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A new class of projectively flat Finsler metrics

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Abstract: A class of projectively flat Finsler metrics with three parameters was constructed, which generalizes a result obtained by Mo and Yang.

Key words: Finsler metric; projectively flat; flag curvature

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关于一类新的射影平坦 Finsler 度量

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摘要：构造了一类带有三参数的射影平坦 Finsler 度量，推广了莫和杨的结论.

关键词：Finsler 度量；射影平坦；旗曲率

0 Introduction

Finsler geometry is more colorful than Riemannian geometry because there are several non-Riemannian quantities on a Finsler manifold besides the Riemannian quantities. One of the important problems in Finsler geometry is to study and characterize the projectively flat metrics on an open domain $U\subset\mathbb{R}^n$. Projectively flat metrics on U are Finsler metrics whose geodesics are straight lines. This is the Hilbert's 4th problem in the regular case[1]. In 1903, Hamel[2] found a system of partial differential equations

$$F_{x^k y^l} y^k = F_{x^l} \tag{1}$$

which can characterize the projectively flat metrics $F=F(x,y)$ on an open subset $U\subset\mathbb{R}^n$. And we know that Riemannian metrics form a special and important class in Finsler geometry. Beltrami's theorem[3] tells us that a Riemannian metric is locally projectively flat if and only if it is of constant sectional curvature. The flag curvature in Finsler geometry is a natural extension of the sectional curvature in Riemannian geometry. Besides, every locally projectively flat Finsler metrics F on a manifold M is of scalar flag curvature, i.e., the flag curvature $K=K(x,y)$ is

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a scalar function on $TM\setminus\{0\}$. Many projectively flat Finsler metrics with constant flag curvature are obtained in Refs. [4-7]. Besides, there are a lot of locally projectively flat Finsler metrics that are not of constant flag curvature[8-10]. Thus, the Beltrami's theorem is no longer true for Finsler metrics.

In this paper, we construct a new class of Finsler metrics with three parameters which are not of constant sectional curvatures. Let ζ be an arbitrary constant and $\Omega=\mathbb{B}^n(r)\in\mathbb{R}^n$ where $r=\sqrt{-\zeta}$ if $\zeta<0$ and $r=+\infty$ if $\zeta>0$, $|\cdot|$ and $\langle,\rangle$ be the standard Euclidean norm and inner product in $\mathbb{R}^n$, respectively.

Define F: $T\Omega\to[0,+\infty)$ by

$$\alpha=\frac{\sqrt{\kappa^2\langle x,y\rangle^2+\varepsilon|y|^2(1+\zeta|x|^2)}}{1+\zeta|x|^2} \tag{2}$$

$$\beta=\frac{\kappa\langle x,y\rangle}{1+\zeta|x|^2}+\frac{\kappa\langle a,y\rangle}{1+\varepsilon\langle a,x\rangle} \tag{3}$$

where ε is an arbitrary positive constant, κ is an arbitrary constant, and $a\in\mathbb{R}^n$ is constant vector with sufficiently small norm.

Remark 0.1 When $\kappa^2=1$, $\zeta=-1$, $\varepsilon=1$, F is a general Funk metric[11]. If $a=0$, F is Mo and Yang's metric[8]. They not only proved that F is a Randers metric but also showed that F is a projectively flat Finsler metric if and only if $\kappa^2+\varepsilon\zeta=0$.

As a natural prolongation, we obtain the following results:

Theorem 0.1 Let $F=\alpha+\beta$: $T\Omega\to[0,+\infty)$ be a function given by (2) and (3). Then, it has the following properties.

① When $\kappa^2+\varepsilon\zeta=0$, F is a Randers metric.

② F is a projectively flat Finsler metric if and only if $\kappa^2+\varepsilon\zeta=0$.

③ When $\kappa^2+\varepsilon\zeta=0$, the projectively flat Finsler metric's flag curvature is given by

$$K=\frac{P^2-P_{x^k}y^k}{F^2}=\frac{1}{4\kappa^2F^4}\{-[4\zeta^2(F-\psi)^3+4\varepsilon^2\psi^3]F+3[\zeta(F-\psi)^2+\varepsilon\psi^2]^2\},$$

where $\psi=\dfrac{\kappa\langle a,y\rangle}{1+\varepsilon\langle a,x\rangle}$.

1 Preliminaries

A Minkowski norm $\Psi(y)$ on a vector space V is a C^∞ function on $V\setminus\{0\}$ with the following properties:

① $\Psi(y)\geqslant 0$ and $\Psi(y)=0$ if and only if $y=0$;

② $\Psi(y)$ is positively homogeneous function of degree one, i.e., $\Psi(ty)=t\Psi(y)$, $t\geqslant 0$;

③ $\Psi(y)$ is strongly convex, i.e., for any $y\neq 0$, the matrix $g_{ij}(x,y):=\frac{1}{2}[F^2]_{y^iy^j}(x,y)$ is positively definite.

A Finsler metric F on a manifold M is C^∞ function on $TM\setminus\{0\}$ such that $F_x:=F|_{T_xM}$ is a Minkowski norm on T_xM for any $x\in M$. The fundamental tensor $g_{ij}(x,y):=\frac{1}{2}[F^2]_{y^iy^j}(x,y)$ is positively definite. If $g_{ij}(x,y)=g_{ij}(x)$, F is a Riemannian metric. If $g_{ij}(x,y)=g_{ij}(y)$, F is a Minkowski metric. If all geodesics are straight lines, F is projectively flat. This is equivalent to $G^i=P(x,y)y^i$ are geodesic coefficients of F, where G^i are given by

$$G^i=\frac{g^{il}}{4}\{[F^2]_{x^my^l}y^m-[F^2]_{x^l}\}.$$

For each tangent plane $\Pi\subset T_xM$ and $y\in\Pi$, the flag curvature of (Π,y) is defined by

$$K(\Pi,y)=\frac{g_{im}R^i_ku^ku^m}{F^2g_{ij}u^iu^j-[g_{ij}y^iu^j]^2},$$

where $\Pi=\mathrm{span}\{y,u\}$, and

$$R^i_k=2\frac{\partial G^i}{\partial x^k}-y^i\frac{\partial^2G^i}{\partial y^i\partial y^k}+2G^j\frac{\partial^2G^i}{\partial y^j\partial y^k}-\frac{\partial G^i}{\partial y^j}\frac{\partial G^j}{\partial y^k}.$$

We need the following lemma for later use.

Lemma 1.1[2] Let F be a Finlser metric on an open domain $U\subset\mathbb{R}^n$, F is projectively flat on this domain if and only if

$$F_{x^ky^l}y^k=F_{x^l}.$$

In this case, the flag curvature K of F is given by

$$K=\frac{P^2-P_{x^k}y^k}{F^2},$$

where the projective factor can be expressed as

$$P=\frac{F_{x^m}y^m}{2F}.$$

2 Proof of Theorem 0.1

Let $\omega=1+\zeta|x|^2$, $\rho^2=\kappa^2+\varepsilon\zeta$, $\alpha^2=a_{ij}y^iy^j$, $\beta=b_iy^i$.

① From (2) and (3), we obtain

$$a_{ij}=\frac{\varepsilon\delta_{ij}}{\omega}+\frac{\kappa^2x^ix^j}{\omega^2},$$

$$b_i=\frac{\kappa x^i}{\omega}+\frac{\kappa a^i}{1+\langle a,x\rangle} \quad (4)$$

$$b_j=\frac{\kappa x^j}{\omega}+\frac{\kappa a^j}{1+\langle a,x\rangle} \quad (5)$$

Noting that $\omega=1+\zeta|x|^2>0$, $(a^{ij})=(a_{ij})^{-1}$, we get

$$a^{ij}=\frac{\omega}{\varepsilon}\left(\delta^{ij}-\frac{\kappa^2x^ix^j}{\varepsilon+\rho^2|x|^2}\right) \quad (6)$$

Combining (4), (5) and (6), we have

$$\|\beta\|_\alpha^2=a^{ij}b_ib_j=\frac{\omega}{\varepsilon}\left(\delta^{ij}-\frac{\kappa^2x^ix^j}{\varepsilon+\rho^2|x|^2}\right)\cdot\left(\frac{\kappa x^i}{\omega}+\frac{\kappa a^i}{1+\langle a,x\rangle}\right)\left(\frac{\kappa x^j}{\omega}+\frac{\kappa a^j}{1+\langle a,x\rangle}\right)=$$
$$\frac{\kappa^2|x|^2}{\varepsilon\omega}+\frac{2\kappa^2\langle a,x\rangle}{\varepsilon(1+\varepsilon\langle a,x\rangle)}+\frac{\omega\kappa^2|a|^2}{\varepsilon(1+\varepsilon\langle a,x\rangle)^2}-\frac{2\kappa^4|x|^2\langle a,x\rangle}{\varepsilon(1+\varepsilon\langle a,x\rangle)(\varepsilon+\rho^2|x|^2)}-$$
$$\frac{\kappa^4\langle a,x\rangle^2\omega}{\varepsilon(1+\langle a,x\rangle^2)(\varepsilon+\rho^2|x|^2)}-\frac{\kappa^4|x|^4}{\varepsilon\omega(\varepsilon+\rho^2|x|^2)}.$$

Suppose $\rho^2=\kappa^2+\varepsilon\zeta=0$ and a has sufficiently small norm, we have

$$\|\beta\|_\alpha^2=-\zeta|x|^2-\frac{2\zeta\omega\langle a,x\rangle}{1+\varepsilon\langle a,x\rangle}-\frac{\zeta\omega|a|^2}{(1+\varepsilon\langle a,x\rangle)^2}-\frac{\zeta^2\omega\langle a,x\rangle^2}{(1+\varepsilon\langle a,x\rangle)^2}=$$
$$1-\frac{\omega[(\zeta\langle a,x\rangle+1+\varepsilon\langle a,x\rangle)^2+\zeta|a|^2]}{(1+\varepsilon\langle a,x\rangle)^2}<1.$$

So F is a Randers metrics.

② Let

$$\Delta=\kappa^2\langle x,y\rangle^2+\varepsilon|y|^2(1+\zeta|x|^2)=\kappa^2\langle x,y\rangle^2+\varepsilon\omega|y|^2,$$

by direct computations, we have

$$\alpha_{x^l}=\frac{1}{\omega^2}[\Delta^{-\frac12}\omega(\kappa^2\langle x,y\rangle y^l+\varepsilon\zeta|y|^2x^l)-2\zeta\Delta^{\frac12}x^l] \quad (7)$$

$$\beta_{x^l}=\frac{1}{\omega^2}[\kappa\omega y^l-2\zeta\kappa\langle x,y\rangle x^l]-\frac{\varepsilon\kappa\langle a,y\rangle a^l}{(1+\varepsilon\langle a,x\rangle)^2} \quad (8)$$

$$\alpha_{x^ky^l}y^k=\frac{1}{\omega^2}\{\Delta^{-\frac32}\varepsilon\rho^2\omega|y|^4\omega x^l+\Delta^{-\frac12}\kappa^2\omega\langle a,x\rangle y^l-\Delta^{-\frac32}\varepsilon\rho^2\omega|y|^2\langle x,y\rangle y^l-2\Delta^{\frac12}\zeta x^l+\varepsilon\zeta\Delta^{-\frac12}\omega|y|^2x^l\} \quad (9)$$

$$\beta_{x^ky^l}y^k=\frac{1}{\omega^2}[\kappa\omega y^l-2\zeta\kappa\langle x,y\rangle x^l]-\frac{\varepsilon\kappa\langle a,y\rangle a^l}{(1+\varepsilon\langle a,x\rangle)^2} \quad (10)$$

From (7) and (8), we obtain

$$F_{x^l}=\alpha_{x^l}+\beta_{x^l}=\frac{1}{\omega^2}[\Delta^{-\frac12}\omega(\kappa^2\langle x,y\rangle y^l+\varepsilon\zeta|y|^2x^l)-2\zeta\Delta^{\frac12}x^l]+\frac{1}{\omega^2}[\kappa\omega y^l-2\zeta\kappa\langle x,y\rangle x^l]-\frac{\varepsilon\kappa\langle a,y\rangle a^l}{(1+\varepsilon\langle a,x\rangle)^2} \quad (11)$$

We also have

$$F_{x^ky^l}y^k=\alpha_{x^ky^l}y^k+\beta_{x^ky^l}y^k=\frac{1}{\omega^2}\{\Delta^{-\frac32}\varepsilon\rho^2\omega|y|^4\omega x^l+\Delta^{-\frac12}\kappa^2\omega\langle x,y\rangle y^l-\Delta^{-\frac32}\varepsilon\rho^2\omega|y|^2\langle x,y\rangle y^l-2\Delta^{\frac12}\zeta x^l+\varepsilon\zeta\Delta^{-\frac12}\omega|y|^2x^l+\kappa\omega y^l-2\zeta\kappa\langle x,y\rangle x^l\}-\frac{\varepsilon\kappa\langle a,y\rangle a^l}{(1+\varepsilon\langle a,x\rangle)^2} \quad (12)$$

here we use (9) and (10).

By (11) and (12), we get

$$F_{x^ky^l}y^k-F_{x^l}=\frac{1}{\omega^2}\{\Delta^{-\frac32}\varepsilon\rho^2\omega|y|^4\omega x^l-\Delta^{-\frac32}\varepsilon\rho^2\omega|y|^2\langle x,y\rangle y^l\}=\frac{1}{\omega}\Delta^{-\frac32}\varepsilon\rho^2|y|^2(|y|^2x^l-\langle x,y\rangle y^l)$$

If $\rho^2=\kappa^2+\varepsilon\zeta=0$, we have $F_{x^ky^l}y^k-F_{x^l}=0$, so F is a projectively flat Finsler metric. The converse is obvious, so the proof is omitted here.

③ From Lemma 1.1, projectively flat Finsler metri's projective factor and flag curvature are given by

$$P=\frac{F_{x^m}y^m}{2F} \quad (13)$$

$$K=\frac{P^2-P_{x^k}y^k}{F^2} \quad (14)$$

By a simple calculation for F, we get

$$F_{x^m}y^m=\frac{1}{\omega^2}[\Delta^{-\frac12}\rho^2|y|^2\langle x,y\rangle-2\Delta^{\frac12}\zeta\langle x,y\rangle+\kappa\omega^2|y|^2-2\zeta\kappa\langle x,y\rangle^2]-\frac{\varepsilon\kappa\langle a,y\rangle^2}{1+\varepsilon\langle a,x\rangle^2} \quad (15)$$

For $\Delta=\kappa^2\langle x,y\rangle^2+\varepsilon\omega|y|^2$, we have

$$\omega|y|^2=\frac{1}{\varepsilon}(\Delta-\kappa^2\langle x,y\rangle^2) \quad (16)$$

By (13), (15) and (16), we get

$$P=\frac{F_{x^m}y^m}{2F}=$$

$$\frac{1}{2F}\left\{\frac{1}{\omega^2}\left[\Delta^{-\frac{1}{2}}\rho^2\mid y\mid^2\langle x,y\rangle-2\Delta^{\frac{1}{2}}\zeta\langle x,y\rangle+\kappa\Delta-\frac{\kappa^3\langle x,y\rangle^2}{\varepsilon}-2\zeta\kappa\langle x,y\rangle^2\right]-\frac{\varepsilon\kappa\langle a,y\rangle^2}{(1+\varepsilon\langle a,x\rangle)^2}\right\}.$$

When $\rho^2=\kappa^2+\varepsilon\zeta=0$, P can be expressed as the following form.

$$P=\frac{1}{2F}\left[\frac{\kappa}{\varepsilon}(F-\psi)^2-\frac{\varepsilon\psi^2}{\kappa}\right]=-\frac{\zeta(F-\psi)^2-\varepsilon\psi^2}{2\kappa F}\tag{17}$$

Using (17), we have

$$P^2=\frac{1}{4\kappa^2F^2}[\zeta(F-\psi)^2+\varepsilon\psi^2]^2\tag{18}$$

$$P_{x^k}y^k=\frac{1}{4\kappa^2F^2}\left\{[4\zeta^2(F-\psi)^3+4\varepsilon^2\psi^3]F-2[\zeta(F-\psi)^2+\varepsilon\psi^2]^2\right\}\tag{19}$$

Substituting (18) and (19) into (14), we get

$$K=\frac{P^2-P_{x^k}y^k}{F^2}=$$

$$\frac{1}{4\kappa^2F^2}\left\{-[4\zeta^2(F-\psi)^3+4\varepsilon^2\psi^3]F+3[\zeta(F-\psi)^2+\varepsilon\psi^2]^2\right\},$$

where $\psi=\dfrac{\kappa\langle a,y\rangle}{1+\varepsilon\langle a,x\rangle}$.

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HRS optics optimization method for Coulomb sum rule (CSR) experiment in JLab Hall-A

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Abstract: In order to study the acceptance and calculate cross sections for CSR experiment, one needs to obtain target variables on the target plane of the spectrometer. Target variables can be reconstructed from focal plane variables with optics. In principle, the optics is the mathematical expression of the magnetic field of HRS. The optics optimization method which minimizes χ^2 of $TgVar_{data} - TgVar_{theory}$ to determine the better optics matrix elements for CSR experiment was studied. The optimization results show a good agreement between the data and theoretical values.

Key words: Coulomb sum rule; optics; acceptance; target coordinate

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JLab Hall-A 库仑求和规则实验高精度谱仪光学性能优化方法

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摘要:在库仑求和实验中,为了研究接收度和截面计算,需要在高精度谱仪的靶平面上获得一些靶变量.这些靶变量可以通过谱仪的光学性能从聚集平面上的变量反推重建出来.原则上,谱仪的光学性能就是高精度谱仪内部磁场的数学表达,可以用光学矩阵来表示.研究了谱仪光学性能优化方法,就是对变量 $\chi^2(TgVar_{data} - TgVar_{theory})$进行最小化计算.通过这个计算过程,可以为库仑求和实验获得更好的光学矩阵数据.从最后的优化结果来看,实际数据和理论值符合得很好.

关键词:库仑求和规则;光学;接收度;靶坐标系

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0 Introduction

There are two high resolution spectrometers (HRS) in Hall A at Jefferson Lab, both with QQDQ (Q: quadrapole, D: dipole) configurations[1]. The HRSs have focusing properties that are point-to-point in the dispersive direction. The scattered particles passing through the HRS are bent and focused on the VDC (vertical drift chamber) plane. All signals are translated into physical variables on the VDC plane, also called "focal plane variables". However, in order to study the acceptance and calculate cross sections, one needs to obtain "target variable" on the target plane. Target variables can be reconstructed from focal plane variables with "optics". In principle, the optics is the mathematical expression (i. e. optics matrix)[2] of magnetic field of HRS. The optics matrix elements allow the reconstruction of the interaction vertex from the coordinates of the detected particles on the focal plane. Data obtained with a set of foil targets (which define a set of well-defined interaction points along the beam) and a sieve-slit collimator were used to determine the optical matrix elements. For Coulomb sum rule (CSR)[3-7] experiment, the nominal momentum range for optics runs for both arms is 0.4～2.0 GeV/c. The optimization method to determine the optics matrix elements will be studied as follows.

1 HRS optics system

1.1 Optics optimization process

During a normal calibration procedure, we can obtain the theoretical and experimental values from one measurement and then try to change calibration constants to make theoretical values as close to experimental values as possible. For example, we know V_{theory} and V_{data}, assuming $V_{data} = C \cdot V_{theory}$. With optimizing C, we finally get a linear fit for V_{theory} vs V_{data}. And C is the result we want to get.

It is the same idea for optics optimization. What we have are target variables, such as Z_{react}, ϕ_{tg}, θ_{tg}, y_{tg}, calculated from sieve slit and survey information. We can define these variables as $TgVar_{theory}$. The experimental values for target variables are reconstructed from focal plane variables (FpVar) detected by VDC with the help of optics matrix (OP). In general, the relation between them is $TgVar_{data} = OP \cdot FpVar$. By changing OP and getting smallest χ^2 of $(TgVar_{data} - TgVar_{theory})$, we can get new optics matrix (OP) for better optics optimization.

1.2 Target and focal plane coordinate

Target coordinate system is one of the coordinates of Hall-A at Jefferson Lab used to define the target variables. Fig. 1 shows the coordinates for electrons scattered from a thin foil target. The target coordinate center is shown in the figure as a black cross. L is the distance from Hall center to the sieve slit plane. The black line tagged with z_{tg} is along the central line of the spectrometer and the one with y_{tg} is perpendicular to it. Θ_0 is the setting central angle of the spectrometer. ϕ_{tg}, y_{tg} and z_{react} are defined in the figure[8]. D is the horizontal displacement of the spectrometer axis from its ideal position. And D is also the D_y defined later in Eq. (1). Note that x_{tg}, θ_{tg}, D_x and x_{sieve} are vertically down (into the page).

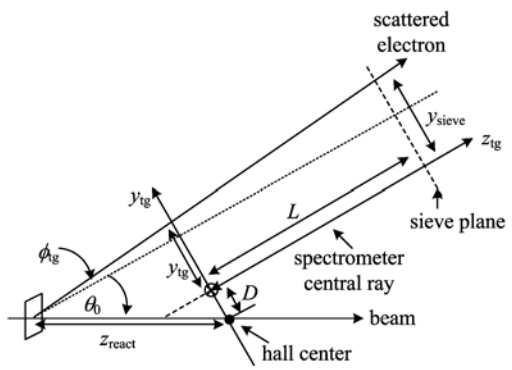

Fig. 1 Target coordinate system schematic

The HRS sieve slit hole pattern is shown in Fig. 2. The distance between holes and the diameter of different holes are defined in the figure.

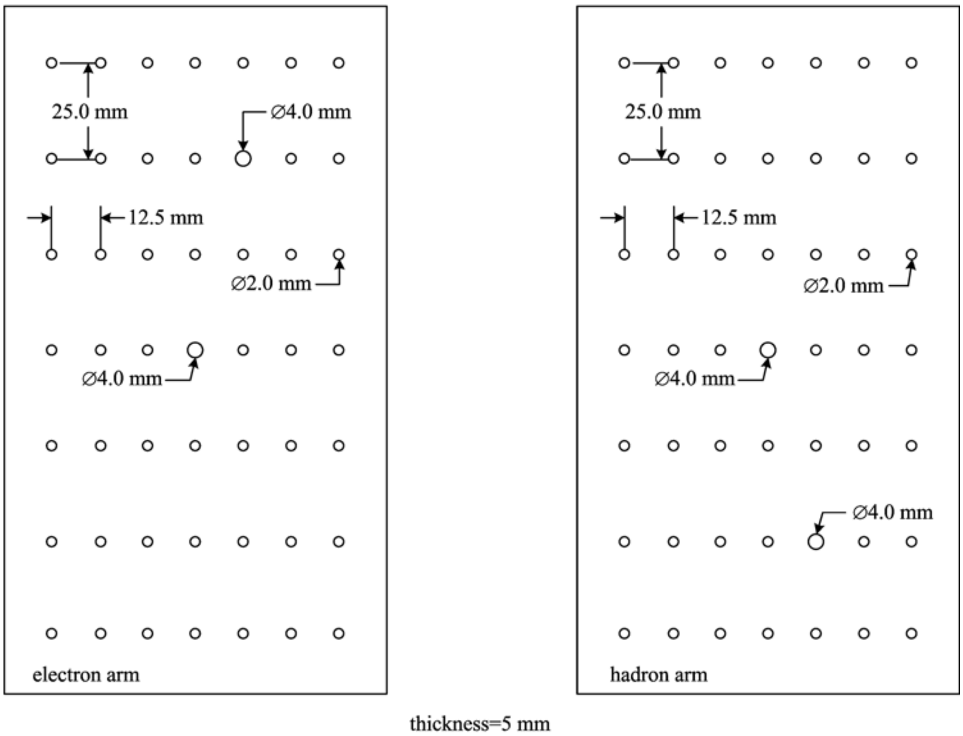

Fig. 2　Sieve slit structure in Hall-A

Tab. 1　L and sieve slit survey

	location	Z/mm	X/mm	Y/mm	yaw angle/degree	3D dist/mm
2006-07-03	left arm sieve slit	357.72	1 109.13	−0.12	72.124	1 165.39
	left arm 6 Msr	335.89	1 038.08	0.05	72.070	1 091.08
	right arm sieve slit	994.57	−657.87	−0.15	−33.483	1 192.46
	right arm 6Msr	933.01	−615.17	−0.18	−33.398	1 117.56
2008-02-08	left arm sieve slit	948.05	677.59	2.05	35.554	1 165.30
	left arm 6Msr	888.31	633.35	2.53	35.488	1 090.98

Survey information on the distance from the Hall center to the front surface of the sieve slits, L, is shown in Tab. 1. Sieve survey information is in Fig. 2 and Tabs. 1, 2. The survey informations of Z_{react}, D_x, D_y of the left and right arm were also obtained. The distance between each optics foil is 4 cm.

In Tab. 1, the coordinates are relative to the ideal Hall-A target and beamline, with $+Z$ along the beam, $+X$ to the beam left and $+Y$ up. Measurements are to the upstream face of the collimators. In Tab. 2, the first sets of coordinates are to the center of the slits relative to the spectrometer center line. The second set is relative to the Hall-A target and beam line as reported in the DT A1102. A $+X$ is to the beam left, A $+Z$ is downstream and a $+Y$ is up.

Tab. 2　Sieve slit offset

location	Z/mm	X/mm	Y/mm
relative to the spectrometer center line			
left (electron slit)	974	1.48	2.76
right (hadron slit)	1 002	−1.31	−2.67
relative to the Hall A beam line			
left (electron slit)	949	213.93	2.12
right (hadron slit)	977	−215.76	−3.19

1.3　$TgVar_{theory}$ and $TgVar_{data}$ calculation

The Formulas to calculate $TgVar_{theory}$ are defined as follows.

$$
\left.\begin{aligned}
\phi_{\text{tg}} &= \frac{y_{\text{sieve}} + D_y - x_{\text{beam}}\cos(\theta_0) + z_{\text{react}}\sin(\theta_0)}{L - z_{\text{react}}\cos(\theta_0) - x_{\text{beam}}\sin(\theta_0)}, \\
\theta_{\text{tg}} &= \frac{x_{\text{sieve}} + D_x - y_{\text{beam}}}{L - z_{\text{react}}\cos(\theta_0) - x_{\text{beam}}\sin(\theta_0)}, \\
y_{\text{tg}} &= y_{\text{sieve}} - L\phi_{\text{tg}}, \\
x_{\text{tg}} &= x_{\text{sieve}} - L\theta_{\text{tg}}, \\
\theta_{\text{scat}} &= \arccos\left(\frac{\cos(\theta_0) - \phi_{\text{tg}}\sin(\theta_0)}{\sqrt{(1+\theta_{\text{tg}}^2+\phi_{\text{tg}}^2)}}\right), \\
p(M,\theta) &= E' = \frac{E}{1 + E/M(1-\cos(\theta))}, \\
dp_{\text{kin}} &= dp - \frac{p(M,\theta_{\text{scat}}) - p(M,\theta_0)}{p_0}
\end{aligned}\right\} \tag{1}
$$

Making use of the survey values, Fig. 2 and Eq. (1), we can calculate $\text{TgVar}_{\text{theory}}$.

VDC detector and focal plane coordinate system is shown in Figs. 3 and 4, respectively.

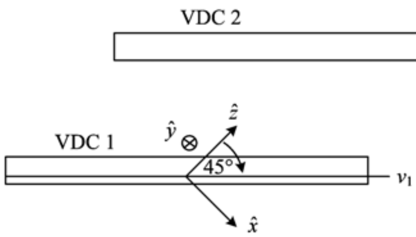

Fig. 3 VDC detector coordinate system

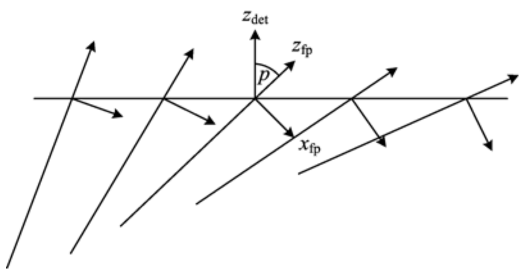

Fig. 4 Focal plane coordinate system

The formulas to calculate $\text{TgVar}_{\text{data}}$ are defined as follows.

$$
\left.\begin{aligned}
\delta &= \sum_{j,k,l} D_{jkl}\theta_{\text{fp}}^j y_{\text{fp}}^k \phi_{\text{fp}}^l, \\
\theta_{\text{tg}} &= \sum_{j,k,l} T_{jkl}\theta_{\text{fp}}^j y_{\text{fp}}^k \phi_{\text{fp}}^l, \\
y_{\text{tg}} &= \sum_{j,k,l} Y_{jkl}\theta_{\text{fp}}^j y_{\text{fp}}^k \phi_{\text{fp}}^l, \\
\phi_{\text{tg}} &= \sum_{j,k,l} P_{jkl}\theta_{\text{fp}}^j y_{\text{fp}}^k \phi_{\text{fp}}^l
\end{aligned}\right\} \tag{2}
$$

where

$$
D_{jkl} = \sum_{i=0}^{m} C_{ijkl}^{D} x_{\text{fp}}^{i} \tag{3}
$$

So after obtaining the δ, θ_{tg}, y_{tg} and ϕ_{tg} value from theory and data, we can extract the calibration coefficients (D_{jkl}, T_{jkl}, Y_{jkl}, P_{jkl}) with χ^2 minimization method for VDC and get better optics conditions.

2 The optics optimization results

After the optimization calculation procedure, we obtained some results. Fig. 5 shows y_{tg} vs ϕ_{tg} distribution before optics calibration, from which we can see the peak location or the center of events cluster can not match well with the theoretical line or center initially.

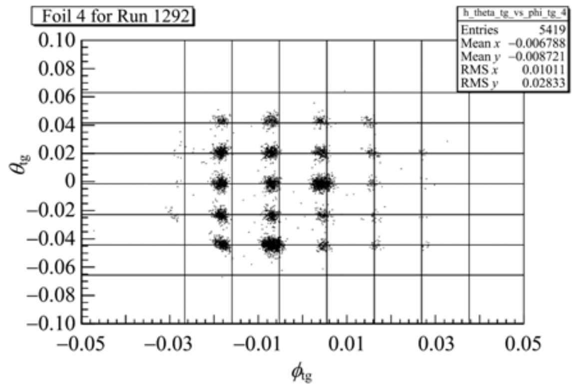

Fig. 5 y_{tg} vs ϕ_{tg} distribution before optics calibration

Fig. 6 shows y_{tg} vs ϕ_{tg} distribution after optics calibration, from which we can see that the peak location or the center of events cluster are aligned well to the theoretical line or center after the optics calibration well.

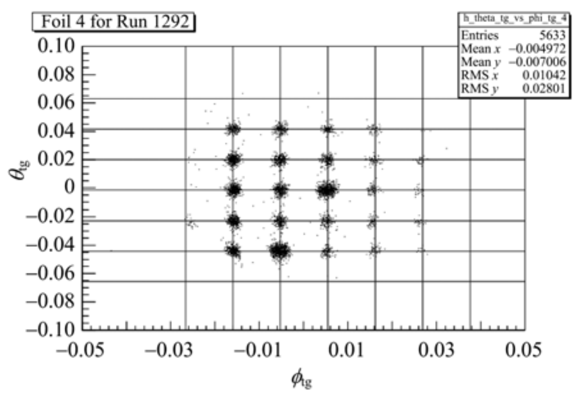

Fig. 6 y_{tg} vs ϕ_{tg} distribution after optics calibration