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异质量两粒子的纠缠态表象的特性分析与压缩态生成

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摘要:发现用 Weyl 编序以及有序算符内的积分技术可以直接推导出异质量两粒子的纠缠态表象,其纠缠特性可以通过 Schmidt 分解得以显现.用此表象给出了一类新的单-双模组合压缩态以及相应的压缩算符.

关键词:异质量的纠缠态表象;Schmidt 分解;压缩态

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Entangled state representation for two particles with unequal masses and the squeezed state generation

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Abstract: It was found that the entangled state representation of two particles with unequal masses can be directly derived by the Weyl correspondence and the technique of integration within an ordered product of operators. The entanglement properties can be achieved through its Schmidt decomposition. With this representation, a kind of new one- and two- combination squeezed state and corresponding squeezing operator was obtained.

Key words: entangled state representation for unequal mass; Schmidt decomposition; squeezed state

0 引言

量子力学的表象是量子力学态矢的“坐标架”,态矢在表象中的投影即为相应的波函数.Dirac 首先将波函数 $\psi(x)$ 改写成 $\langle x \mid \psi\rangle$,引入了坐标表象 $\{\mid x\rangle, \int_{-\infty}^{+\infty}\mathrm{d}x \mid x\rangle\langle x \mid =1\}$.选择合适的表象不仅能够使量子力学中的问题简化,同时也能更好理解量子态的物理意义.

常见的连续表象仅有坐标表象、动量表象、相干态表象[1],由于缺乏解析的纠缠表象理论,因此人们对量子纠缠的研究往往局限在偏振、自旋以及轨道角动量等分离的物理量上[2-7],对连续物理量的纠缠态研究相对较少.为描述量子纠缠,范洪义等对两个

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质量相等的粒子建立了两体纠缠态表象[8]，但对异质量的两体纠缠表象的性质未详细研究.本文利用 Weyl 编序以及相空间中 Wigner 算符的 Radon 变换来引入广义双模纠缠态表象，即质量不相等的两粒子纠缠态表象.在此基础上，通过对广义双模纠缠态的 Schmidt 分解来研究其纠缠特性，并用此表象找出了一类新的单-双模组合压缩态以及相应的压缩算符.

1 广义双模纠缠态表象引入

Fan 等首次提出纠缠态表象[8,9]

$$|\eta\rangle=\exp\left[-\frac{1}{2}|\eta|^2+\eta a_1^\dagger+\eta^* a_2^\dagger-a_1^\dagger a_2^\dagger\right]|00\rangle \tag{1}$$

$|\eta\rangle$ 为对易算符 X_1+X_2，P_1-P_2 共同的本征态，

$$(X_1+X_2)|\eta\rangle=\sqrt{2}\eta_1|\eta\rangle \tag{2}$$

$$(P_1-P_2)|\eta\rangle=\sqrt{2}\eta_2|\eta\rangle \tag{3}$$

式中，$\eta=\eta_1+\mathrm{i}\eta_2$，$X_i=\frac{a_i+a_i^\dagger}{\sqrt{2}}$，$P_i=\frac{a_i-a_i^\dagger}{\sqrt{2}\mathrm{i}}$.

$|\eta\rangle$ 满足完备性

$$\int\frac{\mathrm{d}^2\eta}{\pi}|\eta\rangle\langle\eta|=1 \tag{4}$$

与正交性

$$\langle\eta'|\eta\rangle=\pi\delta(\eta'-\eta)\delta(\eta'^*-\eta^*) \tag{5}$$

$|\eta\rangle$ 为质量相等的两粒子纠缠态表象中的态矢.现我们将其推广到质量不相等的两粒子情形.注意到质心坐标

$$X_c=\mu_1X_1+\mu_2X_2 \tag{6}$$

及质量权重相对动量

$$P_r=\mu_2P_1-\mu_1P_2 \tag{7}$$

式中，$\mu_1=\frac{m_1}{m_1+m_2}$，$\mu_2=\frac{m_2}{m_1+m_2}$，$X_c$，$P_r$ 是对易的，$[X_c,P_r]=0$，故应该具有共同的本征态 $|\xi\rangle$：

$$X_c|\xi\rangle=\sqrt{\frac{\sigma}{2}}\xi_1|\xi\rangle \tag{8}$$

$$P_r|\xi\rangle=\sqrt{\frac{\sigma}{2}}\xi_2|\xi\rangle \tag{9}$$

式中，$\xi=\xi_1+\mathrm{i}\xi_2$，$\sigma=2(\mu_1^2+\mu_2^2)$.

由式(8)和(9)，可将 $|\xi\rangle\langle\xi|$ 写成 δ 函数的形式：

$$|\xi\rangle\langle\xi|=\frac{\sigma}{2}\int\mathrm{d}^2\xi'\delta\left[\sqrt{\frac{\sigma}{2}}\xi_1-\sqrt{\frac{\sigma}{2}}\xi_1'\right]\cdot$$

$$\delta\left[\sqrt{\frac{\sigma}{2}}\xi_2-\sqrt{\frac{\sigma}{2}}\xi_2'\right]|\xi'\rangle\langle\xi'|=$$

$$\frac{\sigma}{2}\int\mathrm{d}^2\xi'\delta\left[\sqrt{\frac{\sigma}{2}}\xi_1-X_c\right]\delta\left[\sqrt{\frac{\sigma}{2}}\xi_2-P_r\right]|\xi'\rangle\langle\xi'| \tag{10}$$

另一方面，经典函数 $(\mu_1x_1+\mu_2x_2)(\mu_2p_1-\mu_1p_2)$ 的 Weyl 量子化算符为

$$\vdots(\mu_1X_1+\mu_2X_2)(\mu_2P_1-\mu_1P_2)\vdots,$$

其中记号 $\vdots\ \ \vdots$ 代表 Weyl 编序[10-12].根据 Weyl 编序的定义，有

$$(\mu_1X_1+\mu_2X_2)(\mu_2P_1-\mu_1P_2)=$$

$$\vdots(\mu_1X_1+\mu_2X_2)(\mu_2P_1-\mu_1P_2)\vdots \tag{11}$$

利用式(11)，可将 $|\xi\rangle\langle\xi|$ 化为 Weyl 编序形式：

$$|\xi\rangle\langle\xi|=\frac{\sigma}{2}\int\mathrm{d}^2\xi'\cdot$$

$$\vdots\delta\left[\sqrt{\frac{\sigma}{2}}\xi_1-X_c\right]\delta\left[\sqrt{\frac{\sigma}{2}}\xi_2-P_r\right]\vdots|\xi'\rangle\langle\xi'| \tag{12}$$

又由 Wigner 算符的 Weyl 编序形式

$$\triangle_i(x_i,p_i)=\vdots\delta[x_i-X_i]\delta[p_i-P_i]\vdots \tag{13}$$

可将式(10)进一步化简为[13-15]

$$|\xi\rangle\langle\xi|=\frac{\sigma}{2}\int\mathrm{d}^2\xi'\mathrm{d}p_1\mathrm{d}x_1\mathrm{d}p_2\mathrm{d}x_2\cdot$$

$$\delta\left[\sqrt{\frac{\sigma}{2}}\xi_1-(\mu_1x_1+\mu_2x_2)\right]\cdot$$

$$\delta\left[\sqrt{\frac{\sigma}{2}}\xi_2-(\mu_2p_1-\mu_1p_2)\right]\cdot$$

$$\triangle_1(x_1,p_1)\triangle_2(x_2,p_2)|\xi'\rangle\langle\xi'|=$$

$$\frac{\pi\sigma}{2}\int\mathrm{d}p_1\mathrm{d}x_1\mathrm{d}p_2\mathrm{d}x_2\delta\left[\sqrt{\frac{\sigma}{2}}\xi_1-(\mu_1x_1+\mu_2x_2)\right]\cdot$$

$$\delta\left[\sqrt{\frac{\sigma}{2}}\xi_2-(\mu_2p_1-\mu_1p_2)\right]\cdot$$

$$\triangle_1(x_1,p_1)\triangle_2(x_2,p_2) \tag{14}$$

上式推导中已假定了 $\int\frac{\mathrm{d}^2\xi}{\pi}|\xi\rangle\langle\xi|=1$，后面式(18)表明此假定是正确的.

将 Wigner 算符 $\triangle_i(x_i,p_i)$ 的正规乘积形式

$$\triangle_i(x_i,p_i)=$$

$$\frac{1}{\pi}:\exp[-(x_i-X_i)^2-(p_i-P_i)^2]: \tag{15}$$

代入式(14)，并进行有序算符内积分[16]，同时利用双粒子真空投影算符的正规乘积[17]

$$:\exp[-a_1^\dagger a_1-a_2^\dagger a_2]:=|00\rangle\langle00| \tag{16}$$

得到不同质量的双粒子的纠缠态表象 $|\xi\rangle$：

$$|\xi\rangle=\exp\{-\frac{1}{2}|\xi|^2+\frac{[\xi+(\mu_1-\mu_2)\xi^*]\ a_1^\dagger}{\sqrt{\sigma}}+\frac{[\xi^*-(\mu_1-\mu_2)\xi]\ a_2^\dagger}{\sqrt{\sigma}}+\frac{\mu_2-\mu_1}{\sigma}(a_1^{\dagger2}-a_2^{\dagger2})-\frac{4\mu_1\mu_2}{\sigma}a_1^\dagger a_2^\dagger\}|00\rangle \quad (17)$$

当粒子质量相等时，$\mu_1=\mu_2=\frac{1}{2}$，$|\xi\rangle\Rightarrow|\eta\rangle_{\eta=\xi}$ 利用有序算符内积分技术，可以证明 $|\xi\rangle$ 满足完备性

$$\int\frac{d^2\xi}{\pi}|\xi\rangle\langle\xi|=1 \quad (18)$$

及正交性

$$\langle\xi|\xi'\rangle=\pi\,\delta(\xi_1-\xi'_1)\delta(\xi_2-\xi'_2) \quad (19)$$

2 $|\xi\rangle$的纠缠特性

2.1 $|\xi\rangle$的 Schmidt 分解

对式(17)中的 $|\xi\rangle$ 作单侧 Fourier 变换，有

$$\int\frac{d\xi_2}{2\pi}|\xi\rangle e^{-i\xi_2 x}=\sqrt{\frac{1}{2\pi}}\exp\Big\{-\frac{a_1^{\dagger2}-a_2^{\dagger2}}{2}+\frac{2}{\sqrt{\sigma}}(\xi_1\mu_1+\mu_2 x)a_1^\dagger+\frac{2}{\sqrt{\sigma}}(\xi_1\mu_2-\mu_1 x)a_2^\dagger-\frac{1}{2}(\xi_1^2+x^2)\Big\}|00\rangle \quad (20)$$

将坐标表象 $|x\rangle_i$ 的算符表示

$$\left.\begin{aligned}&|x\rangle_i=\pi^{-1/4}\exp\left(-\frac{x_i^2}{2}+\sqrt{2}xa_i^\dagger-\frac{a_i^{\dagger2}}{2}\right)|0\rangle_i,\\&i=1,2\end{aligned}\right\} \quad (21)$$

代入式(20)，得

$$\int\frac{d\xi_2}{2\pi}|\xi\rangle e^{-i\xi_2 x}=\frac{1}{\sqrt{2}}\Big|\frac{x}{\sqrt{1+\frac{1}{g^2}}}+\frac{\xi_1}{\sqrt{1+g^2}}\Big\rangle_1\otimes\Big|-\frac{x}{\sqrt{1+g^2}}+\frac{\xi_1}{\sqrt{1+\frac{1}{g^2}}}\Big\rangle_2 \quad (22)$$

式中，$g=\frac{\mu_2}{\mu_1}$.

对式(22)作变量代换，令 $\frac{x}{\sqrt{1+\frac{1}{g^2}}}\rightarrow x$，则

$$\int\frac{d\xi_2}{2\pi}|\xi\rangle\exp(-i\xi_2 x\sqrt{1+\frac{1}{g^2}})=\frac{1}{\sqrt{2}}\Big|x+\frac{\xi_1}{\sqrt{1+g^2}}\Big\rangle_1\otimes\Big|-\frac{x}{g}+\frac{\xi_1}{\sqrt{1+\frac{1}{g^2}}}\Big\rangle_2 \quad (23)$$

对式(23)作 Fourier 逆变换，得

$$|\xi\rangle=\sqrt{\frac{1+g^2}{2g^2}}e^{-i\frac{\xi_1\xi_2}{g}}\int dx\,|x\rangle_1\otimes\Big|-\frac{x}{g}+\frac{\xi_1}{\sqrt{1+\frac{1}{g^2}}}\Big\rangle_2\exp(i\xi_2 x\sqrt{1+\frac{1}{g^2}}) \quad (24)$$

式(24)即为坐标表象中广义双模纠缠态 $|\xi\rangle$ 的 Schmidt 分解.

上式表明，当对量子态 $|\xi\rangle$ 中第一个粒子进行测量时，若测量算符为 $|x'\rangle_{11}\langle x'|$，第一个粒子测得的坐标值为 x' 时，无论两个粒子相距多远，则第二个粒子必定塌缩到量子态

$${}_1\langle x'|\xi\rangle=\sqrt{\frac{1+g^2}{2g^2}}\Big|-\frac{x'}{g}+\xi_1\sqrt{1+\frac{1}{g^2}}\Big\rangle_2\cdot\exp\left[i\xi_2(x'\sqrt{1+\frac{1}{g^2}}-\frac{\xi_1}{g})\right] \quad (25)$$

同理，可得动量表象下广义双模纠缠态 $|\xi\rangle$ 的 Schmidt 分解为

$$|\xi\rangle=\sqrt{\frac{1+g^2}{2}}e^{ig\xi_1\xi_2}\int dp\,|-p\rangle_1\otimes|-gp-\sqrt{1+g^2}\xi_2\rangle_2\exp(i\xi_1 p\sqrt{1+g^2}) \quad (26)$$

当对量子态 $|\xi\rangle$ 中第一个粒子进行测量时，若测量算符为 $|p'\rangle_{11}\langle p'|$，第一个粒子测得的动量值为 p'，则第二个粒子必定塌缩到量子态

$${}_1\langle p'|\xi\rangle=\sqrt{\frac{1+g^2}{2}}e^{ig\xi_1\xi_2}|gp'-\sqrt{1+g^2}\xi_2\rangle_2\cdot\exp\left[i\xi_1(-p'\sqrt{1+g^2}+g\xi_2)\right] \quad (27)$$

以上这些讨论反映了 $|\xi\rangle$ 内禀的纠缠特性.

2.2 纠缠算符

利用

$$P|x\rangle=i\frac{d}{dx}|x\rangle,\ \exp(-iPy)|x\rangle=|x+y\rangle \quad (28)$$

以及

$$|p\rangle=\frac{1}{\sqrt{2\pi}}\int dx\,|x\rangle e^{ipx} \quad (29)$$

得

$$e^{-iP_2X_1/g}|\xi\rangle=\sqrt{\frac{1+g^2}{2g^2}}e^{-i\frac{\xi_1\xi_2}{g}}\int dx\,|x\rangle_1\otimes$$

$$e^{-iP_2x/g}\ \left|-\frac{x}{g}+\xi_1\sqrt{1+\frac{1}{g^2}}\right\rangle_2\exp\left(i\sqrt{1+\frac{1}{g^2}}\xi_2x\right)=$$
$$\sqrt{\frac{1+g^2}{2g^2}}e^{-i\frac{\xi_1\xi_2}{g}}\int dx\ |x\rangle_1\otimes$$
$$\left|\sqrt{1+\frac{1}{g^2}}\xi_1\right\rangle_2\exp\left(i\sqrt{1+\frac{1}{g^2}}\xi_2x\right)=$$
$$\sqrt{\frac{2\pi\ (1+g^2)}{2g^2}}e^{-i\frac{\xi_1\xi_2}{g}}\ \Big|\ p=$$
$$\left.\sqrt{1+\frac{1}{g^2}}\xi_2\right\rangle_1\otimes\left|\sqrt{1+\frac{1}{g^2}}\xi_1\right\rangle_2 \tag{30}$$

对式(30)两边同乘以 $e^{iP_2X_1/g}$,得

$$e^{iP_2X_1/g}\sqrt{\frac{2\pi\ (1+g^2)}{2g^2}}e^{-i\frac{\xi_1\xi_2}{g}}\ \Big|\ p=\left.\frac{\sqrt{1+g^2}}{g}\xi_2\right\rangle_1\otimes$$
$$\left|\sqrt{1+\frac{1}{g^2}}\xi_1\right\rangle_2=|\xi\rangle \tag{31}$$

式(31)表明,$e^{iP_2X_1/g}$ 作用于可分态 $e^{-i\frac{\xi_1\xi_2}{g}}\ \big|\ p=\sqrt{1+\frac{1}{g^2}}\xi_2\big\rangle_1\otimes\big|\sqrt{1+\frac{1}{g^2}}\xi_1\big\rangle_2$ 时,得纠缠态 $|\xi\rangle$,所以 $e^{iP_2X_1/g}$ 是一个纠缠算符.

同理利用

$$X\ |p\rangle=-i\frac{d}{dp}\ |p\rangle,\exp(iXp')\ |p\rangle=|p+p'\rangle \tag{32}$$

以及

$$|x\rangle=\frac{1}{\sqrt{2\pi}}\int dp\ |p\rangle e^{-ixp} \tag{33}$$

得

$$e^{-iP_1X_2g}\ |\xi\rangle=\sqrt{\pi(1+g^2)}e^{ig\xi_1\xi_2}\ \Big|\ x=$$
$$\left.\xi_1\sqrt{1+g^2}\right\rangle_1\otimes\left|-\sqrt{1+g^2}\xi_2\right\rangle_2 \tag{34}$$

因此,$e^{iP_1X_2g}$ 也是一个纠缠算符,它将可分态 $e^{ig\xi_1\xi_2}\ \big|\ x=\xi_1\sqrt{1+g^2}\big\rangle_1\otimes\big|-\sqrt{1+g^2}\xi_2\big\rangle_2$ 转化为纠缠态 $|\xi\rangle$:

$$e^{iP_1X_2g}\sqrt{\pi\ (1+g^2)}e^{ig\xi_1\xi_2}\ \Big|\ x=$$
$$\left.\xi_1\sqrt{1+g^2}\right\rangle_1\otimes\left|-\sqrt{1+g^2}\xi_2\right\rangle_2=|\xi\rangle \tag{35}$$

3 $|\xi\rangle$的压缩

将 $\xi\rightarrow\frac{\xi}{\lambda}$,构造压缩算符

$$S=\int\frac{d^2\xi}{\pi\ \lambda}\ \left|\frac{\xi}{\lambda}\right\rangle\langle\xi|=\frac{2\lambda}{(1+\lambda^2)}\exp\left[\frac{(\mu_1-\mu_2)(a_1^{\dagger2}-a_2^{\dagger2})+4\mu_1\mu_2a_1^{\dagger}a_2^{\dagger}}{\sigma}\left(\frac{1-\lambda^2}{1+\lambda^2}\right)\right]\times$$
$$:\exp\left[-\frac{(1-\lambda\ )^2}{(1+\lambda^2)}(a_1^{\dagger}a_1+a_2^{\dagger}a_2)\right]:\times\exp\left[\frac{(\mu_1-\mu_2)(a_1^2-a_2^2)+4\mu_1\mu_2a_1a_2}{\sigma}\left(\frac{\lambda^2-1}{\lambda^2+1}\right)\right] \tag{36}$$

将 S 作用于双模真空态 $|00\rangle$ 上,得

$$S\ |00\rangle=\frac{2\lambda}{(1+\lambda^2)}\exp\left[\frac{(\mu_1-\mu_2)(a_1^{\dagger2}-a_2^{\dagger2})+4\mu_1\mu_2a_1^{\dagger}a_2^{\dagger}}{\sigma}\left(\frac{1-\lambda^2}{1+\lambda^2}\right)\right]|00\rangle \tag{37}$$

令 $A=\frac{1-\lambda^2}{1+\lambda^2}\frac{(\mu_1-\mu_2)}{\sigma}$, $B=\frac{1-\lambda^2}{1+\lambda^2}\frac{4\mu_1\mu_2}{\sigma}$, $L=\frac{4}{1-4A^2-B^2}=\frac{(1+\lambda^2)^2}{\lambda^2}$,则

$$S\ |00\rangle=\frac{2}{\sqrt{L}}\exp\left[A(a_2^{\dagger2}-a_1^{\dagger2})+Ba_1^{\dagger}a_2^{\dagger}\right]\ |00\rangle \tag{38}$$

式(38)为单双模组合压缩真空态.因此 S 为单双模组合压缩算符.

我们利用 $|\xi\rangle$ 表象,还可以构造 $|\xi\rangle$ 的单边双模压缩算符,将 $(\mu_2X_1-\mu_1X_2)$,$(\mu_1P_1+\mu_2P_2)$ 作用于 $|\xi\rangle$ 上,有

$$(\mu_2X_1-\mu_1X_2)\ |\xi\rangle=\sqrt{\frac{1+g^2}{2g^2}}e^{-i\frac{\xi_1\xi_2}{g}}\int dx\frac{\mu_1}{g}\sqrt{1+g^2}(\sqrt{1+g^2}X_1-\xi_1)\ |x\rangle_1\otimes$$
$$\left|-\frac{x}{g}+\xi_1\sqrt{1+\frac{1}{g^2}}\right\rangle_2\exp\left(i\xi_2x\sqrt{1+\frac{1}{g^2}}\right)=\frac{1+g^2}{2g^2}\mu_1\left(-i\sqrt{2}\frac{\partial}{\partial\xi_1}\right)|\xi\rangle \tag{39}$$

$$(\mu_1P_1+\mu_2P_2)\ |\xi\rangle=\sqrt{\frac{1+g^2}{2}}e^{ig\xi_1\xi_2}\int dp(-\mu_1)\sqrt{1+g^2}(\sqrt{1+g^2}p+g\xi_2)\ |-p\rangle_1\otimes$$
$$\left|-gp-\sqrt{1+g^2}\xi_2\right\rangle_2\exp(i\xi_1p\sqrt{1+g^2})=\frac{1+g^2}{2}\mu_1i\sqrt{2}\frac{\partial}{\partial\xi_1}\ |\xi\rangle \tag{40}$$

所以在 $|\xi\rangle$ 表象下，

$$\left.\begin{aligned}(\mu_2 X_1-\mu_1 X_2) &\to \frac{1+g^2}{2g^2}\mu_1\left(\mathrm{i}\sqrt{2}\,\frac{\partial}{\partial\xi_1}\right),\\ (\mu_1 P_1+\mu_2 P_2) &\to \frac{1+g^2}{2}\mu_1\left(-\mathrm{i}\sqrt{2}\,\frac{\partial}{\partial\xi_1}\right)\end{aligned}\right\} \tag{41}$$

因此

$$\langle\xi|(\mu_2 P_1-\mu_1 P_2)(\mu_2 X_1-\mu_1 X_2)=\frac{1+g^2}{2g}\mu_1\sqrt{\frac{\sigma}{2}}\,\xi_2\left(\mathrm{i}\sqrt{2}\,\frac{\partial}{\partial\xi_1}\right)\langle\xi| \tag{42}$$

令 $\xi_2=\mathrm{e}^y$，则

$$\begin{aligned}\langle\xi|(\mu_2 P_1-\mu_1 P_2)(\mu_2 X_1-\mu_1 X_2)&=\mathrm{i}\,\frac{1+g^2}{2g}\mu_1\sqrt{\sigma}\,\mathrm{e}^y\,\frac{\partial y}{\partial\xi_1}\,\frac{\partial}{\partial y}\langle\xi_1,\xi_2=\mathrm{e}^y|=\\ &\mathrm{i}\,\frac{1+g^2}{2g}\mu_1\sqrt{\sigma}\,\frac{\partial}{\partial y}\langle\xi_1,\xi_2=\mathrm{e}^y|\end{aligned} \tag{43}$$

利用平移性质

$$\exp\left(-\lambda\,\frac{\partial}{\partial y}\right)f(y)=f(y-\lambda) \tag{44}$$

得

$$\langle\xi|\exp\left[\frac{\mathrm{i}\lambda}{2}(\mu_2 P_1-\mu_1 P_2)(\mu_2 X_1-\mu_1 X_2)\right]=\frac{1+g^2}{2g}\mu_1\sqrt{\sigma}\,\langle\xi_1,\mathrm{e}^{-\lambda}\xi_2| \tag{45}$$

令

$$S'=\exp\left[\frac{\mathrm{i}\lambda}{2}(\mu_2 P_1-\mu_1 P_2)(\mu_2 X_1-\mu_1 X_2)-\frac{\lambda}{2}\right] \tag{46}$$

则

$$\langle\xi|S'=\frac{1+g^2}{2g}\mu_1\sqrt{\sigma}\exp\left(-\frac{\lambda}{2}\right)\langle\xi_1,\mathrm{e}^{-\lambda}\xi_2| \tag{47}$$

幺正算符 S' 称之为单边压缩算符.

同理可证

$$S''=\exp\left[-\frac{\mathrm{i}\lambda}{2}(\mu_1 X_1+\mu_2 X_2)(\mu_1 P_1+\mu_2 P_2)-\frac{\lambda}{2}\right] \tag{48}$$

也为单边压缩算符，满足

$$\langle\xi|S''=\frac{1+g^2}{2}\mu_1\sqrt{\sigma}\exp\left(-\frac{\lambda}{2}\right)\langle\mathrm{e}^{-\lambda}\xi_1,\xi_2| \tag{49}$$

4　结论

本文利用 Weyl 编序对应以及有序算符内的积分技术可以直接导出异质量两粒子的纠缠态表象.通过 Schmidt 分解来展现其纠缠特性.用此表象我们找到了新的单-双模组合压缩态、相应的压缩算符以及 $|\xi\rangle$ 的单边压缩算符.使用这个表象在求解两个不同质量粒子之间既有坐标耦合，也有动量耦合相互作用时，可以直接导出哈密尔顿量本征态的波函数。当我们研究有电容-电感耦合的量子介观电路时，也可以建立这类表象以明确相应的纠缠特征.

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