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Multi-valued indicators in DEA in the presence of undesirable outputs: A goal-directed approach

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Abstract: The data envelopment analysis (DEA) is an important data-driven method for the performance evaluation and performance improvement of a set of peer decision making units (DMUs), involving multiple inputs and multiple outputs which are identified as performance indicators. However, some performance indicators, unlike conventional DEA models with one single value, may have more than one value because of different definitions or measurement standards referring to multi-valued indicators. In addition, the performance indicators reflect the current status of DMUs, which ignore the goals of decision-makers. We first propose two modified slacks-based DEA models to deal with multi-valued indicators and provide the Pareto-optimal solution in two common decision-making scenarios, namely the decentralized and centralized decision-making cases. Furthermore, we extend the models by incorporating with the goals of decision-makers to help the DMUs improve their performance and get close to the goals of decision-makers as much as possible. The slacks-based approaches and integration of goals enhance the discriminability of the models to DMUs and provide more practical improvement for some indicators. A case study of 22 cities in the Yangtze River delta region in China is used to illustrate the effectiveness and practicality of our proposed models.

Keywords: Data envelopment analysis, multi-valued indicators, goals, decentralized decision-making, centralized decision-making

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1 Introduction

The data envelopment analysis (DEA), first proposed by Charnes et al[1], is a well-known non-parametric data-driven tool for building a composite index (e.g., performance, benchmarking) of a set of homogeneous DMUs consuming multiple inputs to produce multiple outputs[2,3]. As one of the most important evaluation tools, DEA has been developed rapidly in both theory and application over the past four decades[4,5]. People now pay more and more attention to the environment. The DEA is a widely used method on energy and environment, where it usually involves the undesirable outputs in the researches[6], such as air pollutants. The decision-makers usually prefer to a smaller amount of undesirable outputs and the ways to deal with the undesirable outputs are widely studied in existing literatures[7,8].

In DEA modeling and applications, the selection of performance indicators (inputs/outputs) is crucial to the robustness of the evaluation results since the evaluation results may change with the selection of input and output indicators[9]. Therefore, some studies on the selection of performance indicators have been developed in DEA literatures and have been employed in different ways, such as the principal component analysis[10] and the aggregation method[11]. Usually, two considerations occur in the selection of performance indicators. One is that the data of some performance indicators may be missing. The other is that the traditional DEA methods assume that the inputs/outputs are respectively independent corresponding to one value. However, some input/output indicators may have more than one single value in practice because of various measurement standards or definitions; such indicators are identified as multi-valued indicators[12]. For example, both PM 10 and PM 2.5 are selected to measure the concentration of the particulate matter in the air. Accordingly, one challenge in the application of DEA is to select an appropriate value for multi-valued inputs/outputs. To date, only a few research efforts, such as Toloo and Hančlová[12], have proposed selecting methods based

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on the directional distance function (DDF) to solve the problem of multi-valued indicators.

Furthermore, the decision-makers' goals, which reflect their preferences for performance evaluation and improvement direction, also play a decisive role in the selection of performance indicators. The performance indicators are directly related with the results of performance evaluation in DEA methods. The multi-valued indicators under different standards refer to different decision-makers' goals. In other words, selecting an appropriate value for a multi-valued indicator is inevitably influenced by decision-makers' goals, which further affects the results of performance evaluation and improvements. Sales targets, target yield for corporations, and air pollutant concentration limits are common examples of decision-makers' goals, seen as the expected level. Such goals significantly impact performance evaluation in real-world situations. However, traditional DEA methods focus on comparisons among peer DMUs to provide evaluation and benchmarks without considering the goals of decision-makers. To fill this gap, indirect and direct DEA based approaches taking into account the goals of decision-makers have been proposed[13-15]. The former approaches replace the decision-makers' goals with other values such as utility. Lozano et al[16] propose a bargaining based DEA approach considering the utility instead of goals to improve the performance of inefficient DMUs, while the complicated calculation process limits its utilization. The latter approaches handle the decision-makers' goals in a direct way. For example, Stewart[17] proposes a new DEA model which constructs new reference points with the goals of top managers as benchmarks. Azadi et al[18] apply a goal-directed benchmarking method for supplier selection. Ruiz and Sirvent[19] introduce a DEA method to generate strongly efficient targets which satisfy the requirement of minimum distance to the goals and to the current performance. Besides, the goals are often established to plan the improvement. However, there are some goals that cannot be achieved at current production situation, referring to overly high goals, and there are also some goals that are unambitious, which cannot effectively guide the improvement, referring to overly low goals[20]. The DEA targets provide the best practices[19]. Accordingly, we further incorporate the decision-makers' goals into our approaches to handle the problem of multi-valued indicators.

Based on the above analysis, we build on the following works in our current study. Following the work of Toloo and Hančlová[12], we first tackle the problem of multi-valued indicators by proposing modified slacks-based models, which allows different proportional improvement for all inputs and outputs to obtain one suitable single value for each multi-valued indicators. Furthermore, we incorporate the decision-makers' goals into the proposed models. The new models guide the DMUs to reach points on the best practice frontier, which are close to the goals. Our approaches employ the absolute distance to measure the gaps between projection points and the goals due to the overly high/low goals in practice. To be more practical, this work takes the realities of decentralized and centralized decision-making cases into consideration to expand the scope of application.

The rest of the paper is organized as follows. In the next section, we provide preliminaries. Section 3 introduces our proposed models to deal with the problem of multi-valued indicators and the extended models considering the goals of decision-makers in decentralized and centralized decision-making cases. In Section 4, we apply our models to evaluate the environmental performance of the cities in the Yangtze River delta region in China. Finally, we conclude the paper and discuss further extensions.

2 Preliminaries

In this section, we introduce notations and the preliminaries of multi-valued indicators, the slacks-based model, and the DDF method of Toloo and Hančlová[12]. We first list the related notations shown in the following Table 1, which mainly involves the single-valued indicators and multi-valued indicators. The notations in Table 1 help better understand our methods.

We note the following relationship: $I^S \cup I^M = I$, $I^S \cap I^M = \varnothing R^S \cup R^M = R$, $R^S \cap R^M = \varnothing F^S \cup F^M = F$, and $F^S \cap F^M = \varnothing$ In addition, we have $I_p^M \subseteq I^M$, $R_q^M \subseteq R^M$, and $F_u^M \subseteq F^M$, and respectively use the binary variables δ_i^x, δ_r^y, and δ_f^b to denote the selected situation for i^{th} multi-valued input, r^{th} multi-valued desirable output, and f^{th} multi-valued undesirable output, which are defined as follows:

$$\delta_i^x = \begin{cases} 1, & \text{if } i^{\text{th}} \text{multi-valued input is selected} \\ 0, & \text{otherwise} \end{cases}$$
$$\delta_r^y = \begin{cases} 1, & \text{if } r^{\text{th}} \text{multi-valued desirable output is selected} \\ 0, & \text{otherwise} \end{cases}$$
$$\delta_f^b = \begin{cases} 1, & \text{if } f^{\text{th}} \text{multi-valued undesirable output is selected} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

We usethe following three vectors $x \in \Re^m$, $y \in \Re^s$ and $b \in \Re^l$ to represent inputs, desirable outputs, and undesirable outputs. Constructed by the inputs and outputs of all DMUs, the production technology T is defined as follows:

$$T = \{(x, y, b) \mid x \text{ can produce } y \text{ and } b\} \tag{2}$$

Denoted matrices$X$, Y and B as $X = [x_{ij}] = [x_{11}, \cdots, x_{mn}] \in \Re^{m\times n}$, $Y = [y_{rj}] = [y_{11}, \cdots, y_{sn}] \in \Re^{s\times n}$

Table 1. Illustration of the notations.

Constants					
n	number of DMUs				
m	number of inputs				
s	number of desirable outputs				
l	number of undesirable outputs				
P	number of multi-valued inputs				
Q	number of multi-valued desirable outputs				
U	number of multi-valued undesirable outputs				
$\lvert I_p^M \rvert$	number of values for the $p^{th}(p = 1,\cdots,P)$ multi-valued input				
$\lvert R_q^M \rvert$	number of values for the $q^{th}(q = 1,\cdots,Q)$ multi-valued desirable output				
$\lvert F_u^M \rvert$	number of values for the $u^{th}(u = 1,\cdots,U)$ multi-valued undesirable output				
S^I	number of single-valued inputs				
S^R	number of single-valued desirable outputs				
S^F	number of single-valued undesirable outputs				
DEA variables					
x_{ij}	$i^{th}(i = 1,\cdots,m)$ input of $DMU_j(j = 1,\cdots,n)$				
y_{rj}	$r^{th}(r = 1,\cdots,s)$ desirable output of $DMU_j(j = 1,\cdots,n)$				
b_{fj}	$f^{th}(f = 1,\cdots,l)$ undesirable output of $DMU_j(j = 1,\cdots,n)$				
Sets					
I	input set	I^S	single-valued input set	I^M	multi-valued input set
R	desirable output set	R^S	single-valued desirable output set	R^M	multi-valued desirable output set
F	undesirable output set	F^S	single-valued undesirable output set	F^M	multi-valued undesirable output set
I_p^M	set of all available values for the $p^{th}(p = 1,\cdots,P)$ multi-valued input				
R_q^M	set of all available values for the $q^{th}(q = 1,\cdots,Q)$ multi-valued desirable output				
F_u^M	set of all available values for the $u^{th}(u = 1,\cdots,U)$ multi-valued undesirable output				

and $B = [b_{fj}] = [b_{11},\cdots,x_{ln}] \in \Re^{l\times n}$. The technology set under variable returns to scale (VRS) is given as: $T(x) = \{(y,b) \mid x \geqslant \lambda X, y \leqslant \lambda Y, b \geqslant \lambda B, \geqslant 0, \lambda^T e = 1\}$, where an vector representing intensity variable. The VRS assumption is a broader scenario in reality since the full proportionality assumption under CRS assumption is not often satisfied[21]. We deal with undesirable outputs following the strong disposability assumption[22]. The reason is that the implicit assumption of the weak disposability is that all DMUs use the same abatement factor, which is inconsistent with the practice of focusing emission reduction efforts on DMUs with less emission reduction costs[23]; some outputs are also inappropriate for weak disposability assumption like SO_2 emissions[24].

2.1 Modified slack-based model

According to the slacks-based measure (SBM), which is first proposed by Tone[25] and later extended to the situation with undesirable outputs. We use the following modified slacks-based model to measure the inefficiency of a specific DMU_o.

$$\max\rho_o = \frac{1}{m+s+l}\left(\sum_{i=1}^{m}\frac{s_{io}^-}{x_{io}} + \sum_{r=1}^{s}\frac{s_{ro}^+}{y_{ro}} + \sum_{f=1}^{l}\frac{s_{fo}^-}{b_{fo}}\right),$$

$$\text{s.t.}\quad x_{io} = \sum_{j\in J}\lambda_j x_{ij} + s_{io}^-, i \in I \quad (3.1)$$

$$y_{ro} = \sum_{j\in J}\lambda_j y_{rj} - s_{ro}^+, r \in R \quad (3.2)$$

$$b_{fo} = \sum_{j\in J}\lambda_j b_{fj} + s_{fo}^-, f \in F \quad (3.3)$$

$$\sum_{j\in J}\lambda_j = 1,$$

$$\lambda_j, s_{io}^-, s_{ro}^+, s_{fo}^- \geqslant 0, j \in J, i \in I, r \in R, f \in F \quad (3.4)$$
(3)

The objective function is different from the model proposed by Tone[25], which is also widely used in practice[26,27] and avoid the non-linear problem of the SBM model. We see that model (3) meets the null-joint assumption in dealing with undesirable outputs, which means that there can be no desirable outputs if there are no undesirable outputs. In model (3), s_i^-, s_r^+, and s_f^- are respectively the slacks of the i^{th} input, r^{th} desirable output, and f^{th} undesirable output, which are first defined by Charnes et al[28]. The ρ_o satisfies the properties of unit invariance and monotonicity, which is consistent with the classic slacks-based measure[25]. Denote $\rho_o^*(s_i^{-*}, s_r^{+*}, s_f^{-*}, \lambda_j^*), i \in I, r \in R, f \in F, j \in J$ as the optimal objective function value of model (3). With this notation, $(\mathbf{0}_m, \mathbf{0}_s, \mathbf{0}_l, e_o)$ is one feasible solution of model (3), where $e \in \Re^n$ represents the o^{th} component is 1 and the rest are 0. Then we have $\rho_o^* \geqslant 0$ since this is a maximization problem. Obviously, DMU_o is efficient if $\rho_o^* = 0$, otherwise it is inefficient, which means that ρ_o measures the inefficiency score of DMU_o.

DMU_o is Pareto-Koopmans efficient when $\rho_o^* = 0$. Following the definition proposed by Scheel[29], if $\text{DMU}_o(x_o, y_o, b_o)$ is Pareto-Koopmans efficient, there is no $\text{DMU}_a(x_a, y_a, b_a)$ in technolody set such that $x_a \leqslant x_o$, $y_a \geqslant y_o$, and $b_a \leqslant b_o$ with at least one strict inequality. In model (3), $\rho_o^* = 0$ means $s_{ro}^{+*} = s_{io}^{-*} = s_{fo}^{-*} = 0, i \in I, r \in R, f \in F$, which indicates DMU_o is located on the efficient frontier. If a point (x_a, y_a, b_a) exists which satisfies $x_a < x_o$ or $y_a > y_o$ or $b_a < b_o$ or any combinations of these three strict inequalities, then (x_a, y_a, b_a) is not in the technology set and the slacks will be negative. The negative slacks contradict $(x_a, y_a, b_a) \in T$ and $s_{io}^- \geqslant 0$, $s_{ro}^+ \geqslant 0$, $s_{fo}^- \geqslant 0$. There is no strict inequalities, which implies $\text{DMU}_o(x_o, y_o, b_o)$ is Pareto-Koopmans efficient when the $\rho_o = 0$.

2.2 Multi-valued measures selection based on DDF

Toloo and Hančlová[12] propose the DDF model to select suitable value for the multi-valued indicators, presented as

$$\max \beta$$

$$\text{s.t.} \sum_{j \in J} \lambda_j x_{ij} \leqslant x_{io}, i \in I^S \quad (4.1)$$

$$\sum_{j \in J} \lambda_j x_{ij} \leqslant x_{io} + M(1 - \delta_{io}^x), i \in I_p^M, p = 1, \cdots P \quad (4.2)$$

$$\sum_{j \in J} \lambda_j y_{rj} \geqslant (1 + \beta) y_{ro}, r \in R^S \quad (4.3)$$

$$\sum_{j \in J} \lambda_j y_{rj} \geqslant (1 + \beta) y_{ro} - M(1 - \delta_{ro}^y), r \in R_q^M, q = 1, \cdots Q \quad (4.4)$$

$$\sum_{j \in J} \lambda_j b_{fj} = (1 - \beta) b_{fo}, f \in F^S \quad (4.5)$$

$$\sum_{j \in J} \lambda_j b_{fj} \leqslant (1 - \beta) b_{fo} + M(1 - \delta_{fo}^b), f \in F_u^M, u = 1, \cdots U \quad (4.6)$$

$$\sum_{j \in J} \lambda_j b_{fj} \geqslant (1 - \beta) b_{fo} - M(1 - \delta_{fo}^b), f \in F_u^M, u = 1, \cdots U \quad (4.7)$$

$$\sum_{i \in I_p^M} \delta_{io}^x = 1, p = 1, \cdots P \quad (4.8)$$

$$\sum_{r \in R_q^M} \delta_{ro}^y = 1, q = 1, \cdots Q \quad (4.9)$$

$$\sum_{f \in F_u^M} \delta_{fo}^b = 1, u = 1, \cdots U,$$

$$\delta_{io}^x, \delta_{ro}^y, \delta_{fo}^b \in \{0, 1\}, i \in I^M, r \in R^M, f \in F^M \quad (4.10)$$

$$\lambda_j \geqslant 0, j \in J, i \in I, r \in R, f \in F \quad (4.11)$$
(4)

where M is a large positive number. Denote the β^* as the optimal objective function value, where $\beta^* \geqslant 0$. Constraint (4.5) ensures $\beta^* \leqslant 1$. Model (4) ignores the slacks, which may not guarantee the Pareto optimal solution. Besides, it does not consider the inefficiency of inputs, thus over estimating the performance.

3 Methodology

In this section, we introduce two modified slacks-based models to handle the problems of multi-valued input/output indicators. Then we incorporate the decision-makers' goals to extend the models. Our models consider decentralized decision-making cases and centralized decision-making cases.

3.1 Modified slacks-based models without goals in the presence of multi-valued indicators

3.1.1 Decentralized decision-making case without goals

In the decentralized decision-making case, DMUs make decisions independently of each other. Therefore, each DMU is evaluated by using its own preferred standards referring to specific value of multi-valued performance indicators. Here, we define ρ_o^M for a specific DMU_o as follows:

$$\rho_0^M = \frac{\left(\sum_{i \in I^S} \frac{s_i^-}{x_{i0}} + \sum_{p=1}^{P} \sum_{i \in I_p^M} \frac{s_i^- \delta_i^x}{x_{io}} + \sum_{r \in R^S} \frac{s_r^+}{y_{r0}} + \sum_{q=1}^{Q} \sum_{r \in R_q^M} \frac{s_r^+ \delta_r^y}{y_{ro}} + \sum_{f \in F^S} \frac{s_f^-}{b_{f0}} + \sum_{u=1}^{U} \sum_{f \in F_u^M} \frac{s_f^- \delta_f^b}{b_{fo}}\right)}{S^I + P + S^R + Q + S^F + U},$$

which measure the inefficiency of the DMU_o through the ratio of slacks and the original value of input/output indicators, and $\rho_0^M \geq 0$. The following model (5) is applied to yield the maximum improvements for DMU_o, given the ρ_0^M.

$$
\begin{aligned}
&\max \rho_o^M \\
&s.t. \sum_{j \in J} \lambda_j x_{ij} = x_{io} - s_{io}^-, i \in I^S \quad (5.1)\\
&\sum_{j \in J} \lambda_j x_{ij} \leq x_{io} - s_{io}^- + M(1 - \delta_{io}^x), \\
&\quad i \in I_p^M, p = 1, \cdots P \quad (5.2)\\
&\sum_{j \in J} \lambda_j x_{ij} \geq x_{io} - s_{io}^- - M(1 - \delta_{io}^x), \\
&\quad i \in I_p^M, p = 1, \cdots P \quad (5.3)\\
&\sum_{j \in J} \lambda_j y_{rj} = y_{ro} + s_{ro}^+, r \in R^S \quad (5.4)\\
&\sum_{j \in J} \lambda_j y_{rj} \leq y_{ro} + s_{ro}^+ + M(1 - \delta_{ro}^y), \\
&\quad r \in R_q^M, q = 1, \cdots Q \quad (5.5)\\
&\sum_{j \in J} \lambda_j y_{rj} \geq y_{ro} + s_{ro}^+ - M(1 - \delta_{ro}^y), \\
&\quad r \in R_q^M, q = 1, \cdots Q \quad (5.6)\\
&\sum_{j \in J} \lambda_j b_{fj} = b_{fo} - s_{fo}^-, f \in F^S \quad (5.7)\\
&\sum_{j \in J} \lambda_j b_{fj} \leq b_{fo} - s_{fo}^- + M(1 - \delta_{fo}^b), \\
&\quad f \in F_u^M, u = 1, \cdots U \quad (5.8)\\
&\sum_{j \in J} \lambda_j b_{fj} \geq b_{fo} - s_{fo}^- - M(1 - \delta_{fo}^b), \\
&\quad f \in F_u^M, u = 1, \cdots U \quad (5.9)\\
&\sum_{i \in I_p^M} \delta_{io}^x = 1, p = 1, \cdots P \quad (5.10)\\
&\sum_{r \in R_q^M} \delta_{ro}^y = 1, q = 1, \cdots Q \quad (5.11)\\
&\sum_{f \in F_u^M} \delta_{fo}^b = 1, u = 1, \cdots U \quad (5.12)\\
&\delta_{io}^x, \delta_{ro}^y, \delta_{fo}^b \in \{0,1\}, i \in I^M, \\
&\quad r \in R^M, f \in F^M \quad (5.13)\\
&\sum_{j \in J} \lambda_j = 1, \\
&\lambda_j, s_{io}^-, s_{ro}^+, s_{fo}^- \geq 0, j \in J, i \in I, r \in R, f \in F \quad (5.14)
\end{aligned}
\tag{5}
$$

where M is a large positive number. (In our application, M is set equal to 10^6). In model (5), the inequalities ensure that the appropriate value of multi-valued input/output indicators can be selected for DMU_o. To be specific, for the multi-valued inputs, $\delta_{io}^x = 1$ means $\sum_{j \in J} \lambda_j x_{ij} \leq x_{i0} - s_{io}^-$, and $\sum_{j \in J} \lambda_j x_{ij} \geq x_{io} - s_{io}^-$, so that the constraints $\sum_{j \in J} \lambda_{jt} x_{ij} = x_{i0} - s_i^-$ are always satisfied for DMU_o, that is, the i^{th} input indicator is selected. Otherwise, $\delta_{io}^x = 0$ implies that the i^{th} input indicator is abandoned. Analogous meanings are applied to multi-valued desirable outputs and multi-valued undesirable outputs.

Let $\rho^{(5)*}(s_{i \in I^S \cup I_p^M}^{-*}, s_{r \in R^S \cup R_q^M}^{+*}, s_{f \in F^S \cup F_u^M}^{-*}, \delta_{i \in I_p^M}^{x*}, \delta_{r \in R_q^M}^{y*}, \delta_{f \in F_u^M}^{b*}, ^*)$ denote the optimal objective function value of model (5). DMU_o is identified as efficient if $\rho^{(5)*} = 0$; otherwise, it is inefficient. Also, we respectively obtain the selected values for multi-valued inputs, multi-valued desirable outputs, and multi-valued undesirable outputs when $\delta_{i \in I_p^M}^{x*} = 1$, $\delta_{r \in R_q^M}^{y*} = 1$, and $\delta_{f \in F_u^M}^{b*} = 1$. Additionally, for any inefficient DMU_o, the targets of inputs, undesirable outputs, and undesirable outputs can be expressed as follows.

$$
\left.
\begin{aligned}
x_{io}^* &= x_{io} - s_{io}^{-*}, i \in I^S \\
x_{io}^* &= x_{io} - s_{io}^{-*} * \delta_{io}^{x*}, i \in I_p^M, p = 1, \cdots P \\
y_{ro}^* &= y_{ro} + s_{ro}^{+*}, r \in R^S \\
y_{ro}^* &= y_{ro} + s_{ro}^{+*} * \delta_{ro}^{y*}, r \in R_q^M, q = 1, \cdots Q \\
b_{fo}^* &= b_{fo} - s_{fo}^{-*}, f \in F^S \\
b_{fo}^* &= b_{fo} - s_{fo}^{-*} * \delta_{fo}^{b*}, f \in F_u^M, u = 1, \cdots U
\end{aligned}
\right\} \quad (6)
$$

Then, in order to demonstrate the advantages of our model, we also use the model (3) to deal with the multi-valued indicators. We can see that our proposed model (5) deals with multi-valued indicators problem to obtain the inefficiency score of the evaluated DMU with one time calculation. Accordingly, model (3) must be solved K times to find the optimal combination of multi-valued performance indicators to ensure the maximum improvement for each DMU, where $K = \prod_{p=1}^{P} |I_p^M| * \prod_{q=1}^{Q} |R_q^M| * \prod_{u=1}^{U} |F_u^M|$; that is, K is the number of combinations of multi-valued indicators with each such indicators taking one of the respective value.

Specifically, model (3) can be converted into model(7) to obtain the inefficiency score of DMU_o by solving the $k^{th}(k = 1, \cdots, K) \in \mathbb{K}$combination of multi-valued performance indicators.

$$
\left.
\begin{aligned}
&\max \rho_{ko} = \frac{1}{m+s+l}\left(\sum\nolimits_{i=1}^{m} \frac{s_{i0}^-}{x_{i0}} + \right.\\
&\qquad \left.\sum\nolimits_{r=1}^{s} \frac{s_{r0}^+}{y_{r0}} + \sum\nolimits_{f=1}^{l} \frac{s_{f0}^+}{b_{f0}}\right) \\
&\text{s.t.} \quad x_{io} = \sum_{j \in J} \lambda_j x_{ij} + s_{io}^-, i \in I^S \cup I^{Mk} \\
&y_{ro} = \sum_{j \in J} \lambda_j y_{rj} - s_{ro}^+, r \in R^S \cup R^{Mk} \\
&b_{fo} = \sum_{j \in J} \lambda_j b_{fj} + s_{fo}^-, f \in F^S \cup F^{Mk} \\
&\sum_{j \in J} \lambda_j = 1, \\
&\lambda_j \geq 0, s_{io}^- \geq 0, s_{ro}^+ \geq 0, s_{fo}^- \geq 0, j \in J
\end{aligned}
\right\} \quad (7)
$$

In model (7), I^{Mk}, R^{Mk}, and F^{Mk}respectively represent the set containing the selected multi-valued input/ desirable output/ undesirable output indicators in the $k^{th}(k = 1,\cdots,K)$ combination. In addition, $m = S^I + P$, $s = S^R + Q$, and $l = S^F + U$ denote the number of inputs, the number of desirable outputs, and the number of undesirable outputs, respectively. By calculating model (7) K times, we obtain the optimal objective function value $\rho_{ko}^{*}(k = 1,\cdots,K)$ for each combination. Assume $\rho_{co}(s_i^{-*}, s_r^{+*}, s_f^{-*}, \lambda^*) = \max_{k=1,\ldots,K} \rho_{ko}^{*}$; that is, the $c^{th}(c = 1,\cdots,K)$ combination is selected. DMU_o is identified as efficient if $\rho_{co} = 0$.

Theorem 3.1 Denote the optimal objective function value of model (5) and the optimal objective function value of model (7) with multi-valued performance indicators in $k^{th}(k = 1,\cdots,K)$ combination as $\rho^{(5)*}$ and $\rho_k^{(7)*}$ respectively, then it satisfies $\rho^{(5)*} = \max\{\rho_k^{(7)*}, k \in \mathbb{K}\}$.

Proof We assume the optimal objective function value of model (5) is $\rho^{(5)*}(s^{-*}_{i\in I^S\cup I_p^M}, s^{+*}_{r\in R^S\cup R_q^M}, s^{-*}_{f\in F^S\cup F_u^M}, \delta^{x*}_{i\in I_p^M}, \delta^{y*}_{r\in R_q^M}, \delta^{b*}_{f\in F_u^M}, ^*)$. Let the optimal solution of model (7) be the $d^{th}, d \in \mathbb{K}$ combination of the multi-valued indicators, that is $\rho_d^{(7)*} = \max\{\rho_k^{(7)*}, k \in \mathbb{K}\}$. The optimal solution of model (5) with $\delta_i^{'x*} = 1, i \in I^{Md}$, $\delta_r^{'y*} = 1, r \in R^{Md}$ and $\delta_f^{'b*} = 1$, $f \in F^{Md}$ referring to the selected multi-valued indicators is also a feasible solution to model (7). We get the $\rho^{(5)*} \leqslant \rho_d^{(7)*}$ since model (7) is a maximization problem.

Then we assume the I^{Mc}, R^{Mc} , and F^{Mc} respectively indicate the set of selected multi-valued inputs, multi-valued desirable outputs, and multi-valued undesirable outputs in the $c^{th}, c \in \mathbb{K}$ combination. Assume the maximum optimal objective function value of model (7) is $\rho_c^{(7)*}(s_i^{-*}, s_r^{+*}, s_f^{-*}, \lambda_j^*)$ with $\rho_c^{(7)*} = \max\{\rho_k^{(7)*}, k \in \mathbb{K}\}$. The solution of model (7) incorporating with $\delta_i^{'x*} = 1, i \in I^{Mc}$, $\delta_r^{'y*} = 1, r \in R^{Mc}$, and $\delta_f^{'b*} = 1, f \in F^{Mc}$ is also feasible solution to model (5). We get $\rho^{(5)*} \geqslant \rho_c^{(7)*} = \max\{\rho_k^{(7)*}, k \in K\}$ since model (5) is maximization problem.

If the optimal objective function value of model (7) is zero in all combinations of selected multi-valued indicators, the optimal objective function value of the model (5) is zero, according to Theorem 3.1, $\rho^{(5)*} = \max\{\rho_k^{(7)*}, k \in \mathbb{K}\}$. In addition, when the optimal objective function value of the model (5) is equal to zero, the DMU is Pareto-Koopmans efficient. Through model (5), we get the selected value of multi-valued inputs/outputs with binary variables equal to one. Then the combination of selected multi-valued input/output is feasible solution to model (7), and we have the maximum optimal objective function value of model (7) since $\rho^{(5)*} = \max\{\rho_k^{(7)*}, k \in \mathbb{K}\}$. When the objective function value of model (5) is zero, the objective function value of model (7) is zero under all the different combinations of selected multi-valued indicators. According to the discussion in Preliminaries, $\rho^{(7)*} = 0$ is a Pareto-optimal solution, that is the DMU achieve Pareto-Koopmans efficient when $\rho^{(5)*} = 0$. To sum up, our proposed model (5) is easier to calculate the performance of DMUs through one time calculation in the presence of multi-valued indicators.

However, model (5) is a nonlinear programming problem. Model (5) can be converted into a linear programming model following the way of Cook et al[3]; The detailed transformation process is shown in Appendix A.

3.1.2 Centralized decision-making case without goals

In the centralized decision-making case, all DMUs are controlled by central decision-makers. The central decision-makers make decisions from an overall perspective rather than any individual DMU's point of view. In other words, all DMUs are assessed by using one consistent standard on performance indicators, which is selected by the central decision-makers.

In the case of centralized decision-making, we want to achieve the overall maximum improvements with one consistent set of input/output indicators for all DMUs. The model (8) is introduced and is shown as

$$\max \sum_{t=1}^{n} \rho_t^M$$

$$\text{s.t.} \sum_{j\in J} \lambda_{jt} x_{ij} = x_{it} - s_{it}^-, t \in J, i \in I^S \quad (8.1)$$

$$\sum_{j\in J} \lambda_{jt} x_{ij} \leqslant x_{it} - s_{it}^- + M(1-\delta_i^x), \quad t \in J, i \in I_p^M, p = 1,\cdots P \quad (8.2)$$

$$\sum_{j\in J} \lambda_{jt} x_{ij} \geqslant x_{it} - s_{it}^- - M(1-\delta_i^x), \quad t \in J, i \in I_P^M, P = 1,\cdots P \quad (8.3)$$

$$\sum_{j\in J} \lambda_{jt} y_{rj} = y_{rt} + s_{rt}^+, t \in J, r \in R^S \quad (8.4)$$

$$\sum_{j\in J} \lambda_{jt} y_{rj} \leqslant y_{rt} + s_{rt}^+ + M(1-\delta_r^y), \quad t \in J, r \in R_q^M, q = 1,\cdots Q \quad (8.5)$$

$$\sum_{j\in J} \lambda_{jt} y_{rj} \geqslant y_{rt} + s_{rt}^+ - M(1-\delta_r^y), \quad t \in J, r \in R_q^M, q = 1,\cdots Q \quad (8.6)$$

$$\sum_{j\in J} \lambda_{jt} b_{fj} = b_{ft} - s_{ft}^-, t \in J, f \in F^S \quad (8.7)$$

$$\sum_{j\in J} \lambda_{jt} b_{fj} \leqslant b_{ft} - s_{ft}^- + M(1-\delta_f^b), \quad t \in J, f \in F_u^M, u = 1,\cdots U \quad (8.8)$$

$$\sum_{j\in J} \lambda_{jt} b_{fj} \geqslant b_{ft} - s_{ft}^- - M(1-\delta_f^b), \quad t \in J, f \in F_u^M, u = 1,\cdots U \quad (8.9)$$

$$\sum_{i\in I_p^M} \delta_i^x = 1, p = 1,\cdots P \quad (8.10)$$

$$\sum_{r\in R_q^M} \delta_r^y = 1, q = 1,\cdots Q \quad (8.11)$$

$$\sum_{f\in F_u^M} \delta_f^b = 1, u = 1,\cdots U \quad (8.12)$$

$$\delta_i^x, \delta_r^y, \delta_f^b \in \{0,1\}, i \in I^M, r \in R^M, f \in F^M \quad (8.13)$$

$$\sum_{j\in J}\lambda_{jt} = 1, t \in J, \qquad \lambda_{jt}, s_{it}^-, s_{rt}^+, s_{ft}^- \geqslant 0, t, j \in J, i \in I, r \in R, f \in F \tag{8.14}$$

(8)

where

$$\rho_t^M = \frac{\left(\sum_{i\in I^S}\frac{s_{it}^-}{x_{it}} + \sum_{p=1}^{P}\sum_{i\in I_p^M}\frac{s_{it}^-\delta_i^x}{x_{it}} + \sum_{r\in R^S}\frac{s_{rt}^+}{y_{rt}} + \sum_{q=1}^{Q}\sum_{r\in R_q^M}\frac{s_{rt}^+\delta_r^y}{y_{rt}} + \sum_{f\in F^S}\frac{s_{ft}^-}{b_{ft}} + \sum_{f\in F_u^M}\frac{s_{ft}^-\delta_f^b}{b_{ft}}\right)}{S^I + P + S^R + Q + S^F + U}, t \in J \tag{9}$$

measures the inefficiency of each DMU and $\rho_t^M \geqslant 0, t \in J$. A DMU belongs to the reference set if $\lambda_{jt} > 0$, where $\lambda_{jt}, t, j \in J$ are the intensity vectors. In addition, all the DMUs have the same value on the binary variables $\delta_{i\in I_p^M}^{x*}$, $\delta_{r\in R_q^M}^{y*}$ and $\delta_{f\in F_u^M}^{b*}$referringto one consistent standard.

Note that each evaluated DMU selects its preferred input/output indicators in model (5), whereas model (8) uses the same performance indicators for all assessed DMUs. The different selected standards lead to different performance evaluation results for the DMUs. Here, we discuss the relationship between models (5) and (8).

Theorem 3.2 The optimal objective function value of model (8) denoted as $\rho^{M'(8)*}$ is no greater than the sum of optimal objective function values of model (5) for each DMU denoted as $\rho_t^{M(5)*}, t \in J$, that is $\rho^{M'(8)*} \leqslant \sum_{t=1}^{n}\rho_t^{M(5)*}$.

Proof Assume that the optimal objective function value of model (5) is $\rho_c^{M(5)*}$ for DMU_c, and the optimal objective function value of model (8) is $\rho^{M'(8)*}$. Then, the solution for DMU_c with $\rho_c^{M'(8)*}$ obtained from model (8) is a feasible solution of model (5). Since the maximization problem of model (5), we have $\rho_c^{M(5)*} \geqslant \rho_c^{M'(8)*}$. Therefore, considering all DMUs, it is plain that $\sum_{t=1}^{n}\rho_t^{M(5)*} \geqslant \sum_{t=1}^{n}\rho_t^{M''(8)} = \rho^{M''(8)*}$.

The model (8) has an advantage in computation faced with multi-valued indicators, which selects suitable value for multi-valued indicators in one time calculation for all DMUs. Besides, model (8) can be similarly transformed into a linear programming model; Appendix B gives the details.

In conclusion, comparing with the method of Toloo and Hančlová[12], we propose the slacks-based models to deal with multi-valued indicators, which consider the inefficiency of all inputs and outputs and can obtain Pareto solution. Besides, we consider the performance improvement and guide the adjustment of inputs and outputs.

3.2 Modified slacks-based models with goals in the presence of multi-valued indicators

In real-world practice, the selection of inputs/outputs may depend on the decision-makers' preferences[30]. Goals such as the five-year economic development plan in China, the expected sales in a company, and the concentration limit on PM 10 represent the preferences of decision-makers. Goals are often set in organizational planning, which should not be ignored in performance evaluation and improvement[7, 31, 32]. As a result, the goals may affect the selection of the multi-valued indicators and thus influence the performance evaluation results. Therefore, the goals of decision-makers should be considered. However, the goals set by the decision-makers may be unachievable or unambitious in practice[19]. Besides, the established goals may not be on best practice frontier. The DEA method provides the best practice frontier, which can be seen as the benchmark for the inefficient DMUs[17]. Accordingly, the DEA method can be used to guide the target setting. Therefore, we extend the above proposed slacks-based models by incorporating decision-makers' goals. To be specific, our proposed models consider the best practice benchmark and decision-makers' goals at the same time, which aim to find targets on best practice frontier as close as possible to decision-makers' goals for DMUs. In this section, we unfold from two cases, including decentralized decision-making and centralized decision-making, to illustrate the effect of decision-makers' goals.

3.2.1 Decentralized decision-making case with goals

We first take into account the goals from the perspective of decentralized decision-making. Based on model (5), we provide model (11) to help DMU_0 move towards the best practice frontier and closer to decision-makers' goal simultaneously. In this work, the objective function related to goal ρ_o^g is as follows:

$$\rho_o^g = \frac{\left(\sum_{i\in I^S}\frac{|s_i^{g-}|}{x_{io}} + \sum_{p=1}^{P}\sum_{i\in I_p^M}\frac{|s_i^{g-}|\delta_i^x}{x_{io}} + \sum_{r\in R^S}\frac{|s_r^{g+}|}{y_{ro}} + \sum_{q=1}^{Q}\sum_{r\in R_q^M}\frac{|s_r^{g+}|\delta_r^y}{y_{ro}} + \sum_{f\in F^S}\frac{|s_f^{g-}|}{b_{fo}} + \sum_{u=1}^{U}\sum_{f\in F_u^M}\frac{|s_f^{g-}|\delta_f^b}{b_{fo}}\right)}{S^I + P + S^R + Q + S^F + U} \tag{10}$$

which measures the inefficiency about the goals for DMU_o, representing the gaps between the targets (projection points) and the goals, and $\rho_o^g \geqslant 0$. The $|s_{io}^{g-}|$, $|s_{ro}^{g+}|$ and $|s_{fo}^{g-}|$ in ρ_o^g respectively represent the deviations between input targets and goals, desirable output targets and goals, undesirable output targets and goals, where the targets refer to the projection points on the effective practice frontier and the goals refer to the decision-makers' goals.

We want to achieve maximum improvement, which maximize ρ_o^M defined in Section 3.1.1 and get closer to the goals, which need to minimize ρ_o^g. Then the model incorporating with the goals is as follows:

$$\max \rho_o^M$$
$$\min \rho_o^g$$
$$\text{s.t.}\ (5.1-5.9)$$
$$\sum_{j\in J}\lambda_j x_{ij} = g_{io}^x - s_{io}^{g-}, i \in I^S \quad (11.1)$$
$$\sum_{j\in J}\lambda_j x_{ij} \leqslant g_{io}^x - s_{io}^{g-} + M(1-\delta_{io}^x), i \in I_p^M, p = 1,\cdots P \quad (11.2)$$
$$\sum_{j\in J}\lambda_j x_{ij} \geqslant g_{io}^x - s_{io}^{g-} - M(1-\delta_{io}^x), i \in I_p^M, p = 1,\cdots P \quad (11.3)$$
$$\sum_{j\in J}\lambda_j y_{rj} = g_{ro}^y + s_r^{g+}, r \in R^S \quad (11.4)$$
$$\sum_{j\in J}\lambda_j y_{rj} \leqslant g_{ro}^y + s_{ro}^{g+} + M(1-\delta_{ro}^y), r \in R_q^M, q = 1,\cdots Q \quad (11.5)$$
$$\sum_{j\in J}\lambda_j x_{ij} \geqslant g_{ro}^y + s_{ro}^{g+} - M(1-\delta_{ro}^y), r \in R_q^M, q = 1,\cdots Q \quad (11.6)$$
$$\sum_{j\in J}\lambda_j b_{fj} = g_{fo}^b - s_{fo}^{g-}, f \in F^S \quad (11.7)$$
$$\sum_{j\in J}\lambda_j b_{fj} \leqslant g_{fo}^b - s_{fo}^{g-} + M(1-\delta_{fo}^b), f \in F_u^M, u = 1,\cdots U \quad (11.8)$$
$$\sum_{j\in J}\lambda_j b_{fj} \geqslant g_{fo}^b - s_{fo}^{g-} - M(1-\delta_{fo}^b), f \in F_u^M, u = 1,\cdots U \quad (11.9)$$
$$\sum_{j\in J}\lambda_j = 1,$$
$$\lambda_j \geqslant 0, s_{io}^- \geqslant 0, s_{ro}^+ \geqslant 0, s_{fo}^- \geqslant 0,$$
$$j \in J, i \in I, r \in R, f \in F,$$
$$s_{io}^{g-}, s_{ro}^{g+}, s_{fo}^{g-}\ \text{free}, i \in I, r \in R, f \in F \quad (11.10)$$
$$(11)$$

The model involves two objective function. It can be written as $\max(\rho_o^M - \rho_o^g)$ when the model is calculated. The ρ_o^M in model (11) is the same as that in model (5) due to the same aim, and $g_{io}^x, i \in I$, $g_{ro}^y, r \in R$ and $g_{fo}^b, f \in F$ are the goals of inputs, desirable outputs, and undesirable outputs, respectively. The goals for each indicator can be determined by the decision-makers depending on administrative policies and the local situation. We can change the values of g_i^x, g_r^y and g_f^b for different DMUs to set corresponding goals. We can also set the same goals for DMUs with the same values of g_i^x, g_r^y, and g_f^b for all the DMUs based on the need of reality. When we consider the two objective functions, the inefficiency scores for DMU_o should represent these two aspects, which is defined as $\rho_o^{OD} = \rho_o^M + \rho_o^g$. Function ρ_o^g aims at minimizing the distance from the targets to the decision-makers' goals, which is expressed by absolute distance. In other words, model (11) simultaneously generates the maximum improvement to the efficient frontier for the inefficient DMUs and helps DMUs to get closer to the goals. The projected point of DMU_o obtained from model (11) is as follows:

$$\left.\begin{array}{l}
x_{io}^* = \sum_{j\in J}\lambda_j x_{ij} = x_{io} - s_{io}^{-*} = g_{io}^x - s_{io}^{g-*}, i \in I^S \\
x_{io}^* = \sum_{j\in J}\lambda_j x_{ij} = x_{io} - s_{io}^{-*} * \delta_{io}^{x*} = g_{io}^x - s_{io}^{g-*} * \delta_{io}^{x*}, \\
\qquad i \in I_p^M, p = 1,..P \\
y_{ro}^* = \sum_{j\in J}\lambda_j y_{rj} = y_{ro} + s_{ro}^{+*} = g_{ro}^y + s_{ro}^{g+*}, r \in R^S \\
y_{ro}^* = \sum_{j\in J}\lambda_j y_{rj} = y_{ro} + s_{ro}^{+} * \delta_{ro}^{y*} = g_{ro}^y + s_{ro}^{g+*} * \delta_{ro}^{y*}, \\
\qquad r \in R_q^M, q = 1,\cdots Q \\
b_{fo}^* = \sum_{j\in J}\lambda_j b_{fj} = b_{fo} - s_{fo}^{-*} = g_{fo}^b - s_{fo}^{g-*}, f \in F^S \\
b_{fo}^* = \sum_{j\in J}\lambda_j b_{fj} = b_{fo} - s_{fo}^{-*} * \delta_{fo}^{b*} = g_{fo}^b - s_{fo}^{g-*} * \delta_{fo}^{b*}, \\
\qquad f \in R_u^M, u = 1,\cdots U
\end{array}\right\} \quad (12)$$

In addition to improving the input/output, the results also guide the targets setting.

Note that model (11) is a nonlinear programming model. We transform model (11) into a linear programming model based on the way of Cook et al[3]; details are in Appendix C.

3.2.2 Centralized decision-making case with goals

We now put forward our model for the centralized decision-making case. We define the ρ_t^g for each DMU as follows:

$$\rho_t^g = \frac{\left(\sum_{i\in I^S}\frac{|s_{it}^{g-}|}{x_{it}} + \sum_{p=1}^{P}\sum_{i\in I_p^M}\frac{|s_{it}^{g-}|\delta_i^x}{x_{it}} + \sum_{r\in R^S}\frac{|s_{rt}^{g+}|}{y_{rt}} + \sum_{q=1}^{Q}\sum_{r\in R_q^M}\frac{|s_{rt}^{g+}|\delta_r^y}{y_{rt}} + \sum_{f\in F^S}\frac{|s_{ft}^{g-}|}{b_{ft}} + \sum_{u=1}^{U}\sum_{f\in F_u^M}\frac{|s_{ft}^{g-}|\delta_f^b}{b_{ft}}\right)}{(S^I + P + S^R + Q + S^F + U)} \quad (13)$$

It implies the gaps between the targets (projection points) and the goals for $DMU_t, t \in J$.

Based on model (8), we propose the following model (14), which encompasses the decision-makers' goals, given the ρ_t^g and the ρ_t^M defined in Section 3.1.2.

$$\max \sum_{t=1}^{n} \rho_t^M$$
$$\min \sum_{t=1}^{n} \rho_t^g$$
s.t. (8.1 − 8.9)
$$\sum_{j \in J} \lambda_{jt} x_{ij} = g_i^x - s_{it}^{g-}, t \in J, i \in I^S \quad (14.1)$$
$$\sum_{j \in J} \lambda_{jt} x_{ij} \leqslant g_i^x - s_{it}^{g-} + M(1 - \delta_i^x), \quad t \in J, i \in I_p^M, p = 1, \cdots P \quad (14.2)$$
$$\sum_{j \in J} \lambda_{jt} x_{ij} \geqslant g_i^x - s_{it}^{g-} - M(1 - \delta_i^x), \quad t \in J, i \in I_p^M, p = 1, \cdots P \quad (14.3)$$
$$\sum_{j \in J} \lambda_{jt} y_{rj} = g_r^y + s_{rt}^{g+}, t \in J, r \in R^S \quad (14.4)$$
$$\sum_{j \in J} \lambda_{jt} y_{rj} \leqslant g_r^y + s_{rt}^{g+} + M(1 - \delta_r^y), \quad t \in J, r \in R_q^M, q = 1, \cdots Q \quad (14.5)$$
$$\sum_{j \in J} \lambda_{jt} y_{rj} \geqslant g_r^y + s_{rt}^{g+} - M(1 - \delta_r^y), \quad t \in J, r \in R_q^M, q = 1, \cdots Q \quad (14.6)$$
$$\sum_{j \in J} \lambda_{jt} b_{fj} = g_f^b - s_{ft}^{g-}, t \in J, f \in F^S \quad (14.7)$$
$$\sum_{j \in J} \lambda_{jt} b_{fj} \leqslant g_f^b - s_{ft}^{g-} + M(1 - \delta_f^b), \quad t \in J, f \in F_u^M, u = 1, \cdots U \quad (14.8)$$
$$\sum_{j \in J} \lambda_{jt} b_{fj} \geqslant g_f^b - s_{ft}^{g-} - M(1 - \delta_f^b), \quad t \in J, f \in F_u^M, u = 1, \cdots U \quad (14.9)$$
(8.10 − 8.13)
$$\sum_{j \in J} \lambda_{jt} = 1, t \in J$$
$$\lambda_{jt} \geqslant 0, s_{it}^- \geqslant 0, s_{rt}^+ \geqslant 0, s_{ft}^- \geqslant 0, t, j \in J, i \in I, r \in R, f \in F,$$
$$s_{it}^{g-}, s_{rt}^{g+}, s_{ft}^{g-} \text{free}, t \in J, i \in I, r \in R, f \in F \quad (14.10)$$
(14)

We use the similar way in Section 3.2.1 to represent the objective functions of model (14). The objective function can be represented as $\max(\sum_{t=1}^{n} \rho_t^M - \sum_{t=1}^{n} \rho_t^g)$ in the calculation process. We define the overall inefficiency score for each DMU as $\rho_t^{OC} = \rho_t^M + \rho_t^g, t \in J$ The model provides a consistent standard for all the DMUs, as required by centralized decision-making case. By solving model (14), the optimal objective function value is obtained as
$$\rho^{(20)*}(s_{j,i \in I^S \cup I^M}^{-*}, s_{j,r \in R^S \cup R^M}^{+*}, s_{j,f \in F^S \cup F^M}^{-*}, s_{j,i \in I^S \cup I^M}^{g-*}, s_{j,r \in R^S \cup R^M}^{g+*}, s_{j,f \in F^S \cup F^M}^{g-*}, \delta_{j,i \in I_p^M}^{x*}, \delta_{j,r \in R_q^M}^{y*}, \delta_{j,f \in F_u^M}^{b*}, {}_j^*),$$
where $j \in J$, The binary variables ($\delta_{j,i \in I_p^M}^{x*}$, $\delta_{j,r \in R_q^M}^{y*}$, $\delta_{j,f \in F_u^M}^{b*}$), $j \in J$ have same values for all DMUs. The relationship of the performance between the decentralized and centralized decision-making cases, as Theorem 3.2 stated, is changed after considering goals, which indicates that the goals impact the performance measurement and improvements.

We use similar way to transform this nonlinear programming model to a linear programming model for the ease of calculation, giving details in Appendix D.

In conclusion, comparing the method of Toloo and Hančlová[12], we further propose the slack-based models with the integration of goals in presence of multi-valued indicators. The models get Pareto solution. Besides, we consider the adjustment of inputs and outputs, and the guidance of targets setting according to the solution of our models.

4 Application to cities in the Yangtze River delta (YRD) region

The Yangtze River delta (YRD) region, locating at the strategic hub of China's "Belt and Road" plan, plays an important role in China's economic development[33]. The strategic concept of "Yangtze River delta Integration" was first proposed in 1982 and becomes a national strategy. The cities in the YRD region carry out extensive cooperation and use the same rules and regulations for management in some field. Increasing attention is paid to energy conservation and environmental protection in China, and the cities in the YRD region are significant for China to convert to a green economy. Therefore, in this section, our proposed models are used on the environmental performance evaluation and improvement for the cities in the Yangtze River delta (YRD) region of China in 2017.

4.1 Dataset

Referring to prior studies[34,35], we select capital, labor, and energy consumption as three inputs. GDP (Gross Domestic Product) and GVA (Gross value added) are two commonly used desirable outputs to assess economic development[36,37]. We define the multi-valued desirable output as economic development, including GDP (y_1) and GVA (y_2), which are also used by Toloo and Hančlová[12]. For the undesirable outputs, we chose SO_2 emissions and NO_2 emissions[38,40]. Furthermore, we consider the particulate matter since the serious air pollution situation exists in China. There are two standards for particulate matter in the atmosphere reflecting air quality: PM 10 and PM 2.5, which are line with the aforementioned definition of multi-valued indicators. Primary and secondary concentration limits

Table 2. Input/output indicators

Input/output	Variables			Units
Inputs	Energy consumption: Total electricity consumption	x_1	Single-valued	100 million kwh
	Capital: Total investment in fixed assets	x_2	Single-valued	100 million RMB
	Labor: Number of employed persons at each year's end	x_3	Single-valued	10 thousand people
Desirable outputs	Economic development: GDP	y_1	Multi-valued	100 million RMB
	Economic development: GVA	y_2		100 million RMB
Undesirable outputs	Particulate matter: PM10	b_1	Multi-valued	$/\mu m^3$
	Particulate matter: PM2.5	b_2		$/\mu m^3$
	NO_2 emissions	b_3	Single-valued	$/\mu m^3$
	SO_2 emissions	b_4	Single-valued	$/\mu m^3$

Table 3. Cities in Yangtze River delta.

Provinces/municipalities	Cities
Jiangsu	Nanjing, Zhenjiang, Yangzhou, Changzhou, Suzhou, Wuxi, Nantong
Zhejiang	Hangzhou, Huzhou, Shaoxing, Ningbo, Jinhua, Taizhou
Shanghai	Shanghai
Anhui	Hefei, Wuhu, Chuzhou, Maanshan, Tongling, Chizhou, Anqing, Xuancheng

Table 4. Descriptive statistics of the data.

	Energy consumption	Capital	Labor	GDP	GVA	PM 10	PM 2.5	NO_2	SO_2
Mean	432.38	3589.85	377.36	6808.42	6594.50	73.65	46.69	40.45	14.82
Median	303.30	3228.95	312.90	4736.15	4509.89	76.00	45.00	40.00	14.50
Min	61.21	714.59	76.49	624.35	621.42	54.00	32.00	22.00	7.00
Max	1526.77	7246.60	1372.65	30632.99	30122.98	93.00	60.00	52.00	27.00
S.D.	395.45	1859.34	276.62	6751.48	6579.56	10.85	7.41	7.18	4.30

for PM10 and PM2.5 are set in China's Ambient air quality standard GB3095-2012. Our example uses a multi-valued undesirable output measure for particulate matter, including PM10 (b_1) and PM2.5 (b_2) [41,42]. Table 2 summarizes the input/output indicators.

The data is collected from the China Statistical Yearbook, Urban-level Statistical Yearbook, and Urban Environment Bulletin. Based on the YRD urban agglomeration development plan released in 2016, 26 cities are included in the YRD region. Because of data availability, Yancheng, Taizhou, Jiaxing, and Zhoushan are excluded. Table 3 presents the provinces/municipalities and their constituents. The data statistics description is reported in Table 4. There are gaps among the cities shown in the line "S. D." of Table 4.

4.2 Results and analysis

We set the same goals for 22 cities in both decentralized and centralized decision-making cases for easily calculation and comparison. In order to illustrate the effect of decision-makers' goals, the values of goals are obtained following the way of Ruiz et al[43]. The goals of inputs, the goals of desirable outputs, and the goals of undesirable outputs are respectively set as $x_{ij}^{av}-(x_{ij}^{av}-x_{ij}^{min})/2$, $y_{rj}^{av}+(y_{rj}^{max}-y_{rj}^{av})/2$, and $b_{fj}^{av}-(b_{fj}^{av}-b_{fj}^{min})/2$, where x_{ij}^{av}, y_{ij}^{av}, and b_{ij}^{av} respectively denote the average value of the i^{th} inputs, average value of the r^{th} desirable outputs, and average value of the f^{th} undesirable outputs; y_{rj}^{max}, x_{ij}^{min}, and b_{ij}^{min} respectively denote the maximum value of the r^{th} desirable output, minimum values of the i^{th} input and minimum values of the f^{th} undesirable output. The initial values of the decision-makers' goals are listed in column 2 of Table 10.

On the one hand, under the decentralized decision-making case, the frequency of selected multi-valued

Table 5. Frequency of selected multi-valued indicators in decentralized case.

Without goals				With goals			
y_1	y_2	b_1	b_2	y_1	y_2	b_1	b_2
9	13	11	11	11	11	16	6

Table 6. Frequency of selected combinations of multi-valued indicators in decentralized case.

Without goals				With goals			
$\{y_1,b_1\}$	$\{y_1,b_2\}$	$\{y_2,b_1\}$	$\{y_2,b_2\}$	$\{y_1,b_1\}$	$\{y_1,b_2\}$	$\{y_2,b_1\}$	$\{y_2,b_2\}$
3	6	8	5	8	3	8	3

indicators and the change of the selected combinations of multi-valued indicators, obtained from models (5) and (11), are respectively shown in Tables 5 and 6. We can identify the indicators that need improvements through the frequency of selection. The higher the frequency of being selected, the poorer performance of most cities on this indicator. For instance, the most frequently selected indicators is PM 10 (b_1) (16 times) if considering the goals, which means most cities need pay more attention to the PM 10 (b_1) in view of the current level and targets setting. On the other hand, in the centralized decision-making situation, all cities are evaluated with the same standards, i. e. , they have the same selected combination of the multi-valued indicators, that is, $\{y_2,b_1\}$ and $\{y_2,b_2\}$ obtained from model (8) and model (14), respectively.

By solving models (5), (8), (11) and (14), the inefficiency scores of 22 cities are obtained and reported in Table 7. Considering the situation without the goals, we draw the following conclusions. Firstly, less than 60% of the cities are efficient, whose inefficiency score equal to zero. It means that some cities are still environmentally inefficient, which calls for more efforts and management strategies in environmental improvement in the YRD region. Secondly, some cities such as Changzhou, whose inefficiency score under the centralized decision-making case (0.2043) is lower than that under the decentralized decision-making case (0.2146). Besides, it is clear that the average environmental performance in the centralized decision-making case is lower than that in the decentralized decision-making case, as seen in the last row of Table 7; this result is consistent with Theorem 3. 2, and implies that characteristics of some cities may be ignored in the centralized decision-making case. Thirdly, under the same decision-making case, there are inter-city gaps in the environmental performance in the YRD region, which is consistent with the initial judgment in our

Table 7. Inefficiency scores of the cities.

Cities	Without goals		With goals	
	Decentralized	Centralized	Decentralized	Centralized
Nanjing	0.0000	0.0000	0.4261	0.4288
Zhenjiang	0.0000	0.0000	0.6336	0.6336
Yangzhou	0.0000	0.0000	0.5975	0.6152
Changzhou	0.2146	0.2043	0.5439	0.5723
Suzhou	0.0000	0.0000	0.3970	0.4095
Wuxi	0.1716	0.1466	0.4758	0.4480
Nantong	0.1063	0.1063	0.5194	0.5194
Hangzhou	0.1681	0.1659	0.4420	0.4222
Huzhou	0.0000	0.0000	1.0450	1.0537
Shaoxing	0.1903	0.1775	0.5703	0.5764
Ningbo	0.1439	0.1388	0.4509	0.4842
Jinhua	0.0000	0.0000	0.8879	0.8879
Taizhou	0.0000	0.0000	0.7428	0.7651
Shanghai	0.0000	0.0000	0.4507	0.4509
Hefei	0.0000	0.0000	0.5388	0.5558
Wuhu	0.1133	0.1019	0.9809	0.9814
Chuzhou	0.1404	0.1397	1.7634	1.8052
Maanshan	0.2187	0.1807	1.7275	1.7468
Tongling	0.0000	0.0000	2.9084	3.0638
Chizhou	0.0000	0.0000	5.0454	5.0542
Anqing	0.0000	0.0000	1.9454	1.9770
Xuancheng	0.0000	0.0000	2.4571	2.4933
Average	0.0667	0.0619	1.1614	1.1793

descriptive analysis in Table 4. Take the decentralized decision-making situation as an example, where the largest inefficiency score is 0.2187 (Ma'anshan) and the lowest inefficiency score is 0 (such as Nanjing) in Table 7.

Considering the situation with goals, the environmental performance results obtained from models (11) and (14) are shown in columns 4−5 of Table 7. We conclude the following results to illustrate the effects of goals. Firstly, compared with the case without goals, the higher inefficiency scores mean worse environmental performance due to the additional constraints of the goals. However, using goals avoids the problem that the performance evaluation under the centralized decision-making case is always higher than that under the

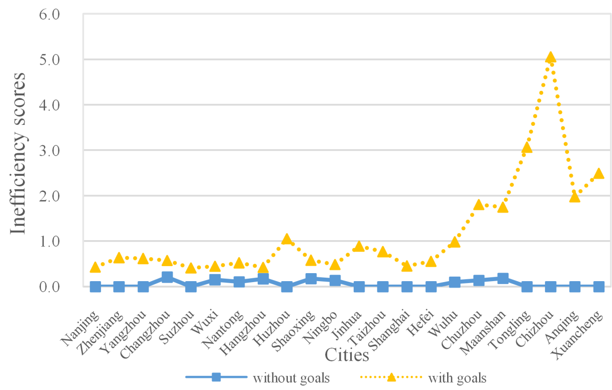

Figure 1. Inefficiency scores for cities under centralized decision-making case.

decentralized decision-making case without goals, e. g. , Nanjing, Changzhou. Secondly, considering goals enhances the discriminability of the models on DMUs. For example, under the centralized decision-making situation, there are more obvious differences among cities, where the gap between the maximum and minimum inefficiency scores is 4. 6447 and 0. 2043 respectively, seen in Table 7. The environmental performance gaps among cities are better distinguished, seen in Figure 1. Cities with more advanced economies often have a better environmental performance, as exemplified by Nanjing, Suzhou, and Shanghai. Besides, some DMUs such as Hefei that are classified as efficient when there are no goals can be distinguished after considering the goals. Besides, Suzhou have the best environmental performance with the lowest inefficiency score, which may be attributed to impressive economic development, and its economic level ranks among the top in China. However, Chizhou, as a less developed city, has the worst environmental performance with the maximum inefficiency score, i. e. , 5. 0454 and 5. 0542 respectively under decentralized and centralized decision-making cases. The developed cities often pay more attention to the environment, which accords with the reality of China.

We then discuss the differences across provinces according to the regional division in Table 3. First, under decentralized decision-making case, provincial capitals such as Nanjing and Hefei (excepting Hangzhou) are environmentally efficient. The reason for the low environmental performance of Hangzhou may be that more human activities increase pollution (undesirable outputs). In addition, the capital of provinces often has a better environmental performance. Hefei, the capital of Anhui, has abetter environmental performance compared with some cities in other provinces, such as inefficiency score 0. 5558 versus

Table 8. Average inefficiency scores of 4 provinces/municipalities.

Provinces	Without goals		With goals	
	Decentralized	Centralized	Decentralized	Centralized
Jiangsu	0. 3170	0. 3149	0. 5820	0. 5709
Zhejiang	0. 4216	0. 4195	0. 7655	0. 7314
Shanghai	0. 0000	0. 0000	0. 4507	0. 4726
Anhui	0. 5080	0. 5075	2. 2531	2. 2673

Yangzhou's 0. 6152, as seen in column 5 in Table 7. Second, unbalanced environmental performance exists in YRD region according the average inefficiency scores for different provinces, shown in Table 8. Taking the centralized decision-making case as an example, Shanghai has the best environmental performance, and Anhui has the worst environmental performance. The results are more direct, seen in Figure 1. The cities in the right part of the graph with large fluctuations belong to Anhui province. Compared with other provinces or municipalities in YRD region, the reason behind the situation may be less-advanced economy in Anhui, and the government pays more attention to economic development rather than the environment. This result is not find in the case without goals, which implies the important role of goals. Besides, there are gaps among internal cities in Anhui province. Therefore, Anhui province needs to improve its environmental performance and it is necessary to adopt environmental policies tailored to local conditions for different cities.

The environmental inefficiency scores indicate these cities have the potential to enhance the environmental performance by adjusting their input/output. We consider the situation with goals and the adjustments of input/output can be obtained from models (11) and model (14). Taking Ma'anshan as an example, the adjustments of input/output indicators are listed in Table 9. There are some differences in the selected indicators, which affects the degree of improvement of input/output indicators, such as the adjustments for PM 10 (b_1) with 9. 9 in decentralized decision-making case, and PM 2. 5 (b_2) with 0. 62 in the centralized decision-making case. It means Ma'anshan needs to pay more attention to the governance of PM 10 (b_1), which reflects the short board of Ma'anshan in environment. However, from the centralized decision-making case, the selected PM 2. 5 (b_2) is short board for most cities in YRD. In short, for the purpose of supporting decision-making, it is useful to comprehensively consider different decision scenarios and the effect of goals.

Table 9. Adjustments of input/output indicators for Ma'anshan.

	Initial values	With goals	
		Decentralized	Centralized
s_1^x	455.03	0	0
s_2^x	3896.30	612.43	379.93
s_3^x	281.70	0	0
s_1^y	6618.42	699.07	/
s_2^y	6241.54	/	1176.56
s_1^b	73.00	9.9	/
s_2^b	47.00	/	0.62
s_3^b	41.00	6.7	0.49
s_4^b	17.00	3.8	3.17

[Note] "/" means unselected indicators.

Setting suitable goals is a common policy to guide the environmental governance. Take the cities Nanjing, Jinhua, Shanghai and Hefei as examples, which are identified as efficient DMUs under the case without goals and are respectively located in Jiangsu, Zhejiang, Shanghai, Anhui, corresponding to the regional division of Table 3. Unlike the situation without goals, considering goals distinguishes these DMUs, and it guides the targets setting for these cities, as shown in Table 10. For inputs and undesirable outputs, the less is the better. The negative adjustments show that the goals are unachievable based on current situation, and it is more reasonable to set a higher value of goals. The positive adjustments show that the lower value of goals are suggested. In light of Table 10, for example, we take the SO_2 emission (b_4) as an example. The negative adjustments for Nanjing (−15.77) means that the goal of decision-makers on SO_2 emission (b_4) is currently unreachable, and higher value of the goal is suggested. However, for Jinhua (2.23), the positive adjustment means that lower value of goal is suggested. The analysis results for desirable outputs are contrary since the more is the better. The negative adjustments mean the goals are too high to reach currently and lower value of goals are more appropriate; the positive adjustments imply that it suggests to set higher value of goals. For instance, negative adjustments for Nanjing, Jinhua, Hefei under the centralized decision-making case indicate that the goals on GVA(y_2) need to be set lower value. However, the goals on GVA (y_2) for Shanghai with a positive adjustment under the centralized decision-making case is overly low and higher value of goal is suggested.

In conclusion, our approach evaluates the environmental performance and guides the improvement in the presence of multi-valued indicators, and further provide the insight in incorporating with decision-makers' goals. Given the aforementioned analysis, the environmental performance of cities has room to improve in the YRD region; the cities need pay more attention to energy conservation and environmental protection policies, and prevent short-sighted goals which ignore the environmental protection. Besides, the decision-makers can keep the whole picture of the centralized and decentralized decision-making cases in mind to support decisions on the environmental protection. Furthermore, adjustments in the environmental policies should be in line with the situation in each city due to gaps among the YRD cities, and the cities need to learn from successful practice and strengthen inter-provincial cooperation to achieve an integrated development strategy according to the practice. Additionally, apart from the environmental protection regulations and laws, appropriate goals should also be correlated with environmental protection efforts.

Table 10. Adjustments of goals for four cities.

	Goals	Nanjing		Jinhua		Shanghai		Hefei	
		Decentralized	Centralized	Decentralized	Centralized	Decentralized	Centralized	Decentralized	Centralized
$s_{x_1}^g$	246.80	−310.16	−310.16	−90.88	−90.88	−1279.97	−1279.97	−49.29	−49.29
$s_{x_2}^g$	2152.22	−4062.98	−4062.98	−48.30	−48.30	−5094.38	−5094.38	−4199.21	−4199.21
$s_{x_3}^g$	226.93	−230.67	−230.67	150.44	150.44	−1145.72	−1145.72	−311.17	−311.17
$s_{y_1}^g$	18720.71	−7005.61	/	/	/	11912.28	/	−11717.66	/
$s_{y_2}^g$	18358.74	/	−7004.97	−14474.45	−14474.45	/	11764.24	/	−11551.03
$s_{b_1}^g$	63.83	/	/	/	/	/	/	−16.17	/
$s_{b_2}^g$	39.35	−0.65	−0.65	−2.65	−2.65	0.35	0.35	/	−16.65
$s_{b_3}^g$	31.23	−15.77	−15.77	2.23	2.23	−12.77	−12.77	−20.77	−20.77
s_4^g	10.91	−5.09	−5.09	0.91	0.91	−1.09	−1.09	−1.09	−1.09

[Note] "/" mean unselected indicator.

5 Conclusion

Unlike the traditional DEA methods considering single-valued performance indicators, in this work, we first propose two modified slacks-based models to select a suitable value for the multi-valued indicators. Then we further incorporate goals into the proposed models. Aiming for more practicality, our proposed models consider two cases of the decentralized and centralized decision-making. Using an empirical example of 22 cities in the Yangtze River delta region in China, we demonstrate the applicability and practicality of our models.

To sum up, our models make the following contributions to the literature on multi-valued indicators in DEA. First, the new models not only provide insight into performance measurement, but also guide adjustments on input/output indicators and targets setting. Secondly, our models with the goals demonstrate that the performance evaluation and improvements are more in line with the expectations of decision-makers. Third, comparing with the modified SBM model, our models are easier to deal with multi-valued indicators through one time calculation. Fourth, our models are practical considering the goals of decision-makers in both the decentralized and centralized decision-making cases, which provide multi-faceted support for decision-makers.

This study can be extended as follows. First, the goals in this work are virtual values calculated from the original inputs/outputs data set. The goals are usually set by the interaction among decision-makers, considering many factors such as polices, prior performance in practice and are usually established before the performance evaluation. Future research can use the actual existed goals in specific practices. Besides, our models guide the targets setting, and the projection points obtained from our models can be set as new goals for the next production period according to the practical need. Second, our models assume that all the DMUs and all inputs/outputs of each DMU have goals. In reality, some indicators have no explicit goals; for these indicators, our models can be modified by removing the corresponding constraints of these indicators on goals. Third, besides the environmental performance evaluation, the models can also be used in other fields based on the actual need.

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Conflict of interest

The authors declare no conflict of interest.

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Appendix A

We first propose the Proposition 1, which is useful for converting the model to a linear programming model. The Proposition 1 is as follows.

Proposition 1 For each $\mathrm{DMU}_j, j \in J$, the following inequalities hold

$$0 \leqslant s_{fj}^{-} \leqslant b_{fj}, 0 \leqslant s_{rj}^{+} \leqslant \max_{t \in J}\{y_{rt}\}, 0 \leqslant s_{ij}^{-} \leqslant x_{ij}, \forall f \in F, r \in R, i \in I \tag{A.1}$$

Proof For a specific DMU_o under VRS, we have $b_{fo} - s_f^{-} \geqslant 0$ because of the constraints $\sum_{j \in J} \lambda_j b_{fj} = b_{fo} - s_{fo}^{-}$, $b_{fo} \geqslant 0$, $s_{fo}^{-} \geqslant 0$, and $\lambda_j \geqslant 0$. Similarly, we have $x_{io} - s_{io}^{-} \geqslant 0$. Besides, it obtains $\max_{t \in J} y_{rt} \geqslant \sum_{j \in J} \lambda_j y_{rj} = y_{ro} + s_{ro}^{+} \geqslant 0$ because of the $\sum_{j \in J} \lambda_j = 1$.

The specific process of converting into a linear programming model is as follows. We use the $z_{io}^{x}, i \in I_p^M$, z_{ro}^{y}, $r \in R_q^M$ and $z_{fo}^{b}, f \in F_u^M$ to replace the variables $s_{io}^{-}\delta_{io}^{x}$, $s_{ro}^{+}\delta_{ro}^{y}$, and $s_{fo}^{-}\delta_{fo}^{b}$, which cause the nonlinear problem. Then the objective function $\rho_o^{M'}$ is rewritten as

$$\frac{\left(\sum_{i \in I^S} \frac{s_{io}^{-}}{x_{i0}} + \sum_{p=1}^{P} \sum_{i \in I_p^M} \frac{z_{io}^{x}}{x_{io}} + \sum_{r \in R^S} \frac{s_{ro}^{+}}{y_{r0}} + \sum_{q=1}^{Q} \sum_{r \in R_q^M} \frac{z_{ro}^{y}}{y_{ro}} + \sum_{f \in F^S} \frac{s_{fo}^{-}}{b_{f0}} + \sum_{u=1}^{U} \sum_{f \in F_u^M} \frac{z_{fo}^{b}}{b_{fo}}\right)}{S^I + P + S^R + Q + S^F + U}.$$

We get the mixed-integer linear programming problem (MILP) model (A.2).

$$\max \rho_o^{M'}$$

$$s.t.\ (5.1 - 5.14)$$

$$0 \leqslant z_{io}^{x} \leqslant \delta_{io}^{x} x_{i0}, i \in I_p^M, p = 1, \cdots P \tag{A.2.1}$$

$$0 \leqslant s_{io}^{-} - z_{io}^{x} \leqslant (1 - \delta_{io}^{x}) x_{i0}, i \in I_p^M, p = 1, \cdots P \tag{A.2.2}$$

$$0 \leqslant z_{ro}^{y} \leqslant \delta_{ro}^{y} \max_{t \in J}\{y_{rt}\}, r \in R_q^M, q = 1, \cdots Q \tag{A.2.3}$$

$$0 \leqslant s_{ro}^{+} - z_{ro}^{y} \leqslant (1 - \delta_{ro}^{y}) \max_{t \in J}\{y_{rt}\}, r \in R_q^M, q = 1, \cdots Q \tag{A.2.4}$$

$$0 \leqslant z_{fo}^{b} \leqslant \delta_{fo}^{b} b_{f0}, f \in F_u^M, u = 1, \cdots U \tag{A.2.5}$$

$$0 \leqslant s_{fo}^{-} - z_{fo}^{b} \leqslant (1 - \delta_{fo}^{b}) b_{f0}, f \in F_u^M, u = 1, \cdots U \tag{A.2.6}$$

$$\lambda_j, s_{io}^{-} \geqslant 0, s_{ro}^{+} \geqslant 0, s_{fo}^{-} \geqslant 0, j \in J, i \in I, r \in R, f \in F$$

$$z_{io}^{x} \geqslant 0, z_{ro}^{y} \geqslant 0, z_{fo}^{b} \geqslant 0, i \in I, r \in R, f \in F$$

(A.2)

Proposition 2 Model (A.2) is equivalent to model (5).

Proof The single-valued indicators are the same for model (A.2) and model (5), so we only discuss the multi-valued indicators. For simplicity, we presume one multi-valued input, one multi-valued desirable output, and one multi-valued undesirable output, which indicates $P = 1$, $Q = 1$, and $U = 1$. We assume that $\delta_a^x = 1, a \in I_1^M$; $\delta_c^y = 1$, $c \in R_1^M$; and $\delta_d^b = 1, d \in F_1^M$, which means that we select the a^{th} value for the multi-valued input, c^{th} value for the multi-valued desirable output, and d^{th} value for the multi-valued undesirable output, respectively. Then, the objective function of model (5) is $\rho_0^M = \frac{\left(\sum_{i \in I^S} \frac{s_{io}^{-}}{x_{i0}} + \frac{s_{ao}^{-}}{x_{a0}} + \sum_{r \in R^S} \frac{s_{ro}^{+}}{y_{r0}} + \frac{s_{co}^{+}}{y_{c0}} + \sum_{f \in F^S} \frac{s_{fo}^{-}}{b_{f0}} + \frac{s_{do}^{-}}{b_{d0}}\right)}{S^I + 1 + S^R + 1 + S^F + 1}$. The corresponding constraints for multi-valued input are $\sum_{j \in J} \lambda_j x_{aj} \leqslant x_{ao} - s_{ao}^{-}$, $\sum_{j \in J} \lambda_j x_{aj} \geqslant x_{ao} - s_{ao}^{-}$, $\sum_{j \in J} \lambda_j x_{ij} \leqslant x_{i0} - s_{io}^{-} + M, i \neq a \in I_1^M$, and $\sum_{j \in J} \lambda_j x_{ij} \geqslant x_{io} - s_{io}^{-} - M, i \neq a \in I_1^M$. By Proposition 1, we have $0 \leqslant s_{io}^{-} \leqslant x_{io}, i \in I_1^M$. The constraints of the multi-valued desirable output are similar: $\sum_{j \in J} \lambda_j y_{cj} \leqslant y_{co} + s_{co}^{+}$, $\sum_{j \in J} \lambda_j y_{cj} \geqslant y_{co} + s_{co}^{+}$; $\sum_{j \in J} \lambda_j y_{rj} \leqslant y_{ro} + s_{ro}^{+} + M, r \neq c \in R_1^M$; and $\sum_{j \in J} \lambda_j y_{rj} \geqslant y_{ro} + s_{ro}^{+} - M, r \neq c \in R_1^M$. By Proposition 1, we get $0 \leqslant s_{ro}^{+} \leqslant \max_{t \in J}\{y_{rt}\}, r \in R_1^M$. For multi-valued undesirable output, we have $\sum_{j \in J} \lambda_j b_{dj} \leqslant b_{do} - s_{do}^{-}$, $\sum_{j \in J} \lambda_j b_{dj} \geqslant b_{do} - s_{do}^{-}$, $\sum_{j \in J} \lambda_j b_{fj} \leqslant b_{f0} - s_{fo}^{-} + M, f \neq d \in F_1^M$, and $\sum_{j \in J} \lambda_j b_{fj} \geqslant b_{fo} - s_{fo}^{-} - M, f \neq d \in F_1^M$. We get $0 \leqslant s_{fo}^{-} \leqslant b_{fo}, f \in F_1^M$ from Proposition 1.

In addition, since $z_{io}^{x} = s_{io}^{-}\delta_{io}^{x}, i \in I_p^M$; $z_{ro}^{y} = s_{ro}^{+}\delta_{ro}^{y}, r \in R_q^M$; and $z_{fo}^{b} = s_{fo}^{-}\delta_{fo}^{b}, f \in F_u^M$, we have $z_{ao}^{x} = s_{ao}^{-}, a \in I_1^M$;

$z_{co}^{y} = s_{co}^{+}, c \in R_1^M$; and $z_{do}^{b} = s_{do}^{-}, d \in F_1^M$. The objective function of the model (A.2) is then rewritten as $\rho_0^{M'} = \frac{\left(\sum_{i \in I^S} \frac{s_{io}^{-}}{x_{i0}} + \frac{s_{ao}^{-}}{x_{i0}} + \sum_{r \in R^S} \frac{s_{ro}^{+}}{y_{r0}} + \frac{s_{co}^{+}}{y_{r0}} + \sum_{f \in F^S} \frac{s_{fo}^{-}}{b_{f0}} + \frac{s_{do}^{-}}{b_{f0}}\right)}{S^I + 1 + S^R + 1 + S^F + 1}$, which is equivalent to the ρ_o^M in model (5).

We first prove the same objective function between model (A.2) and model (5). Then we focus on the constraints of two models. We get the relevant constraints of model (A.2). The constraints of the multi-valued inputs are $\sum_{j \in J} \lambda_j x_{aj} \leq x_{ao} - s_{ao}^{-}$, $\sum_{j \in J} \lambda_j x_{aj} \geq x_{ao} - s_{ao}^{-}, 0 \leq s_{ao}^{-} \leq x_{ao}, 0 \leq x_{ao}$, and $\sum_{j \in J} \lambda_j x_{ij} \leq x_{io} - s_{io}^{-} + M, i \neq a \in I_1^M$, $\sum_{j \in J} \lambda_j x_{ij} \geq x_{io} - s_{io}^{-} - M, i \neq a \in I_1^M, 0 \leq s_{io}^{-} \leq x_{io}, i \neq a \in I_1^M$. For the multi-valued desirable output, the constraints are $\sum_{j \in J} \lambda_j y_{cj} \leq y_{co} + s_{co}^{+}$, $\sum_{j \in J} \lambda_j y_{cj} \geq y_{co} + s_{co}^{+}$, $0 \leq s_{co}^{+} \leq \max_{t \in J}\{y_{ct}\}$, $0 \leq \max_{t \in J}\{y_{ct}\}$, and $\sum_{j \in J} \lambda_j y_{rj} \leq y_{ro} + s_{ro}^{+} + M, r \neq c \in R_1^M$, $\sum_{j \in J} \lambda_j y_{rj} \geq y_{ro} + s_{ro}^{+} - M, r \neq c \in R_q^M$, and $0 \leq s_{ro}^{+} \leq \max_{t \in J}\{y_{rt}\}, r \neq c \in R_q^M$. When it comes to the multi-valued undesirable output, the constraints are $\sum_{j \in J} \lambda_j b_{dj} \leq b_{do} - s_{do}^{-}$, $\sum_{j \in J} \lambda_j x_{dj} \geq b_{do} - s_{do}^{-}$, $0 \leq s_{do}^{-} \leq b_{do}$, $0 \leq b_{do}$, and $\sum_{j \in J} \lambda_j b_{fj} \leq b_{fo} - s_{fo}^{-} + M, f \neq d \in F_1^M$, $\sum_{j \in J} \lambda_j b_{fj} \geq b_{fo} - s_{fo}^{-} - M, f \neq d \in F_1^M$, $0 \leq s_{fo}^{-} \leq b_{fo}, f \neq d \in F_1^M$. These constraints are also the same between the two models. Therefore, we have Proposition 2. When there are more than one multi-valued input/output, the analysis is in a similar way.

Appendix B

Similar to the method in Appendix A, we transform the nonlinear programming model (8) to MILP model (B.1). We employ $z_{it}^{x}, i \in I_p^M$, $z_{rt}^{y}, r \in R_q^M$ and $z_{ft}^{b}, f \in F_u^M$ to replace the variables $s_{it}^{-}\delta_i^{x}$, $s_{rt}^{+}\delta_r^{y}$ and $s_{ft}^{-}\delta_f^{b}$, respectively. We assume $\rho_t^{M'}$ stands for $\frac{\left(\sum_{i \in I^S} \frac{s_{it}^{-}}{x_{it}} + \sum_{p=1}^{P} \sum_{i \in I_p^M} \frac{z_{it}^{x}}{x_{it}} + \sum_{r \in R^S} \frac{s_{rt}^{+}}{y_{rt}} + \sum_{q=1}^{Q} \sum_{r \in R_q^M} \frac{z_{rt}^{y}}{y_{rt}} + \sum_{f \in F^S} \frac{s_{ft}^{-}}{b_{ft}} + \sum_{u=1}^{U} \sum_{f \in F_u^M} \frac{z_{ft}^{b}}{b_{ft}}\right)}{S^I + P + S^R + Q + S^F + U}, t \in J$. Then the model (B.1) is written as

$$
\begin{aligned}
& \max \sum_{t=1}^{n} \rho_t^{M'} \\
& s.t.\ (8.1 - 8.14) \\
& 0 \leq z_{it}^{x} \leq \delta_i^{x} x_{it}, t \in J, i \in I_p^M, p = 1, \cdots P \quad (B.1.1) \\
& 0 \leq s_{it}^{-} - z_{it}^{x} \leq (1 - \delta_i^{x}) x_{it}, t \in J, i \in I_p^M, p = 1, \cdots P \quad (B.1.2) \\
& 0 \leq z_{rt}^{y} \leq \delta_r^{y} \max_{t \in J}\{y_{rt}\}, t \in J, r \in R_q^M, q = 1, \cdots Q \quad (B.1.3) \\
& 0 \leq s_{rt}^{+} - z_{rt}^{y} \leq (1 - \delta_r^{y}) \max_{t \in J}\{y_{rt}\}, t \in J, r \in R_q^M, q = 1, \cdots Q \quad (B.1.4) \\
& 0 \leq z_{ft}^{b} \leq \delta_f^{b} b_{ft}, t \in J, f \in F_u^M, u = 1, \cdots U \quad (B.1.5) \\
& 0 \leq s_{ft}^{-} - z_{ft}^{b} \leq (1 - \delta_f^{b}) b_{ft}, t \in J, f \in F_u^M, u = 1, \cdots U \quad (B.1.6) \\
& \lambda_{jt}, s_{it}^{-} \geq 0, s_{rt}^{+} \geq 0, s_{ft}^{-} \geq 0, j, t \in J, i \in I, r \in R, f \in F \\
& z_{it}^{x} \geq 0, z_{rt}^{y} \geq 0, z_{ft}^{b} \geq 0, i \in I, r \in R, f \in F
\end{aligned}
\quad (B.1)
$$

Proposition 3　Model (B.1) is equivalent to model (8).

Proof　The proof process is similar to that in Proposition 2.

Appendix C

We first propose the Proposition 4, which can be used in the process of converting into a linear programming model.

Proposition 4　For any $DMU_j, j \in J$, the following constraints hold.

$$
\begin{aligned}
& g_i^{x} - \max_{j \in J}\{x_{ij}\} \leq s_i^{g-} \leq g_i^{x} - \min_{j \in J}\{x_{ij}\}, i \in I \\
& \min_{j \in J}\{y_{rj}\} - g_r^{y} \leq s_r^{g+} \leq \max_{j \in J}\{y_{rj}\} - g_r^{y}, r \in R \\
& g_f^{b} - \max_{j \in J}\{b_{fj}\} \leq s_f^{g-} \leq g_f^{b} - \min_{j \in J}\{b_{fj}\}, f \in F
\end{aligned}
\quad (C.1)
$$

Proof　Under VRS assumption, since $\sum_{j \in J} \lambda_j x_{ij} = g_i^{x} - s_i^{g-}$ and $\min_{j \in J}\{x_{ij}\} \leq \sum_{j \in J} \lambda_j x_{ij} \leq \max_{j \in J}\{x_{ij}\}, i \in I$, we have

$g_i^x - \max_{j\in J}\{x_{ij}\} \leqslant s_i^{g-} \leqslant g_i^x - \min_{j\in J}\{x_{ij}\}, i \in I$. Analogously, we get $\min_{j\in J}\{y_{rj}\} - g_r^y \leqslant s_r^{g+} \leqslant \max_{j\in J}\{y_{rj}\} - g_r^y, r \in R$ because of $\sum_{j\in J}\lambda_j y_{rj} = g_r^y + s_r^{g+}$ and $\min_{j\in J}\{y_{rj}\} \leqslant \sum_{j\in J}\lambda_j y_{rj} \leqslant \max_{j\in J}\{y_{rj}\}, r \in R$. We also have $g_f^b - \max_{j\in J}\{b_{fj}\} \leqslant s_f^{g-} \leqslant g_f^b - \min_{j\in J}\{b_{fj}\}, f \in F$ due to $\sum_{j\in J}\lambda_j b_{fj} = g_f^b - s_f^{g-}$ and $\min_{j\in J}\{b_{fj}\} \leqslant \sum_{j\in J}\lambda_j b_{fj} \leqslant \max_{j\in J}\{b_{fj}\}, f \in F$.

Then the detail of converting into a linear programming model is as follows. We make the following conversion for input/output indicators $|s_{io}^{g-}| = u_{io} + v_{io}$, $s_{io}^{g-} = u_{io} - v_{io}$; $|s_{ro}^{g+}| = u_{ro} + v_{ro}$, $s_{ro}^{g+} = u_{ro} - v_{ro}$; and $|s_{fo}^{g-}| = u_{fo} + v_{fo}$, $s_{fo}^{g-} = u_{fo} - v_{fo}$, respectively. The ρ_o^g can be rewritten as

$$\rho_o^{g'} = \frac{\sum_{i\in I^S}\frac{(u_{io}+v_{io})}{x_{i0}} + \sum_{p=1}^{P}\sum_{i\in I_p^M}\frac{(u_{io}+v_{io})\delta_{io}^x}{x_{i0}} + \sum_{r\in R^S}\frac{(u_{ro}+v_{ro})}{y_{r0}} + \sum_{q=1}^{Q}\sum_{r\in R_q^M}\frac{(u_{ro}+v_{ro})\delta_{ro}^y}{y_{r0}} + \sum_{f\in F^S}\frac{(u_{fo}+v_{fo})}{b_{f0}} + \sum_{u=1}^{U}\sum_{f\in F_u^M}\frac{(u_{fo}+v_{fo})\delta_{fo}^b}{b_{fo}}}{S^I + P + S^R + Q + S^F + U}.$$

Because of $(u_{io} + v_{io})\delta_{io}^x$, $(u_{ro} + v_{ro})\delta_{ro}^y$, and $(u_{fo} + v_{fo})\delta_{fo}^b$, the model is still nonlinear, we utilize the method in Appendix A. We replace $u_{io} * \delta_{io}^x$, $v_{io} * \delta_{io}^x$, $u_{ro} * \delta_{ro}^y$, $v_{ro} * \delta_{ro}^y$, $u_{fo} * \delta_{fo}^b$ and $v_{fo} * \delta_{fo}^b$ with z_{io}^{xu}, z_{io}^{xv}, z_{ro}^{yu}, z_{ro}^{yv}, z_{fo}^{bu} and z_{fo}^{bv}, respectively. Then we rewrite $\rho_o^{g'}$ as

$$\rho_o^{g''} = \frac{\sum_{i\in I^S}\frac{(u_{io}+v_{io})}{x_{i0}} + \sum_{p=1}^{P}\sum_{i\in I_p^M}\frac{(z_{io}^{xu}+z_{io}^{xv})}{x_{i0}} + \sum_{r\in R^S}\frac{(u_{ro}+v_{ro})}{y_{r0}} + \sum_{q=1}^{Q}\sum_{r\in R_q^M}\frac{(z_{ro}^{yu}+z_{ro}^{yv})}{y_{r0}} + \sum_{f\in F^S}\frac{(u_{fo}+v_{fo})}{b_{f0}} + \sum_{u=1}^{U}\sum_{f\in F_u^M}\frac{(z_{fo}^{bu}+z_{fo}^{bv})}{b_{f0}}}{S^I + P + S^R + Q + S^F + U}.$$

For the sake of simplicity, we employ UB_{io}^x, LB_{io}^x, UB_{ro}^y, LB_{ro}^y, UB_{fo}^b, and LB_{fo}^b to respectively represent the $g_{io}^x - \min_{j\in J}\{x_{ij}\}$, $g_{io}^x - \max_{j\in J}\{x_{ij}\}$, $\max_{j\in J}\{y_{rj}\} - g_{ro}^y$, $\min_{j\in J}\{y_{rj}\} - g_{ro}^y$, $g_{fo}^b - \min_{j\in J}\{b_{fj}\}$, and $g_{fo}^b - \max_{j\in J}\{b_{fj}\}$ for DMU$_o$ according toProposition 4. Then the constraints are rewritten as

$$\left.\begin{aligned} LB_{io}^x \leqslant s_{io}^{g-} = u_{io} - v_{io} \leqslant UB_{io}^x, i \in I \\ LB_{ro}^y \leqslant s_{ro}^{g+} = u_{ro} - v_{ro} \leqslant UB_{ro}^y, r \in R \\ LB_{fo}^b \leqslant s_{fo}^{g-} = u_{fo} - v_{fo} \leqslant UB_{fo}^b, f \in F \end{aligned}\right\} \tag{C.2}$$

Then we express model (C.3) utilizing Propositions 2 and 4 as follows, given $\rho_o^{g''}$ and $\rho_o^{M'}$ in model (A.2).

$$\begin{aligned}
&\max \rho_o^{M'} \\
&\min \rho_o^{g''} \\
&s.t.\ (5.1 - 5.9) \\
&\sum_{j\in J}\lambda_j x_{ij} = g_{io}^x - (u_{io} - v_{io}), i \in I^S \\
&\sum_{j\in J}\lambda_j x_{ij} \leqslant g_{io}^x - (u_{io} - v_{io}) + M(1 - \delta_{io}^x), i \in I_p^M, p = 1, \cdots P \\
&\sum_{j\in J}\lambda_j x_{ij} \geqslant g_{io}^x - (u_{io} - v_{io}) - M(1 - \delta_{io}^x), i \in I_p^M, p = 1, \cdots P \\
&\sum_{j\in J}\lambda_j y_{rj} = g_{ro}^y + (u_{ro} - v_{ro}), r \in R^S \\
&\sum_{j\in J}\lambda_j y_{rj} \leqslant g_{ro}^y + (u_{ro} - v_{ro}) + M(1 - \delta_{ro}^y), r \in R_q^M, q = 1, \cdots Q \\
&\sum_{j\in J}\lambda_j x_{ij} \geqslant g_{ro}^y + (u_{ro} - v_{ro}) - M(1 - \delta_{ro}^y), r \in R_q^M, q = 1, \cdots Q \\
&\sum_{j\in J}\lambda_j b_{fj} = g_{fo}^b - (u_{fo} - v_{fo}), f \in F^S \\
&\sum_{j\in J}\lambda_j b_{fj} \leqslant g_{fo}^b - (u_{fo} - v_{fo}) + M(1 - \delta_{fo}^b), f \in F_u^M, u = 1, \cdots U \\
&\sum_{j\in J}\lambda_j b_{fj} \geqslant g_{fo}^b - (u_f - v_{fo}) - M(1 - \delta_{fo}^b), f \in F_u^M, u = 1, \cdots U \\
&(5.10 - 5.14) \\
&(A.2.1 - A.2.6) \\
&\delta_{io}^x * LB_{io}^x \leqslant z_{io}^{xu} - z_{io}^{xv} \leqslant \delta_{io}^x * UB_{io}^x, i \in I_p^M, p = 1, \cdots P \\
&(1 - \delta_{io}^x)LB_{io}^x \leqslant (u_{io} - v_{io}) - (z_{io}^{xu} - z_{io}^{xv}) \leqslant (1 - \delta_{io}^x)UB_{io}^x, i \in I_p^M, p = 1, \cdots P \\
&\delta_{ro}^y * LB_{ro}^y \leqslant z_{ro}^{yu} - z_{ro}^{yv} \leqslant \delta_{ro}^y * UB_{ro}^y, r \in R_q^M, q = 1, \cdots Q \\
&(1 - \delta_{ro}^y)LB_{ro}^y \leqslant (u_{ro} - v_{ro}) - (z_{ro}^{yu} - z_{ro}^{yv}) \leqslant (1 - \delta_{ro}^y)UB_{ro}^y, r \in R_q^M, q = 1, \cdots Q \\
&\delta_{fo}^b * LB_{fo}^b \leqslant z_{fo}^{bu} - z_{fo}^{bv} \leqslant \delta_{fo}^b * UB_{fo}^b, f \in F_u^M, u = 1, \cdots U
\end{aligned}$$

$$(1-\delta_{fo}^{b})LB_{fo}^{b} \leqslant (u_{fo}-v_{fo})-(z_{fo}^{bu}-z_{fo}^{bv}) \leqslant (1-\delta_{fo}^{b})UB_{fo}^{b}, f \in F_{u}^{M}, u=1,\cdots U$$
$$\lambda_{j} \geqslant 0, s_{io}^{-} \geqslant 0, s_{ro}^{+} \geqslant 0, s_{fo}^{-} \geqslant 0, j \in J, i \in I, r \in R, f \in F$$
$$z_{io}^{x} \geqslant 0, z_{ro}^{y} \geqslant 0, z_{fo}^{b} \geqslant 0, i \in I, r \in R, f \in F$$
$$z_{io}^{xu}, z_{io}^{xv} \geqslant 0, z_{ro}^{yu}, z_{ro}^{yv} \geqslant 0, z_{fo}^{bu}, z_{fo}^{bv} \geqslant 0, i \in I, r \in R, f \in F$$
$$u_{io} \geqslant 0, v_{io} \geqslant 0, u_{ro} \geqslant 0, v_{ro} \geqslant 0, u_{fo} \geqslant 0, v_{fo} \geqslant 0, i \in I, r \in R, f \in F \tag{C.3}$$

Proposition 5　Model (11) is equivalent to model (C.3).

Proof　Following the proof process of Proposition 2, we mainly focus on the multi-valued indicators and assume that the model has one multi-valued input, one multi-valued desirable output, and one multi-valued undesirable output, which indicates $P=1$, $Q=1$, and $U=1$. We also assume that $\delta_{ao}^{x}=1, a \in I_{1}^{M}$; $\delta_{co}^{y}=1, c \in R_{1}^{M}$; and $\delta_{do}^{b}=1, d \in F_{1}^{M}$. We only need to verify the part about goals because of Proposition 2. The objective function about goals of model (11) is

$$\frac{\sum_{i \in I^{S}} \frac{(u_{io}+v_{io})}{x_{io}}+\frac{(u_{ao}+v_{ao})}{x_{ao}}+\sum_{r \in R^{S}} \frac{(u_{ro}+v_{ro})}{y_{ro}}+\frac{(u_{co}+v_{co})}{y_{co}}+\sum_{f \in F^{S}} \frac{(u_{fo}+v_{fo})}{b_{fo}}+\frac{(u_{do}+v_{do})}{b_{do}}}{S^{I}+1+S^{R}+1+S^{F}+1}.$$

The corresponding constraints about goals for multi-valued inputs are $\sum_{j \in J} \lambda_{j} x_{aj} \leqslant g_{ao}^{x}-s_{ao}^{g-}$, $\sum_{j \in J} \lambda_{j} x_{aj} \geqslant g_{ao}^{x}-s_{ao}^{g-}$, $\sum_{j \in J} \lambda_{j} x_{ij} \leqslant g_{io}^{x}-s_{io}^{g-}+M, i \neq a \in I_{1}^{M}$, and $\sum_{j \in J} \lambda_{j} x_{ij} \geqslant g_{io}^{x}-s_{io}^{g-}-M, i \neq a \in I_{1}^{M}$. By Proposition 4, we have $LB_{io}^{x} \leqslant s_{io}^{g-}=u_{io}-v_{io} \leqslant UB_{io}^{x}, i \in I_{1}^{M}$. For multi-valued desirable outputs, the constraints on goals containing $\sum_{j \in J} \lambda_{j} y_{cj} \leqslant g_{co}^{y}+s_{co}^{g+}$, $\sum_{j \in J} \lambda_{j} y_{cj} \geqslant g_{co}^{y}+s_{co}^{g+}$, $\sum_{j \in J} \lambda_{j} y_{rj} \leqslant g_{ro}^{y}+s_{ro}^{g+}+M, r \neq c \in R_{1}^{M}$ and $\sum_{j \in J} \lambda_{j} y_{rj} \geqslant g_{ro}^{y}+s_{ro}^{g+}-M, r \neq c \in R_{1}^{M}$. Based on Proposition 4, we get $LB_{ro}^{y} \leqslant s_{ro}^{g+}=u_{ro}-v_{ro} \leqslant UB_{ro}^{y}, r \in R_{q}^{M}$. For multi-valued undesirable outputs, we have $\sum_{j \in J} \lambda_{j} b_{dj} \leqslant g_{do}^{b}-s_{do}^{g-}$, $\sum_{j \in J} \lambda_{j} b_{dj} \geqslant g_{do}^{b}-s_{do}^{g-}$, $\sum_{j \in J} \lambda_{j} b_{fj} \leqslant g_{fo}^{b}-s_{fo}^{g-}+M, f \neq d \in F_{1}^{M}$, and $\sum_{j \in J} \lambda_{j} b_{fj} \geqslant g_{fo}^{b}-s_{fo}^{g-}-M, f \neq d \in F_{1}^{M}$. We get $LB_{fo}^{b} \leqslant s_{fo}^{g-}=u_{fo}-v_{fo} \leqslant UB_{fo}^{b}, f \in F_{1}^{M}$ from Proposition 4.

In addition, since $z_{io}^{xu}=u_{io} * \delta_{io}^{x}, z_{io}^{xv}=v_{io} * \delta_{io}^{x}, i \in I_{1}^{M}$; $z_{ro}^{yu}=u_{ro} * \delta_{ro}^{y}, z_{ro}^{yv}=v_{ro} * \delta_{ro}^{y}, r \in R_{1}^{M}$; and $z_{fo}^{bu}=u_{fo} * \delta_{fo}^{b}, z_{fo}^{bv}=v_{fo} * \delta_{fo}^{b}, f \in F_{1}^{M}$, we have $u_{ao}=z_{ao}^{xu}, v_{ao}=z_{ao}^{xv}, a \in I_{1}^{M}$; $u_{co}=z_{co}^{yu}, v_{co}=z_{co}^{yv}, c \in R_{1}^{M}$; and $u_{do}=z_{do}^{bu}$, $v_{do}=z_{do}^{bv}, d \in F_{1}^{M}$. The objective function about goals of model (C.3) can be rewritten as $\frac{\sum_{i \in I^{S}} \frac{(u_{io}+v_{io})}{x_{io}}+\frac{(u_{ao}+v_{ao})}{x_{ao}}+\sum_{r \in R^{S}} \frac{(u_{ro}+v_{ro})}{y_{ro}}+\frac{(u_{co}+v_{co})}{y_{co}}+\sum_{f \in F^{S}} \frac{(u_{fo}+v_{fo})}{b_{fo}}+\frac{(u_{do}+v_{do})}{b_{do}}.}{S^{I}+1+S^{R}+1+S^{F}+1}$, which is the same as that of model (11).

We now discuss the relevant constraints about goals in model (C.3). The constraints of multi-valued inputs are $\sum_{j \in J} \lambda_{j} x_{aj} \leqslant g_{ao}^{x}-(u_{ao}-v_{ao})= g_{ao}^{x}-s_{ao}^{g-}$, $\sum_{j \in J} \lambda_{j} x_{aj} \geqslant g_{ao}^{x}-(u_{ao}-v_{ao})= g_{ao}^{x}-s_{ao}^{g-}$, $LB_{ao}^{x} \leqslant s_{ao}^{g-}=u_{ao}-v_{ao} \leqslant UB_{ao}^{x}$, and $\sum_{j \in J} \lambda_{j} x_{ij} \leqslant g_{io}^{x}-(u_{io}-v_{io})+M= g_{io}^{x}-s_{io}^{g-}+M, i \neq a \in I_{1}^{M}$, $\sum_{j \in J} \lambda_{j} x_{ij} \geqslant g_{io}^{x}-(u_{io}-v_{io})-M= g_{io}^{x}-s_{io}^{g-}-M$, $i \neq a \in I_{1}^{M}$, $LB_{io}^{x} \leqslant s_{io}^{g-}=u_{io}-v_{io} \leqslant UB_{io}^{x}, i \neq a \in I_{1}^{M}$. For the multi-valued desirable outputs, the constraints are $\sum_{j \in J} \lambda_{j} y_{cj} \leqslant g_{co}^{y}+(u_{co}-v_{co})= g_{co}^{y}+s_{co}^{g+}$, $\sum_{j \in J} \lambda_{j} y_{cj} \geqslant g_{co}^{y}+(u_{co}-v_{co})= g_{co}^{y}+s_{co}^{g+}$, $LB_{co}^{y} \leqslant s_{co}^{g+}=u_{co}-v_{co} \leqslant UB_{c}^{y}$, and $\sum_{j \in J} \lambda_{j} y_{rj} \leqslant g_{ro}^{y}+(u_{ro}-v_{ro})+M= g_{ro}^{y}+s_{ro}^{g+}+M, r \neq c \in R_{1}^{M}$, $\sum_{j \in J} \lambda_{j} y_{rj} \geqslant g_{ro}^{y}+(u_{ro}-v_{ro})-M= g_{ro}^{y}+s_{ro}^{g+}-M, r \neq c \in R_{1}^{M}$, $LB_{ro}^{y} \leqslant s_{ro}^{g+}=u_{ro}-v_{ro} \leqslant UB_{r}^{y}, r \neq c \in R_{1}^{M}$. When it comes to the multi-valued undesirable output, the constraints contain $\sum_{j \in J} \lambda_{j} b_{fj} \leqslant g_{do}^{b}-(u_{do}-v_{do})= g_{do}^{b}-s_{do}^{g-}$, $\sum_{j \in J} \lambda_{j} b_{fj} \geqslant g_{do}^{b}-(u_{do}-v_{do})= g_{do}^{b}-s_{do}^{g-}$, $LB_{do}^{b} \leqslant s_{do}^{g-}=u_{do}-v_{do} \leqslant UB_{do}^{b}$, and $\sum_{j \in J} \lambda_{j} b_{fj} \leqslant g_{fo}^{b}-(u_{fo}-v_{fo})+M= g_{fo}^{b}-s_{fo}^{g-}+M, f \neq d \in F_{1}^{M}$, $\sum_{j \in J} \lambda_{j} b_{fj} \geqslant g_{fo}^{b}-(u_{fo}-v_{fo})-M= g_{fo}^{b}-s_{fo}^{g-}-M, f \neq d \in F_{1}^{M}$, $LB_{fo}^{b} \leqslant s_{fo}^{g-}=u_{fo}-v_{fo} \leqslant UB_{fo}^{b}, f \neq d \in F_{1}^{M}$. We prove that both the constraints and the objective function about the goals are the same in the two models. Therefore, combining with Proposition 2, we have Proposition 5. When there are more multi-valued inputs/outputs, the analysis process is analogous.

Appendix D

We consider the $\mathrm{DMU}_t, t \in J$ under centralized decision-making case. We assume that $|s_{it}^{g-}| = u_{it} + v_{it}$, $s_{it}^{g-} = u_{it} - v_{it}$, $|s_{rt}^{g+}| = u_{rt} + v_{rt}$, $s_{rt}^{g+} = u_{rt} - v_{rt}$, $|s_{ft}^{g-}| = u_{ft} + v_{ft}$ and $s_{ft}^{g-} = u_{ft} - v_{ft}$ for input/output indicators. The ρ_t^g can be rewritten as

$$\frac{\sum_{i\in I^S}\frac{(u_{it}+v_{it})}{x_{it}} + \sum_{p=1}^{P}\sum_{i\in I_p^M}\frac{(u_{it}+v_{it})\delta_{it}^x}{x_{it}} + \sum_{r\in R^S}\frac{(u_{rt}+v_{rt})}{y_{rt}} + \sum_{q=1}^{Q}\sum_{r\in R_q^M}\frac{(u_{rt}+v_{rt})\delta_{rt}^y}{y_{rt}} + \sum_{f\in F^S}\frac{(u_{ft}+v_{ft})}{b_{ft}} + \sum_{u=1}^{U}\sum_{f\in F_u^M}\frac{(u_{ft}+v_f t)\delta_f^b}{b_{ft}}}{S^I + P + S^R + Q + S^F + U}.$$

We use z_{it}^{xu}, z_{it}^{xv}, z_{rt}^{yu}, z_{rt}^{yv}, z_{ft}^{bu} and z_{ft}^{bv} to represent $u_{it} * \delta_i^x$, $v_{it} * \delta_i^x$, $u_{rt} * \delta_r^y$, $v_{rt} * \delta_r^y$, $u_{ft} * \delta_f^b$ and $v_{ft} * \delta_f^b$ respectively. Then, we express $\rho_t^{g''}$ as

$$\frac{\sum_{i\in I^S}\frac{(u_{it}+v_{it})}{x_{it}} + \sum_{q=1}^{Q}\sum_{i\in I_p^M}\frac{(z_{it}^{xu}+z_{it}^{xv})}{x_{it}} + \sum_{r\in R^S}\frac{(u_{rt}+v_{rt})}{y_{rt}} + \sum_{q=1}^{Q}\sum_{r\in R_q^M}\frac{(z_{rt}^{yu}+z_{rt}^{yv})}{y_{rt}} + \sum_{f\in F^S}\frac{(u_{ft}+v_{ft})}{b_{ft}} + \sum_{u=1}^{U}\sum_{f\in F_u^M}\frac{(z_{ft}^{bu}+z_{ft}^{bv})}{b_{ft}}}{S^I + P + S^R + Q + S^F + U}.$$

Then, using Propositions 3 and 4, we rewrite the model as follows, given $\rho_t^{M'}$ in model (B.1) and $\rho_t^{g''}$.

$$\begin{gathered}
\max \sum_{t=1}^{n}\rho_t^{M'} \\
\min \sum_{t=1}^{n}\rho_t^{g''} \\
s.t.\ (8.1-8.9) \\
\sum_{j\in J}\lambda_{jt}x_{ij} = g_i^x - (u_{it}-v_{it}), t\in J, i\in I^S \\
\sum_{j\in J}\lambda_{jt}x_{ij} \leqslant g_i^x - (u_{it}-v_{it}) + M(1-\delta_i^x), t\in J, i\in I_p^M, p=1,\cdots P \\
\sum_{j\in J}\lambda_{jt}x_{ij} \geqslant g_i^x - (u_{it}-v_{it}) - M(1-\delta_i^x), t\in J, i\in I_p^M, p=1,\cdots P \\
\sum_{j\in J}\lambda_{jt}y_{rj} = g_r^y + (u_{rt}-v_{rt}), t\in J, r\in R^S \\
\sum_{j\in J}\lambda_{jt}y_{rj} \leqslant g_r^y + (u_{rt}-v_{rt}) + M(1-\delta_r^y), t\in J, r\in R_q^M, q=1,\cdots Q \\
\sum_{j\in J}\lambda_{jt}x_{ij} \geqslant g_r^y + (u_{rt}-v_{rt}) - M(1-\delta_r^y), t\in J, r\in R_q^M, q=1,\cdots Q \\
\sum_{j\in J}\lambda_{jt}b_{fj} = g_f^b - (u_{ft}-v_{ft}), t\in J, f\in F^S \\
\sum_{j\in J}\lambda_{jt}b_{fj} \leqslant g_f^b - (u_{ft}-v_{ft}) + M(1-\delta_f^b), t\in J, f\in F_u^M, u=1,\cdots U \\
\sum_{j\in J}\lambda_{jt}b_{fj} \geqslant g_f^b - (u_{ft}-v_{ft}) - M(1-\delta_f^b), t\in J, f\in F_u^M, u=1,\cdots U \\
(8.10-8.14) \\
(B.1.1-B.1.6) \\
\delta_i^x * LB_i^x \leqslant z_{it}^{xu} - z_{it}^{xv} \leqslant \delta_i^x * UB_i^x, t\in J, i\in I_p^M, p=1,\cdots P \\
(1-\delta_i^x)LB_i^x \leqslant (u_{it}-v_{it}) - (z_{it}^{xu}-z_{it}^{xv}) \leqslant (1-\delta_i^x)UB_i^x, t\in J, i\in I_p^M, p=1,\cdots P \\
\delta_r^y * LB_r^y \leqslant z_{rt}^{yu} - z_{rt}^{yv} \leqslant \delta_r^y * UB_r^y, t\in J, r\in R_q^M, q=1,\cdots Q \\
(1-\delta_r^y)LB_r^y \leqslant (u_{rt}-v_{rt}) - (z_{rt}^{yu}-z_{rt}^{yv}) \leqslant (1-\delta_r^y)UB_r^y, t\in J, r\in R_q^M, q=1,\cdots Q \\
\delta_f^b * LB_f^b \leqslant z_{ft}^{bu} - z_{ft}^{bv} \leqslant \delta_f^b * UB_f^b, t\in J, f\in F_u^M, u=1,\cdots U \\
(1-\delta_f^b)LB_f^b \leqslant (u_{ft}-v_{ft}) - (z_{ft}^{bu}-z_{ft}^{bv}) \leqslant (1-\delta_f^b)UB_f^b, t\in J, f\in F_u^M, u=1,\cdots U \\
\lambda_{jt} \geqslant 0, s_{it}^- \geqslant 0, s_{rt}^+ \geqslant 0, s_{ft}^- \geqslant 0, t\in J, j\in J, i\in I, r\in R, f\in F \\
z_{it}^x \geqslant 0, z_{rt}^y \geqslant 0, z_{ft}^b \geqslant 0, i\in I, r\in R, f\in F, t\in J \\
z_{it}^{xu}, z_{it}^{xv} \geqslant 0, z_{rt}^{yu}, z_{rt}^{yv} \geqslant 0, z_{ft}^{bu}, z_{ft}^{bv} \geqslant 0, i\in I, r\in R, f\in F, t\in J \\
u_{it} \geqslant 0, v_{it} \geqslant 0, u_{rt} \geqslant 0, v_{rt} \geqslant 0, u_{ft} \geqslant 0, v_{ft} \geqslant 0, i\in I, r\in R, f\in F, t\in J
\end{gathered} \tag{D.1}$$

Proposition 6 Model (D.1) is equivalent to model (14).

Proof The process is similar to that of Proposition 5, seen in detail in Appendix C.

非期望产出存在时考虑多值指标的目标导向 DEA 方法

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摘要:数据包络分析(DEA)是一种重要的数据驱动方法,可用于对一组有多个投入和多个产出的同质决策单元(DMU)进行绩效评价和改进,这些投入和产出称为绩效指标. 某些绩效指标与传统 DEA 模型中使用的具有单个值的绩效指标不同,由于其定义或衡量标准不同,可能对应多个值,称为多值指标. 绩效指标通常反映了 DMU 的当前生产状态,忽略了决策者的目标. 为此提出了基于松弛的 DEA 改进模型,用于处理多值指标,可以得到帕累托最优解,并考虑了分散决策和集中决策两种常见决策场景. 此外,我们通过考虑决策者目标得到扩展模型,以帮助 DMU 提高绩效并尽可能达到决策者的目标. 基于松弛的方法和进一步考虑决策者目标增强了模型对 DMU 的区分能力,并为某些指标提供更符合实际的改进. 通过中国长江三角洲 22 个城市的实例应用,说明了我们提出的模型的有效性和实用性.

关键词:数据包络分析;多值指标;目标;分散决策;集中决策