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Effect of output noise in inverse-model-based iterative learning control

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Abstract: Inverse-model-based iterative learning control (ILC) for linear-time invariant, single-input single output (SISO) systems subject to output noise is proposed with the intent of predicting expectation of the underlying "noise-free" mean square error (Euclidean norm) on each iteration. Frequency domain formulae are derived to provide an insight into links between plant characteristics, noise spectra and inverse-model-based ILC parameters. Simulations are used to illustrate the theoretical findings.

Key words: inverse-model-based iterative learning control; output noise; variance; expectation; Euclidean norm; frequency domain

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输出干扰对基于逆模型的迭代学习控制的影响

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摘要:针对具有输出干扰的线性时不变单输入单输出系统,首次运用基于逆模型的迭代学习控制理论分析其轨迹跟踪控制问题.通过严格的数学分析,提出了无干扰输出误差的数学期望计算公式(欧几里得范数)并证明了其收敛性.进而给出了其频域分析公式,证明了系统参数、噪声光谱和基于逆模型的迭代学习控制参数对收敛性的影响.系统仿真实验结果验证了以上的理论发现.

关键词:基于逆模型的迭代学习控制;输出干扰;方差;数学期望;欧几里得范数;频域分析

0 Introduction

Iterative Learning Control is a new control theory that is different from the other classical control theories. The advancements in industrial automation typically require many control systems to perform the same task over and over again. For example, robot manipulators (e.g. a welding robot in car manufacturing) are required to repeat a given motion with high precision. As a result of the repetition of the same operation, the control systems should have an ability to do the same task

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more precisely and accurately when the task is repeated again.

To achieve a near-perfect control theory, ILC is implemented to reduce the error between the output signal and the given reference signal in some repeated systems. It is an iterative update scheme which can improve the quality of system performance from trial to trial. This is the background of ILC.

The original idea was introduced by researchers including Arimoto et al. [1]. Since that time, many papers, surveys and texts on ILC have been published e. g. [2-8] for a variety of algorithms including those considered in this paper and optimization-based methodologies[9-14]. Several application based papers have been published underpinning the practical value of the concepts in a variety of industrial sectors. e. g. [15-17].

The overwhelming majority of these papers consider the noise free cases whereas, in practice, signals are inevitably noisy. Achieving zero tracking error in such circumstances is not possible in reality. This paper considers a simple, practical class of ILC algorithms (inverse-model-based ILC algorithm) for discrete-time systems in the presence of coloured noise and derives formulae for the Expectation of the Euclidean norm of the underlying "noise-free" tracking error on the k^{th} iteration in terms of the matrices in the "lifted model" of the plant. Computational experiments are used to verify the correctness of the proposed properties.

1 Problem statement

To give a precise definition of ILC, consider a linear time invariant, single input and single output, state-space system model defined over a finite discrete time-interval $t \in [0, N]$

$$\left.\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}\right\} \tag{1}$$

where initial states $x(0)=x_0$, $x(t) \in \mathscr{R}^n$, $u(t) \in \mathscr{R}$ and $y(t) \in \mathscr{R}$ denote the state, input and output, respectively. The matrices A, B and C in the state-space function (1) have the appropriate dimensions and it is assumed that the system is controllable and observable and CB is non-zero.

In addition, a reference signal $r(t)$ is specified over a finite time-interval $t \in [0, N]$ and it is assumed that the sampling interval, t_s, is unity. This reference signal is the control objective required in order to find the appropriate input $u(t)$ that is used to compute the output signal $y(t)$. This output signal should track the reference signal $r(t)$ with the least amount errors.

The special feature of the ILC is that after the system has run over the time interval from 0 to N, the system (1) is reset to its initial states to repeat the task. This repetition gives the system the ability to modify the next trial input signal $u(t)$ so that as the number of repetitions increases, the output signal $y(t)$ tracks the given reference signal $r(t)$ more and more accurately. This behaviour is the significant feature of ILC. The question that now arises is how to find a proper control law that alters the input signal. Precisely, the main idea of ILC design is to find a control law

$$u_{k+1} = F(e_{k+1}, e_k, u_k, \cdots) \tag{2}$$

so that

$$\lim_{k\to\infty} e_k = 0 \quad \text{and} \quad \lim_{k\to\infty} u = u^* \tag{3}$$

where e_k is the tracking error calculated by $(r - y_k)$ and u^* is the perfect input which results in perfect tracking. As noted earlier, from the definition of the control problem, it is assumed the perfect input u^* exists. If u^* does not exist, the problem can be modified to find an optimal input u^* where u^* is the answer to the following optimisation problem

$$u^* = arg \min_{u\in U} \| r - G_e u \|^2 \tag{4}$$

where U is the set of the possible input signals.

In order to make the analysis of the system simpler, the system is represented in the matrix form used by many authors[10-12].

$$y_k = Gu_k + d \tag{5}$$

where, if k^* is the relative degree of $G(z)$ (or, equivalently, k^* is the smallest integer such that the Markov parameter $CA^{k^*-1}B\neq 0$)

$$G=\begin{bmatrix} CA^{k^*-1}B & 0 & \cdots & 0 \\ CA^{k^*-2}B & CA^{k^*-1}B & \cdots & 0 \\ CA^{k^*-3}B & CA^{k^*-2}B & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots \\ CA^{N-1}B & CA^{N-2}B & \cdots & CA^{k^*-1}B \end{bmatrix} \quad (6)$$

$$\left.\begin{aligned} y_k &= [y_k(k^*), y_k(k^*+1), y_k(k^*+2), \cdots, y_k(N)]^T \\ u_k &= [u_k(0), u_k(1), u_k(2), \cdots, u_k(N-k^*)]^T \\ d &= [CA^{k^*}x_0, \cdots, CA^N x_0]^T \end{aligned}\right\} \quad (7)$$

The dimension of the lower triangular matrix G is $(N+1-k^*)\times(N+1-k^*)$. It has only one distinct eigenvalue $CA^{k^*-1}B$ with multiplicity $N+1-k^*$. The analysis is unaffected by assuming that $d=0$ so this will be assumed from this point for simplicity. As $CA^{k^*-1}B\neq 0$, G is nonsingular and there always exists a u^* which satisfies $r=Gu^*$ for an arbitrary reference time series $r(t)$ on $[k^*, N]$. That is, defining $r=[r(k^*), \cdots, r(N)]^T$, $r=Gu^*+d$.

Note: As the time series vector y has k^* right shifts relative to u, G is an exact representation of $z^{k^*}G(z)$ on $[0, N-k^*]$.

2 Inverse-model-based ILC algorithm with output noise

2.1 Derivation of the algorithm

This section concentrates on the system in the presence of output noise written in the form

$$y_k = G_e u_k + H_e n_k \quad (8)$$

where n_k is a zero mean, white noise time series which differs on each trial but has the same variance δ. The matrix H_e is the matrix representation of a filter $H(z)$ defining the spectral characteristics of the noise seen at the output. That is,

① $E(n_i^T n_j)=0$ if $i\neq j$.

② $E(n_i n_i^T)=\delta^2 I$ where I is the identity matrix.

The work described in this section is related to that of Butcher et al.[18] but is more generally applicable in the sense that the analysis here is time domain based and hence does not require Butcher's sufficient (but not necessary) frequency domain conditions for convergence to be satisfied for their validity.

The Inverse-model-based ILC control law[19] is now expressed as Eq. (9)

$$u_{k+1} = u_k + \beta G_e^{-1} e_k \quad (9)$$

where β is a learning gain introduced to add flexibility to influence the performance. It is further assumed that the procedure is initiated by the choice of an arbitrary initial control time series u_0, thereby resulting in an initial error e_0.

Two error measures are important to the analysis of the behaviour on trial $k+1$, namely the measured, noise contaminated error e_{k+1} and the underlying, unmeasurable noise-free error $e_{1,k+1}$. These can be computed as follows

$$\left.\begin{aligned} e_{k+1} &= r - y_{k+1} \\ e_{1,k+1} &= r - Gu_{k+1} = e_{k+1} + H_e n_{k+1} \end{aligned}\right\} \quad (10)$$

The noise free tracking error $e_{1,k+1}$ is the one of interest in assessing the real tracking accuracy on trial $k+1$ but the noise contaminated signal e_{k+1} is the only one available for ILC control action. Note that, whilst $e_{1,k+1}$ does not depend on n_{k+1}, it does depend on previous noise signals n_j for iterations $0\leqslant j\leqslant k$ due to the inverse-model-based ILC update law (9) (which depends on e_k).

The noise free tracking error evolution equation needs to be established as reflected in Eq. (11)

$$\left.\begin{aligned} e_{1,k+1} &= r - G_e u_k - \beta e_k \\ e_{1,k+1} &= (1-\beta)e_{1,k} + \beta H_e n_k \end{aligned}\right\} \quad (11)$$

It is useful to make the following observation:

In the absence of noise $e_{1,k+1}=(1-\beta)^{k+1}e_{1,0}$ so that noise free error evolution converges to zero as $k\to\infty$ if, and only if, $\beta\in(0,2)$.

2.2 Properties

It is impossible to predict exact trajectories for the noise-free component of a signal f. It is however possible to predict statistical Expectations of Euclidean norms f^Tf and hence the associated Mean-square values $N^{-1}f^Tf$. And mean-square values is much better to examine the error level.

The following propositions are the first main results in this area.

Proposition 2.1 The expectation $E(\cdot)$ of $\| e_{1,k+1} \|^2$ at the $(k+1)^{th}$ iteration can be computed from

$$E(\| e_{1,k+1} \|^2) = \| (1-\beta)^{k+1} e_{1,0} \|^2 + \delta^2 A_k \tag{12}$$

where A_k is an "amplification factor" given by

$$A_k = \beta^2 \mathrm{tr}(H_e H_e^T) \sum_{j=0}^{k} (1-\beta)^{2j} \tag{13}$$

Note: Here $\mathrm{tr}(A) = \mathrm{tr}(A^T)$ denotes the trace of a square matrix A.

Remark 2.1 That is, the expectation is equal to the value of the noise free error norm (squared) obtained when noise is removed from all iterations increased by the term $\delta^2 A_k$ which is the variance (squared) amplified by a factor A_k dependent on plant dynamics and inverse-model-based ILC parameters. This amplification increases from iteration to iteration as it is easily seen that

$$A_{k+1} \geqslant A_k, \ \forall k \geqslant 0 \tag{14}$$

Proof Applying induction to the noise free error evolution Eq. (11)

$$e_{1,k+1} = (1-\beta) e_{1,k} + \beta H_e n_k \tag{15}$$

results in

$$e_{1,k+1} = (1-\beta)^{k+1} e_{1,0} + \beta (I-\beta)^k H_e n_0 + \beta(1-\beta)^{k-1} H_e n_1 + \cdots + \beta H_e n_k \tag{16}$$

The norm of $e_{1,k+1}$ has the form

$$\| e_{1,k+1} \|^2 = \| (1-\beta)^{k+1} e_{1,0} + \beta(1-\beta)^k H_e n_0 + \beta(1-\beta)^{k-1} H_e n_1 + \cdots + \beta H_e n_k \|^2 \tag{17}$$

Taking the expectation, noting that the cross product terms vanish and using the identity $f^T f = \mathrm{tr}[f f^T]$ gives

$$E(\| e_{1,k+1} \|^2) = \| (1-\beta)^{k+1} e_{1,0} \|^2 + \delta^2 \beta^2 \mathrm{tr}(H_e H_e^T) \sum_{j=0}^{k} (1-\beta)^{2j} \tag{18}$$

This completes the proof.

This formula applies exactly to each iteration. It takes the form of summations of $k+1$ terms and, as a consequence, may become arbitrarily large as $k \to \infty$ if the monotonically increasing sequence $\{A_k\}_{k \geqslant 0}$ diverges. This suggests that the introduction of noise may reduce algorithm performance catastrophically. The following result identifies the conditions where noise amplification remains finite as $k \to \infty$.

Proposition 2.2 If the noise-free inverse-model-based ILC algorithm is convergent (i.e. $n_j = 0$, $\forall j \geqslant 0$) and the learning gain β satisfies $\beta \in (0,2)$, then

$$\lim_{k \to \infty} E(\| e_{1,k+1} \|^2) = \frac{\delta^2 \beta \mathrm{tr}(H_e H_e^T)}{2-\beta} \tag{19}$$

Proof Because the learning gain β satisfies $\beta \in (0,2)$ which implies that $-1 < 1-\beta < 1$, so that

$$\lim_{k \to \infty} (1-\beta)^{2k+2} \| e_{1,0} \|^2 = 0 \tag{20}$$

Now note that

$$A_k = \beta^2 \mathrm{tr}(H_e H_e^T) \sum_{j=0}^{k} (1-\beta)^{2j} = \beta^2 \mathrm{tr}(H_e H_e^T) \frac{1-(1-\beta)^{2k+2}}{1-(1-\beta)^2} \tag{21}$$

As $\beta \in (0,2)$, then

$$A_\infty = \frac{\beta \mathrm{tr}(H_e H_e^T)}{2-\beta} \tag{22}$$

It follows that

$$\lim_{k \to \infty} E(\| e_{1,k+1} \|^2) = \frac{\delta^2 \beta \mathrm{tr}(H_e H_e^T)}{2-\beta} \tag{23}$$

This completes the proof.

3 Frequency domain bounds for amplification

In the most common practical situation when N is large, it would be useful to have formulae that do not need the manipulation of large $(N+1-k^*) \times (N+1-k^*)$ matrices. This section addresses this issue by the derivation of upper bounds on the amplification factors A_k in the frequency domain using contour integration. The results are shown to be very accurate if N is large and have the potential to provide an insight into the effect of plant dynamics and inverse-model-based ILC parameter choice on noise-related performance.

Recall the formula for A_k

$$A_k = \beta^2 \mathrm{tr}(H_e H_e^T) \sum_{j=0}^{k} (1-\beta)^{2j} \tag{24}$$

where H_e is the matrix form of a filter represented by the transfer function $H(z)$ and

$$H_e = \begin{bmatrix} h_1 & 0 & \cdots & 0 \\ h_2 & h_1 & \cdots & 0 \\ h_3 & h_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots \\ h_N & h_{N-1} & \cdots & h_1 \end{bmatrix} \tag{25}$$

Note that $\{h_1, h_2, \cdots, h_N\}$ represents the impulse response of $H(z)$.

It is easily seen that

$$\mathrm{tr}(H_e H_e^T) = N h_1^2 + (N-1) h_2^2 + \cdots + h_N^2 \tag{26}$$

and hence that

$$N^{-1}\mathrm{tr}(H_e H_e^T) = \sum_{j=1}^{N} {}^{2}_{j} - N^{-1}\sum_{j=1}^{N}(j-1)h_j^2 \tag{27}$$

which leads to the result that

Proposition 3.1 Because the filter $H(z)$ is asymptotically stable, then

$$N^{-1}\mathrm{tr}(H_e H_e^T) \leqslant \lim_{N\to\infty} N^{-1}\mathrm{tr}(H_e H_e^T) = \sum_{j=1}^{\infty} h_j^2 < \infty \tag{28}$$

the limit being approached from below.

Proof The proof is equivalent to proving that

$$\lim_{N\to\infty} N^{-1}\sum_{j=1}^{N}(j-1)h_j^2 = 0 \tag{29}$$

Asymptotic stability guarantees the existence of a number $M>0$ and $0<\lambda<1$ such that $|h_j|^2 \leqslant M\lambda^{j-2}$ for all $j \geqslant 1$. Hence

$$\sum_{j=1}^{N}(j-1)h_j^2 \leqslant M\sum_{j=1}^{N}\frac{\mathrm{d}}{\mathrm{d}\lambda}(\lambda^{j-1}) \leqslant M\frac{\mathrm{d}}{\mathrm{d}\lambda}\left[\frac{1}{1-\lambda}\right] < \infty \tag{30}$$

The result follows.

The value of this result is seen by using Parseval's theorem in the form

$$\sum_{j=1}^{\infty} h_j^2 = \oint_{\text{unit circle}} |H(z)|^2 \frac{\mathrm{d}z}{z} \tag{31}$$

which, using the known formula for summation of finite geometric series (13), gives

$$N^{-1}A_k \leqslant \lim_{N\to\infty} N^{-1}A_k = \frac{\beta^2(1-(1-\beta)^{2k+2})}{2\pi i(1-(1-\beta)^2)}\oint |H(z)|^2\frac{\mathrm{d}z}{z} \tag{32}$$

The formulae derived above are upper bounds on amplification factors but, for large k, may be difficult to compute or interpret. Simplifications are possible in the special case where

$$\beta \in (0, 2) \tag{33}$$

Note that,

① This special case assumption is strongly related to the case considered by Butcher et al.[14] underwriting the fact that the results in this paper are distinct and, in their domain, more general.

② It is a sufficient but not necessary condition for convergence of inverse-model-based ILC algorithm. The necessary and sufficient condition is $\beta \in (0, 2)$ as stated previously.

The following bound is then obtained

$$N^{-1}A_k \leqslant \lim_{N\to\infty} N^{-1}A_k = \frac{\beta^2(1-(1-\beta)^{2k+2})}{2\pi i(1-(1-\beta)^2)}\oint |H(z)|^2\frac{\mathrm{d}z}{z} \tag{34}$$

the bound being equal to the following limit

$$\lim_{k\to\infty}\lim_{N\to\infty} N^{-1}A_k = \frac{\beta}{2\pi i(2-\beta)}\oint |H(z)|^2\frac{\mathrm{d}z}{z} \tag{35}$$

This proves the Proposition:

Proposition 3.2 If $\beta \in (0, 2)$, then

$$\lim_{k\to\infty} N^{-1} E[\| e_{1,k+1} \|^2] \leqslant \frac{\delta^2\beta^2(1-(1-\beta)^{2k+2})}{2\pi i(1-(1-\beta)^2)}\oint |H(z)|^2\frac{\mathrm{d}z}{z} < \infty \tag{36}$$

and

$$\lim_{k\to\infty}\lim_{N\to\infty} N^{-1}E(\| e_{1,k+1} \|^2) = \frac{\delta^2\beta}{2\pi i(2-\beta)}\oint_{|z|=1} |H(z)|^2\frac{\mathrm{d}z}{z} \tag{37}$$

The formula (37) is simply a scaled version of the variance of the coloured noise $H_e n$. The scaling factor $\frac{\beta}{2-\beta}$ depends only on the learning gain β. It takes the value unity if $\beta=1$ (i. e. the asymptotic value of the mean square error norm is simply the variance of the coloured noise) and is small only if β is small - a beneficial situation but one that leads to slow convergence.

For the special case where $H(z)$ is a first order system, the Eq. (37) can be rewritten as

$$\lim_{k\to\infty}\lim_{N\to\infty} N^{-1}E(\| e_{1,k+1} \|^2) = \frac{\delta^2\beta C^{*2}B^{*2}}{(2-\beta)(1-A^{*2})} \tag{38}$$

where A^*, B^* and C^* are the state matrices in the state-space function of $H(z)$ (for the first order system, A^*, B^* and C^* are the numbers) and assume $C^*B^* \neq 0$.

To prove this, recall H_e which is the matrix representation of $H(z)$ with the forms

$$H_e = \begin{bmatrix} C^*B^* & 0 & \cdots & 0 \\ C^*A^*B^* & C^*B^* & \cdots & 0 \\ C^*A^{*2}B^* & C^*A^*B^* & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots \\ C^*A^{*N-1}B^* & C^*A^{*N-2}B^* & \cdots & C^*B^* \end{bmatrix} \tag{39}$$

And

$$\frac{1}{2\pi i}\oint_{|z|=1} |H(z)|^2 \frac{dz}{z} = \sum_{j=1}^{\infty} C^{*2}A^{*2j-2}B^{*2} = \lim_{N\to\infty} \frac{C^{*2}B^{*2}(1-A^{*2N})}{1-A^{*2}} \tag{40}$$

Because $H(z)$ is asymptotically stable which implies that $A^{*2}<1$, it follows that

$$\frac{1}{2\pi i}\oint_{|z|=1} |H(z)|^2 \frac{dz}{z} = \frac{C^{*2}B^{*2}}{1-A^{*2}} \tag{41}$$

Hence the Eq. (37) can be rewritten as

$$\lim_{k\to\infty}\lim_{N\to\infty} N^{-1}E(\| e_{1,k+1} \|^2) = \frac{\delta^2\beta C^{*2}B^{*2}}{(2-\beta)(1-A^{*2})} \tag{42}$$

as $H(z)$ is the first order system.

4 Numerical examples

To demonstrate the validity of the theoretical findings, consider continuous-time plant with transfer function

$$G(s) = \frac{s+1}{s^2+5s+6} \tag{43}$$

sampled using a sample interval 0.01s using a Zero-order hold. The corresponding discrete-time system has transfer function

$$G(z) = \frac{0.0098z - 0.0097}{z^2 - 1.951z + 0.9512} \tag{44}$$

and zero initial conditions are assumed. The discrete reference signal chosen is the sampled version of $r=\sin(t)$. And the filter $H(z)$ takes the forms

$$H(z) = \frac{0.136z - 0.13}{z+0.72} \tag{45}$$

where the state matrices of filter $H(z)$ is $A^* = -0.72$, $B^* = 0.5$ and $C^* = -0.4558$. Consider the case of noise variance $\delta = 0.072$ (i.e. $\delta^2 = 0.0052$) equivalent to a noise-to-signal ratio (NSR) of approximately 10%. Noise signals were generated using standard MATLAB routines.

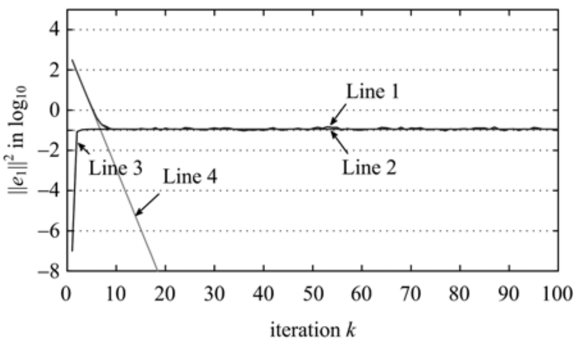
Consider the inverse-model-based ILC algorithm takes the form

$$u_{k+1} = u_k + \beta G_e^{-1} e_k \tag{46}$$

Example 4.1 According to Propositions 2.1 and 2.2, if β is selected as 0.5 which satisfies $\beta \in (0,2)$ and $A^{*2} = 0.5184 < 1$, then

$$\left.\begin{aligned} &\lim_{k\to\infty} E(\| e_{1,k+1} \|^2) = \delta^2 A_\infty = \\ &\qquad \frac{\delta^2\beta\,\mathrm{tr}(H_e H_e^T)}{2-\beta} \approx 0.112 \\ &\log_{10} 0.112 \approx -0.951 \end{aligned}\right\} \tag{47}$$

The noise amplification in this case is $A_\infty \approx 21.534$.



Line 1: $\log_{10}\| e_{1,k+1} \|^2$, Line 2: $\log_{10} E(\| e_{1,k+1} \|^2)$,

Line 3: $\log_{10}\delta^2 A_k$, Line 4: $\log_{10}\| (I-\beta G)^{k+1} e_{1,0} \|^2$

Fig. 1 Evolution of $\| e_{1,k+1} \|^2$ and $E(\| e_{1,k+1} \|^2)$ for NSR≈10%

The results in Fig. 1 indicate the following:

① Although the inverse-model-based ILC algorithm converges to zero error in the absence of noise (Line 4), the presence of noise (Line 1) leaves a residual error as $k\to\infty$.

② The example confirms that $E(\| e_{1,k+1} \|^2)$ (Fig. 1 Line 2) can be a good prediction of observed values of $\| e_{1,k+1} \|^2$ (Line 1) in the presence of noise.

③ The results are consistent with the existence of a finite limit for $E(\| e_{1,k+1} \|^2)$ and indicate the validity of the prediction that it *approaches* $\delta^2 A_k$ as k goes to infinity (Fig. 1 Line 3) e. g. after 20 iterations, $E(\| e_{1,k+1} \|^2)$ and $\delta^2 A_k$ are already very close in value suggesting in Eq. (47) that

$$\left.\begin{aligned} &\lim_{k\to\infty} E(\| e_{1,k+1} \|^2) = \delta^2 A_\infty \approx 0.112 \\ &\log_{10} 0.112 \approx -0.951 \end{aligned}\right\} \quad (48)$$

④ As a consequence, for practical purposes the algorithm performance is dominated by the term $\delta^2 A_k$ if k is large and hence the computation of A_k is central to evaluating algorithm performance.

Example 4. 2 To indicate the validity of the predictions in section 4, N is chosen as 600 in this case and

$$\lim_{k\to\infty} \lim_{N\to\infty} N^{-1} E(\| e_{1,k+1} \|^2) = \frac{\delta^2 \beta C^{*2} B^{*2}}{(2-\beta)(1-A^{*2})} \approx 1.870^{-4} \quad (49)$$

$$\log_{10} 1.563^{-7} \approx -3.728 \quad (50)$$

The information from Fig. 2 confirms the correctness of Proposition 3. 2. In this simulation, the experimental result is very close to the theoretical value, −3. 728, which implies

$$\lim_{k\to\infty} \lim_{N\to\infty} N^{-1} E(\| e_{1,k+1} \|^2) = \frac{\delta^2 \beta C^{*2} B^{*2}}{(2-\beta)(1-A^{*2})} \quad (51)$$

if $H(z)$ is the first order system.

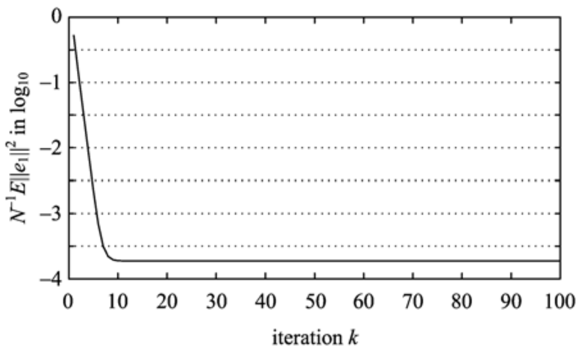


Fig. 2 Evolution of $N^{-1} \| e_{1,k+1} \|^2$ for NSR≈10%

5 Conclusion

In this paper, the main work was to compute the expectation of the noise-free error norm squared on each trial by taking into account the fact that noise from previous iterations will affect dynamics due to the inclusion of previously measured errors from previous iterations in the inverse-model-based ILC algorithm. Explicit formulae have been derived which demonstrate the fact that normally convergent inverse-model-based ILC algorithms of the type considered will again "converge" to limit error time series that have norms equal to the variance of the underlying white noise amplified by a factor dependent on plant dynamics, noise characteristics and inverse-model-based ILC parameter choice.

It has also been shown that these exact formulae can be replaced by frequency domain upper bounds and that these bounds are increasingly less conservative as the time interval for control increases in length. These results have the potential to show the direct link between plant dynamical properties and noise amplification.

Finally, it is noted that, in the event that the noise variance varies from trail to trial but is bounded by δ^2, then noise statistics could be modelled by the inequality $E[n_i n_i^T] \leqslant \delta^2 I \ \forall i \geqslant 0$. Under this circumstance, the Eq. (12) in Proposition 2. 1 is replaced by

$$E(\| e_{1,k+1} \|^2) \leqslant \| (1-\beta)^{k+1} e_{1,0} \|^2 + \delta^2 A_k \quad (52)$$

and it is a simple matter to show then that the formulae given in this paper provide upper bounds on the behaviour of $E(\| e_{1,k+1} \|^2)$.

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