, $h = 20$, $\varepsilon = 0.194$, $k_p = 3.433$, $\beta = 0.009\,6$, $T_i = 25(\tau - h) = 2\,077.5$. Thus we can pick up a P controller with $k_c \in (0.291, 1.5)$ to stabilize the system. As depicted in Fig. 4, the Nyquist plot indicates stability with $k_c = 1$. Otherwise, we pick up $k_c = 1$ and can easily verify that the PI controller stabilizes the UFOPDT system (1) with $k_c \in (0.298, 1.498)$ from Theorem 2.1, which can be illustrated by the Nyquist plot in Fig. 5.

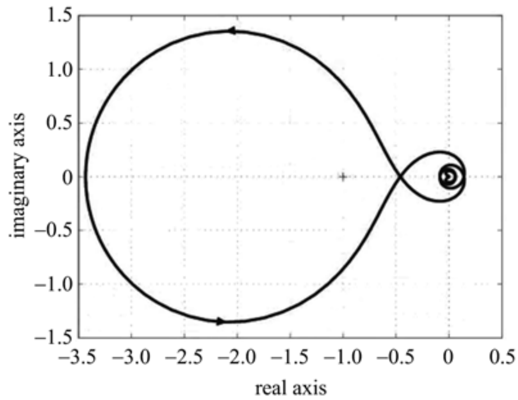

Fig. 4　Using P controller to stabilize the UFOPDT systems

4　Comparison

We can see that the major contributions of this paper include: ① An explicit tuning method for P and PI controllers for UFOPDT systems, which is much simpler compared to existing results; ② The

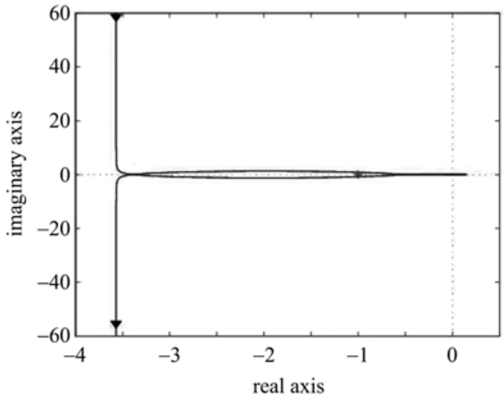

Fig. 5　Using PI controller to stabilize the UFOPDT systems

controllers derived in the present paper will always guarantee stability of the closed loop UFOPDT systems, while most existing results don't. In this section, we will provide various numerical examples to demonstrate the effectiveness of the proposed method, and provide comparisons with other results in the references.

First of all, we will recall a widely discussed tuning method of PI controllers for UFOPDT systems (Ref. [8]). The algorithm of Ref. [8] suggests that the PI controller (14) can be determined by

$$k_{\min}^{*} < k_c < k_{\max}^{*} \tag{37}$$

where

$$k_{\min}^{*} := 0.98\sqrt{1 + \tau^2\omega_c^2}/k_p \tag{38}$$

$$k_{\max}^{*} := 5\alpha\sqrt{\frac{1 + \tau^2\omega_c^2\alpha^2}{1 + 25\alpha^2}}\bigg/k_p \tag{39}$$

with $\alpha = 5\beta(\tau - h)$, $\omega_c = 0.2(\tau - h)$, and $\beta = 1.373$ for $\varepsilon < 0.25$, or $\beta = 0.953$ for $\varepsilon \leqslant 0.25$. Here T_i follows the same definition to Eq. (20).

Now that we recall an example of UFOPDT systems (1), with $k_p = 1$, $\tau = 1$, and $h = 0.1, 0.2 \cdots 0.5$, respectively. The PI controller can be computed based on Eq. (37) and Theorem 2.1. The comparison results are listed in Tab. 1, where we can clearly see that $k_{\min} > k_{\min}^{*}$ and $k_{\max} < k_{\max}^{*}$ for all scenarios.

Actually, we can easily find a counter-example to demonstrate that the algorithm of Ref. [8] is not always stabilizing the system and therefore our algorithm is more precise. For the case of $h = 0.1$, we pick up $k_c = 13$, which agrees with Eq. (37), but outside the stability range calculated from our algorithm (refer to the second row of Tab. 1). The Nyquist plot (see Fig. 6) verifies that the system is indeed unstable.

Tab. 1 Stability range comparisons

ε	$k^*_{\min}$	$k^*_{\max}$	$k_{\min}$	$k_{\max}$
0.1	1.003 9	13.766 3	1.009 5	9.994 8
0.2	1.010 2	6.937 3	1.023 1	4.986 5
0.3	1.019 2	3.329 8	1.043 7	3.306 0
0.4	1.033 0	2.582 8	1.078 3	2.447 2
0.5	1.055 5	2.150 5	1.148 3	1.889 9

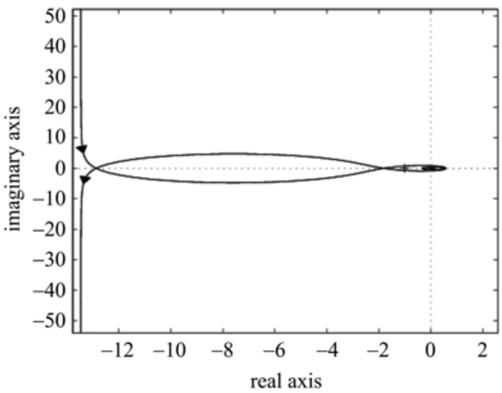

Fig. 6 An example of unstable system with PI controllers from Ref. [8]

It is worth noting that similar counter-examples can be generated for most of the existing results based on various approximation methods, due to the fact that the approximations cannot guarantee sufficiency of the stability when applied to the controller tunings. However, the method presented in this paper has the obvious advantages of sufficiency for the stability condition.

Another interesting question is: what is the conservativeness and inaccuracy of the stability conditions, if sufficiency is achieved. We define $k_c = \sqrt{k_{\min} k_{\max}}$ as the "default" controller gain and will compare the inaccuracy accordingly. As demonstrated in Tab. 2, the differences between column 3 and column 4 (exact calculations based on iterative numerical solutions) indicate the conservativeness of the proposed algorithm.

Tab. 2 Comparisons of controller gains

ε	k^*_c	k_c	exact
0.1	3.72	3.18	3.53
0.2	2.65	2.26	2.48
0.3	1.84	1.86	1.98
0.4	1.63	1.62	1.66
0.5	1.51	1.47	1.48

5 Conclusion

In this paper, we considered the P and PI controller design for UFOPDT systems. The stability conditions and controller tuning algorithms were derived based on the digitized systems with a special sample rate. The results achieved in this paper were compared with existing results and demonstrated better accuracy and can guarantee stability (sufficiency achieved), while most existing results don't. We obtained the stability range of the P and PI controller for UFOPDT systems by the digital control method successfully. Future works along this line of research include extension of the present method to gain/phase margin specifications and design, as well as to other types of time delay systems.

References

[1] Kheirizad I, Jalali A A, Khandani K. Stabilization of fractionalorder unstable delay systems by fractional-order controllers[J]. Journal of Systems and Control Engineering, 2012, 226(9):1 166-1 173.

[2] Luo Y, Chen Y Q. Stabilizing and robust fractional order PI controller synthesis for first order plus time delay systems[J]. Automatica, 2012, 48(9): 2 159-2 167.

[3] Hohenbichler N. All stabilizing PID controllers for time delay systems[J]. Automatica, 2009; 45(11): 2 678-2 684.

[4] YU Tao, LIU Xiang, SUN Youxian. Frequency-domain design of PI Controllers for first-order systems with time delay[J]. Journal of University of Science and Technology of China, 2005, 35(S): 167-173.
余涛，刘翔，孙优贤. 一阶时滞系统 PI 控制器的频率设计法[J]. 中国科学技术大学学报，2005，35(S)：167-173.

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