

Transit-time scattering of radiation belt electrons by off-equatorially generated magnetosonic waves

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Abstract: In the inner magnetosphere, the magnetosonic waves have been proposed to transit-time scatter the radiation belt electrons over a broad range of energy and pitch-angle. Recent observations have shown that magnetosonic waves can be generated in the off-equatorial plasmasphere, whose latitudinal coverage and propagation angle are significantly different from the traditional magnetosonic waves confined near the equator. Using our previously-developed test-particle code, we here investigate the possible transit-time scattering of radiation belt electrons by the off-equatorially generated magnetosonic waves. Our results indicate that the transit-time scattering is primarily related to the perturbation near the edge of the finite wave train. The extending of wave occurrence latitudes causes the shrinking of the equatorial pitch-angle range of the transit-time scattering. Compared to the near-equatorially confined magnetosonic waves, the off-equatorially generated magnetosonic waves produce ignorable pitch-angle perturbations but slightly stronger energy perturbations.

Keywords: magnetosonic wave; radiation belt; transit-time scattering

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1 Introduction

Magnetosonic waves are the low-frequency, highly-compressional electromagnetic emissions in the inner magnetosphere. About 50 years ago, Russell et al.^[1] firstly reported the magnetosonic waves on the basis of OGO 3 observations. In recent years, these waves have received increased attention for their potential contribution to the radiation belt electron dynamics. Horne et al.^[2] showed that magnetosonic waves can cause Landau resonant acceleration of electrons between ~10 keV and several MeV in the outer radiation belt. Shprits^[3] proposed the bounce-resonance between magnetosonic waves and radiation belt electrons. Bortnik et al.^[4] suggested the transit-time scattering of radiation belt electrons by the near-equatorially confined magnetosonic waves. Along with these theoretical insights above, a number of observational studies have linked the radiation belt electron dynamics, such as the formation of electron butterfly distributions^[5-9], to the concurring magnetosonic waves.

The efficiency of all the three mechanisms mentioned above depends on the latitudinal coverage

and propagation angle of magnetosonic waves. In previous studies, the magnetosonic waves are usually considered to be generated by the ion Bernstein mode instability at the quasi-perpendicular normal angle near the equator^[10-13]. In the numerical calculations^[14,15], these waves are assumed to be confined within $\pm 3^\circ$ latitude away from the magnetic equator. Recent observations and ray-tracing simulations^[16] have found that the off-equatorial ion Bernstein mode instability can produce magnetosonic waves near the plasmopause. Compared to the traditional near-equatorially generated magnetosonic waves, these off-equatorially generated waves can bounce over a broader range of latitude and have normal angles further away from 90° . How the off-equatorially generated magnetosonic waves affect the radiation belt electrons remains unclear.

In this study, we perform test-particle simulations to investigate the transit-time scattering of radiation belt electrons by magnetosonic waves. We show that, compared to the near-equatorially confined magnetosonic waves, the off-equatorially generated magnetosonic waves produce the ignorable pitch-angle perturbations but the slightly stronger energy

perturbations.

2 Magnetosonic wave properties

Figure 1 presents a revisit of the magnetosonic waves observed by Van Allen Probe A on 2013-08-13^[16]. These waves appeared as a band of enhanced electromagnetic emissions centering at the local lower hybrid resonance frequency $f_{\text{thr}}^{\text{RBSP}}$, qualitatively different from the traditional magnetosonic waves below the lower hybrid resonance frequency. According to the cold plasma wave dispersion relation, these waves extending above the lower hybrid resonance frequency should have normal angles far away from 90° . Such normal angles allow the magnetosonic waves propagate along the magnetic field lines toward higher latitudes than the observer's latitude of $\sim 15^\circ$.

Our previous linear instability analyses^[16] have shown that these magnetosonic waves mainly grow at latitudes $>15^\circ$ when the normal angles have moved close to 90° . The subsequent ray-tracing simulations^[16] further confirmed that these waves initialized with the "appropriate" normal and azimuthal angles can accumulatively gain energy from the hot protons over latitudes $|\lambda| < 30^\circ$ during the bounce-drift process inside the plasmasphere. As shown in Figure 2 (a), the magnetosonic wave power peaked at ~ 170 Hz around 19:30 UT. Figure 2 (b) demonstrates the normal-angle variations along the latitude for the 170 Hz wave with the "super gain" during bounce-drift propagation^[16]. The normal angle ψ is about 85° at $\lambda = 0^\circ$, and increases to $\sim 90^\circ$ at $\lambda \approx 25^\circ$. The latitudinal dependence of normal angle can be approximately written as

$$\psi = 1.4877674 + 0.43245313\lambda^2 \quad (1)$$

where both ψ and λ are in unit of radian. We next

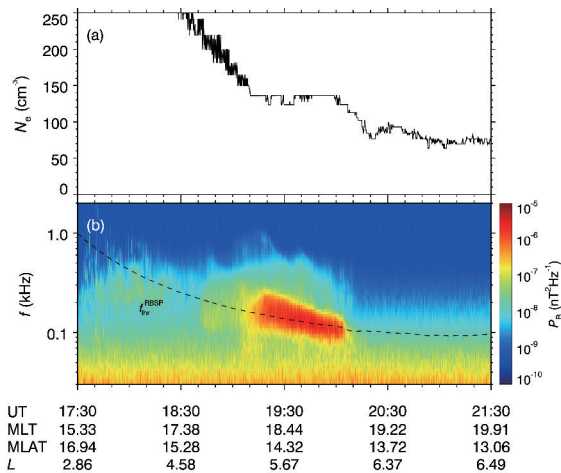


Figure 1. Van Allen Probes observation of magnetosonic waves generated off-equatorially in the high-density plasmasphere^[16]: (a) cold electron density N_e ; (b) wave magnetic power spectral density P_B .

investigate the transit-time scattering of radiation belt electrons by this monochromatic wave of 170 Hz.

3 Test-particle simulations

We use the full test-particle code developed by Su et al^[17] to investigate the interaction between magnetosonic waves and radiation belt electrons. This full test-particle code is an improved version of the early gyro-averaged test-particle code^[18-20]. With this full test-particle code, there have been a series of studies of nonlinear interactions between electromagnetic ion cyclotron waves and magnetospheric particles^[21,22]. For an arbitrary particle with the charge q , rest mass m , Lorentz factor γ , position vector \mathbf{r} and relativistic momentum vector \mathbf{p} , its trajectory is determined by the following equations:

$$\frac{d\mathbf{r}}{dt} = \frac{\mathbf{p}}{\gamma m} \quad (2)$$

$$\frac{d\mathbf{p}}{dt} = q[\mathbf{E}_w + \frac{\mathbf{p}}{\gamma m} \times (\mathbf{B}_0 + \mathbf{B}_w)] \quad (3)$$

To exclude the azimuthal drift of the test-particle, the background magnetic field \mathbf{B}_0 is simplified as a "magnetic bottle" in the Cartesian coordinate system^[23-26]:

$$\mathbf{B}_0 = -\frac{x}{2} \frac{dB_D}{dz} \mathbf{e}_x - \frac{y}{2} \frac{dB_D}{dz} \mathbf{e}_y + B_D \mathbf{e}_z \quad (4)$$

$$B_D = \frac{B_E \sqrt{1 + 3\sin^2 \lambda}}{L^3 \cos^6 \lambda}, \quad B_E = 31200 \text{ nT} \quad (5)$$

$$\frac{dz}{d\lambda} = R_E L \cos \lambda \sqrt{1 + 3\sin^2 \lambda} \quad (6)$$

with the z -component to be equal to the original Earth's magnetic field magnitude depending on L and λ , and the additionally introduced x - and y -components to

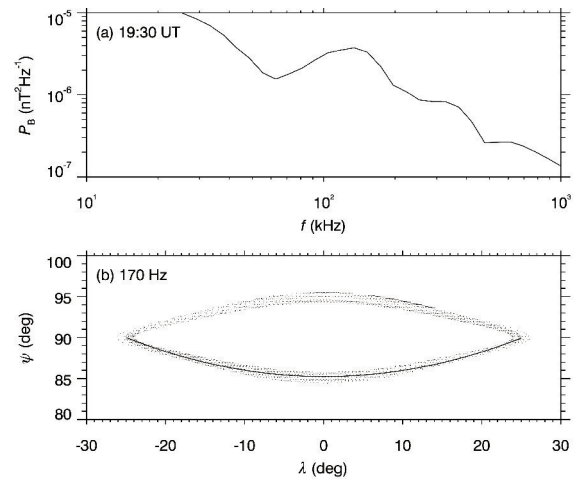


Figure 2. Magnetosonic wave properties around 19:30 UT: (a) frequency-dependent magnetic power spectral density P_B ; (b) latitude-dependent normal-angle ψ of 170 Hz wave from the previous ray-tracing simulations (dots)^[16] and from the polynomial model (line).

maintain the divergence-free condition. Because the x - and y -components are much weaker than the z -component, this model can reproduce the characteristic of wave-particle interaction in the realistic dipole field. For an arbitrary wave, the angular frequency ω , wave vector $\mathbf{k} = k\sin\psi\mathbf{e}_x + k\cos\psi\mathbf{e}_z$ and normal angle ψ obey the well-known cold-plasma wave dispersion relation^[27]. Its electromagnetic fields are listed as follows:

$$\mathbf{E}_w = -E_w^x \sin\phi \mathbf{e}_x - E_w^y \cos\phi \mathbf{e}_y - E_w^z \sin\phi \mathbf{e}_z \quad (7)$$

$$\mathbf{B}_w = B_w^x \cos\phi \mathbf{e}_x - B_w^y \sin\phi \mathbf{e}_y + B_w^z \cos\phi \mathbf{e}_z \quad (8)$$

with the wave phase angle

$$\phi(t, x, z) \equiv \int \mathbf{k} \cdot d\mathbf{r} - \int \omega dt \quad (9)$$

According to the observations in Figure 1^[16], we perform the test-particle simulations at $L = 5.7$ in a dipole field. We have taken into account the latitudinal dependence of background electron density^[28]

$$N_e = N_{\text{eq}} \cos^{-2\eta} \lambda \quad (10)$$

with the latitudinal index $\eta = 0.5$ and the equatorial density $N_{\text{eq}} = 115 \text{ cm}^{-3}$. We select a monochromatic magnetosonic wave of 170 Hz, whose normal angle varies with latitude (Equation (1)). The wave magnitude is assumed to be

$$B_w = B_{w0} \frac{[1 - \tanh(\frac{\lambda - \lambda_{\text{max}}}{\Delta\lambda})][1 + \tanh(\frac{\lambda + \lambda_{\text{max}}}{\Delta\lambda})]}{4} \quad (11)$$

with $B_{w0} = 20 \text{ pT}$ and $\Delta\lambda = 0.02 \text{ rad}$. Specifically, $B_w \approx B_{w0}$ when $|\lambda| < \lambda_{\text{max}}$; B_w drops down toward zero when $|\lambda| \sim \lambda_{\text{max}}$; $B_w = 0$ when $|\lambda| > \lambda_{\text{max}}$.

Figure 3 presents the variations in the equatorial pitch-angle and energy of electrons under the action of

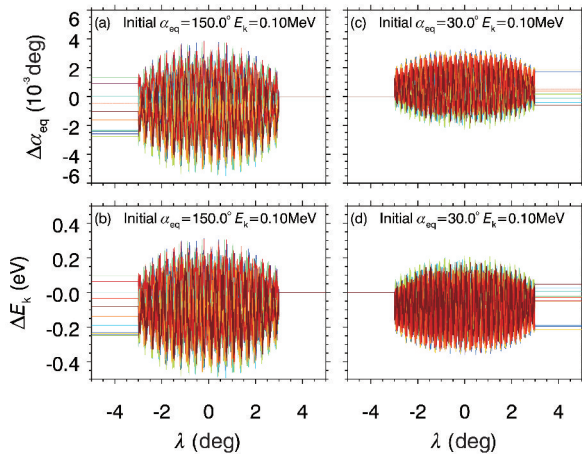


Figure 3. Counter-streaming (left) and co-streaming (right) scattering of radiation belt electrons by magnetosonic waves confined within $|\lambda| \leq 3^\circ$: (a, c) equatorial pitch-angle change $\Delta\alpha_{\text{eq}}$; (b, d) kinetic energy change ΔE_k . The lines are color-coded according to the initial gyro-phase of test particles.

the magnetosonic wave confined near the equator ($\lambda_{\text{max}} = 3^\circ$). The counter-streaming and co-streaming interactions correspond to the situations of $\mathbf{k} \cdot \mathbf{p} < 0$ and $\mathbf{k} \cdot \mathbf{p} > 0$. All the test-electrons are initialized with the energy of $E_k = 0.1 \text{ MeV}$ and the equatorial pitch-angle $\alpha_{\text{eq}} = 30^\circ$, and their initial gyro-phases are uniformly distributed in the range of $0 - 2\pi$. In the high density plasmasphere, the wave parallel phase velocity is much smaller than the electron parallel velocity. For both counter-streaming and co-streaming interactions, the test electrons have experienced the perturbations of waves in about 10 periods. In each full wave period, the non-resonant perturbations average to zero. The net variations of pitch-angle and energy are essentially related to the perturbation near the edge of the finite wave train. Specifically, the net variations in the pitch-angle and energy for both situations are comparable: $\Delta\alpha_{\text{eq}} \approx 0.002^\circ$ and $\Delta E_k \approx 0.25 \text{ eV}$.

In Figure 4, we have enlarged the latitudinal range of magnetosonic waves by setting $\lambda_{\text{max}} = 23^\circ$. Clearly, compared to the near-equatorially confined magnetosonic waves, the off-equatorially generated magnetosonic waves have caused the slightly higher energy variation ΔE_k but ignorable pitch-angle variation $\Delta\alpha_{\text{eq}} \approx 0.0002^\circ$. In our simulation, the wave magnetic field amplitude is assumed to be constant at latitudes $|\lambda| < \lambda_{\text{max}}$. As the latitude increases, the background magnetic field and then the wave electric field increases. Hence, the non-resonant perturbation in energy tends to increase. As discussed in our previous work^[17], the gyro-averaged change rate of the equatorial pitch angle $d\alpha_{\text{eq}}/dt$ is inversely proportional

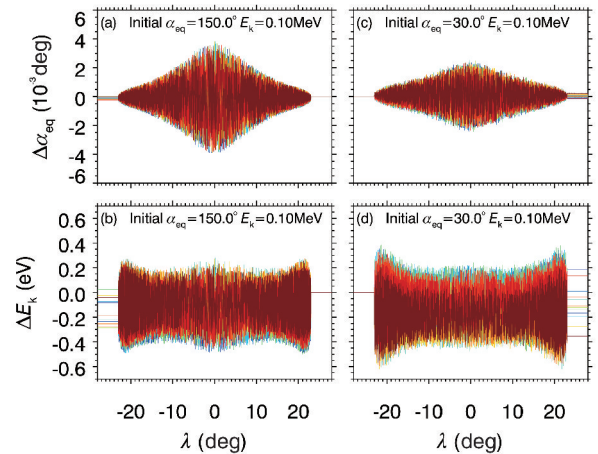


Figure 4. Counter-streaming (left) and co-streaming (right) scattering of radiation belt electrons by magnetosonic waves confined within $|\lambda| \leq 23^\circ$: (a, c) equatorial pitch-angle change $\Delta\alpha_{\text{eq}}$; (b, d) kinetic energy change ΔE_k . The lines are color-coded according to the initial gyro-phase of test particles.

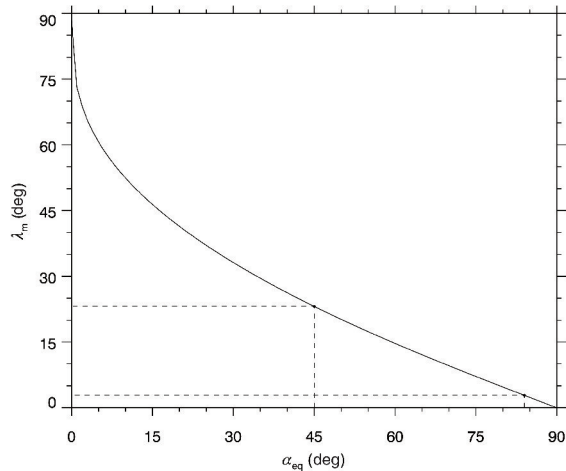


Figure 5. Dependence of mirror latitude λ_m on equatorial pitch-angle α_{eq} .

to $\tan \alpha$, with the local pitch-angle α . At higher latitudes, the local pitch-angle α moves closer to 90° and the resulted non-resonant perturbation of α_{eq} is weaker.

Figure 5 plots the dependence of the mirror latitude λ_m on the equatorial pitch-angle α_{eq} . For the electrons always in the field of wave train, the non-resonant perturbations would not accumulate. To experience the accumulative, non-resonant, transit-time scattering, the electrons have to go through the edges of the wave train. Clearly, the electrons with larger α_{eq} have lower λ_m . For $\lambda_{max} = 3^\circ$ and 23° , the corresponding thresholds are $\alpha_{eq} \approx 83^\circ$ and 45° . In other words, with the occurrence latitude λ_{max} of magnetosonic waves extending from 3° to 23° , the equatorial pitch-angle range of the transit-time scattering shrinks from $\alpha_{eq} < 83^\circ$ to $\alpha_{eq} < 45^\circ$.

4 Conclusions

Our recent study^[16] has illustrated the generation of magnetosonic waves by the off-equatorial hot protons inside the plasmapause. We here investigate the transit-time scattering of radiation belt electrons by such off-equatorially generated magnetosonic waves. The main results are listed as follows:

(I) In the high-density plasmasphere, the so-called transit-time scattering is related to the non-resonant perturbations near the edge of the wave train. In response to the extension of wave occurrence latitude from 3° to 23° , the equatorial pitch-angle range of transit-time scattering shrinks from $< 83^\circ$ to $< 45^\circ$.

(II) Compared to the near-equatorially confined magnetosonic waves, the off-equatorially generated magnetosonic waves cause the ignorable equatorial pitch-angle perturbations but the slightly larger energy perturbations.

Our present simulations are based on a specific event of magnetosonic waves generated off-equatorially^[16]. The change of wave characteristics and background conditions would alter the perturbations in both pitch-angle and energy. For simplicity, our present model includes only a monochromatic wave, rather than a group of waves representing the observed magnetosonic band. As shown in our simulations, the transit-time scattering is mainly related to the non-resonant perturbations near the edge of the wave train. A superposition of many waves would not qualitatively change the picture, and a quantitative study is left for future.

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Conflict of interest

The authors declare no conflict of interest.

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非赤道区域激发的磁声波对辐射带电子的渡越时间散射效应

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摘要: 在内磁层中, 磁声波被认为能够通过渡越时间散射影响很大能量以及投掷角范围的辐射带电子. 近期观测研究发现, 磁声波可以在等离子体层内的非赤道区域被激发并且在纬度覆盖范围、法向角变化等方面都与传统的赤道区域磁声波有显著不同. 利用之前发展的试验粒子模型, 我们研究了非赤道磁声波对辐射带电子的渡

越时间散射效应.发现该效应主要发生在波列的边缘.波动纬度覆盖范围的扩展造成了该效应的有效赤道投掷角范围的收缩.相比于赤道区域磁声波,非赤道激发的磁声波可以产生更强的能量扰动以及可忽略的投掷角扰动.

关键词: 磁声波;辐射带;渡越时间散射