第51卷第2期 中国科学技术大学学报 Vol. 51, No. 2
2021年2月 JOURNAL OF UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA Feb. 2021
Received: 2020-12-09; Revised: 2021-01-30 doi:10.52396/JUST-2020-0019

The asymptotic properties of least square estimators in the linear errors-in-variables regression model with φ-mixing errors

DENG Xin[1], TIAN Chunyu[2], GE Meimei[1], YE Jing[1], DING Yang[1], WU Yi[3*]
1. School of Mathematical and Finance, Chuzhou University, Chuzhou 239000, China;
2. China Electronics Technology Group Corporation No.58 Research Institute, Nanjing 210000, China;
3. School of Big Data and Artificial Intelligence, Chizhou University, Chizhou 247000, China
* Corresponding author. E-mail: wuyi8702@163.com

Abstract: The simple linear errors-in-variables (EV) model with φ-mixing random errors was mainly studied. By using the central limit theorem and the Marcinkiewicz-type strong law of large numbers for the φ-mixing sequence, the asymptotic normality of the least square (LS) estimators for the unknown parameters were established under some mild conditions. In addition, based on the strong convergence for weighted sums of φ-mixing random variables, the strong consistency of the LS estimators were obtained. Finally, the simulation study was provided to verify the validity of the theoretical results.
Keywords: EV model; asymptotic normality; strong consistency; LS estimator; φ-mixing sequence
CLC number: O211.4 **Document code**: A
Mathematics Subject Classification: 62F12; 62F05

1 Introduction

Consider the following simple linear errors-in-variables (EV) model:

$$\eta_i = \theta + \beta x_i + \varepsilon_i,\ \xi_i = x_i + \delta_i,\ 1 \leqslant i \leqslant n \quad (1)$$

where θ and β are unknown parameters; $(\varepsilon_1, \delta_1)$, $(\varepsilon_2, \delta_2), \ldots$ are random errors with mean zero; $x_1, x_2, \ldots$ are unobservable; $\xi_i, \eta_i, i = 1, 2, 3, \ldots$ are observable. From the formula (1), we have

$$\eta_i = \theta + \beta\xi_i + X_i,\ X_i = \varepsilon_i - \beta\delta_i,\ 1 \leqslant i \leqslant n \quad (2)$$

We consider formally (2) as a usual regression model of η_i on ξ_i, and get the least square (LS) estimators of θ and β:

$$\widehat{\beta}_n = \frac{\sum_{i=1}^{n}(\xi_i - \bar{\xi}_n)(\eta_i - \bar{\eta}_n)}{\sum_{i=1}^{n}(\xi_i - \bar{\xi}_n)^2},\ \widehat{\theta}_n = \bar{\eta}_n - \widehat{\beta}_n\bar{\xi}_n \quad (3)$$

where $\bar{\xi}_n = \frac{1}{n}\sum_{i=1}^{n}\xi_i$, and $\bar{\eta}_n, \bar{\delta}_n, \bar{x}_n$ can be similarly defined.

The linear EV model is also called the measurement error model, that is, both independent variables and dependent variables have measurement errors. Since the model was proposed in the 20th century, it has been studied and applied extensively. Based on the finite sample distribution theory, Mittag[1] studied the estimating parameters; Fuller[2] made a detailed study of the linear EV model with measurement errors. Under the independent errors, Liu and Chen[3] discussed the consistency of LS estimators in the linear EV model, and proved the necessary and sufficient condition for $\widehat{\beta}_n$ to be a strong and weak consistent estimator of β: $\lim_{n\to\infty} n^{-1}S_n = \infty$, where $S_n = \sum_{i=1}^{n}(x_i - \bar{x}_n)^2$; Miao et al.[4] and Miao and Yang[5] obtained the central limit theorem and the law of iterated logarithm for the LS estimators $\widehat{\theta}_n$ and $\widehat{\beta}_n$ in the model (1); Miao et al.[6] studied the consistency and asymptotic normality of $\widehat{\beta}_n$ and $\widehat{\theta}_n$ under weaker conditions, which improve the corresponding results of Refs. [3] and [4]; under the dependent errors, Fan et al.[7] established the strong consistency, mean square consistency and the asymptotic normality of $\widehat{\theta}_n$ and $\widehat{\beta}_n$ with stationary α-mixing errors; Yang[8] investigated the asymptotic normality of the LS estimators of unknown parameters under the assumption that the errors are a sequence of stationary positively associated (PA) random variables; Miao et al.[9] considered the asymptotic normality and the strong consistency of the LS estimators of θ and β under negatively associated (NA) random errors; Wang et al.[10] and Wang et al.[11] studied the complete

Citation: DENG Xin, TIAN Chunyu, GE Meimei, et al. The asymptotic properties of least square estimators in the linear errors-in-variables regression model with φ-mixing errors. J. Univ. Sci. Tech. China, 2021, 51(2): 164−172.

consistency, strong and weak consistency in the model (1) with negatively superadditive dependent (NSD), respectively; under weakly negative dependent (WND) random errors, Wang et al. [12] obtained the strong consistency and complete consistency of the LS estimators, which generalize the corresponding ones for the NA random variables; by using the complete convergence for weighted sums of a class of random variables, Shen[13] gave the complete consistency of the LS estimators $\hat{\theta}_n$ and $\hat{\beta}_n$ with martingale difference (MD) errors, and also studied the mean consistency, which generalize the corresponding ones for independent random variables and some dependent random variables.

In this paper, we investigate a much wider dependent error structure: the φ-mixing random errors, and study the asymptotic normality and the strong consistency of the LS estimators (3) for the unknown parameters θ and β in a simple linear EV model (1). Now, let us recall the concept of the φ-mixing random variables.

Let $\{X_n, n\geqslant 1\}$ be a sequence of random variables defined on a fixed probability space (Ω, F, P). Define $F_n^m=\sigma(X_i, n\leqslant i\leqslant m)$.

Definition 1 A sequence $\{X_n, n\geqslant 1\}$ of random variables is said to be a φ-mixing sequence, if

$$\varphi(n) = \sup_{k\geqslant 1} \sup_{A\in F_1^k, B\in F_{k+n}^{\infty}, P(A)>0} |P(B|A) - P(B)| \downarrow 0, \quad n\to\infty.$$

The concept of the φ-mixing random variables was first introduced by Dobrushin[14] in the Markov chain, and has subsequently been studied by many scholars. For example, Badu et al. [15] obtained the uniform and non-uniform Berry-Esseen bounds for standardized sums of non-stationary φ-mixing random variables; Utev[16] studied the central limit theorem for φ-mixing arrays of random variables; Kiesel[17] obtained the almost sure convergence of stationary φ-mixing sequences of random variables by summability methods; Hu and Wang[18] investigated the large deviations of sums of the φ-mixing sequence, and obtained the optimal upper bounds; Yang et al. [19] derived the Berry-Esseen bound of sample quantiles for the φ-mixing random variables under some weak conditions; Shen et al. [20] studied the complete convergence for non-stationary φ-mixing random variables, and got the Baum-Katz-type theorem and Hsu-Robbins-type theorem for φ-mixing random variables. For more details about φ-mixing sequences, one can refer to Refs. [21], [22] and so on. By using the central limit theorem and the Marcinkiewicz-type strong law of large numbers for the φ-mixing sequence, the paper will establish the asymptotic normality of the LS estimators $\hat{\theta}_n$ and $\hat{\beta}_n$ in the model (1) under the assumptions that the random errors are the identically distributed φ-mixing sequence of random variables. Moreover, the strong consistency will be investigated based on the strong convergence for weighted sums of φ-mixing random variables.

Throughout the paper, assume that $\{\varepsilon_i, i\geqslant 1\}$ and $\{\delta_i, i\geqslant 1\}$, which are identically distributed sequences of φ-mixing random variables with mixing coefficients $\varphi(i)$, are independent of each other. And assume that $E\varepsilon_i = E\delta_i = 0$ and $\sum_{i=1}^{\infty}\varphi^{1/2}(i)<\infty$. All limits are taken as the sample size $n\to\infty$, unless it is specially mentioned.

1 Main results

In this section, we give the asymptotic normality and the strong consistency of the LS estimators $\hat{\beta}_n$ and $\hat{\theta}_n$ for unknown parameters β and θ.

Theorem 1 In the model (1), let $S_n = \sum_{i=1}^{n}(x_i - \bar{x}_n)^2$, and assume that the following conditions are satisfied:

$$\lim_{n\to\infty}\frac{n}{\sqrt{S_n}} = 0 \tag{4}$$

$$\lim_{n\to\infty} r_n = 0, \text{ where } r_n = \max_{1\leqslant i\leqslant n}\frac{|x_i - \bar{x}_n|}{\sqrt{S_n}} \tag{5}$$

and there exists a constant $c>0$ such that:

$$|x_i - x_j| \leqslant c|i-j|, \ \forall\ 1\leqslant i<j\leqslant n \tag{6}$$

Furthermore, let $X_i = \varepsilon_i - \beta\delta_i$, $i\geqslant 1$, $E\varepsilon_1^2<\infty$, $E\delta_1^2<\infty$, and assume that

$$\sigma^2 = EX_1^2 + 2\sum_{i=2}^{\infty}EX_1X_i > 0 \tag{7}$$

and

$$\sum_{j=2}^{n}(j-1)|EX_1X_j| = o\left(\frac{1}{l_n}\right) \tag{8}$$

where $l_n = r_n\max\{r_n, n/\sqrt{S_n}\}$. Then,

$$\frac{\sqrt{S_n}}{\sigma}(\hat{\beta}_n - \beta) \xrightarrow{D} N(0,1).$$

Theorem 2 Under the conditions of Theorem 1, assume that

$$\frac{S_n}{n(\bar{x}_n)^2}\to\infty \tag{9}$$

and

$$\sum_{j=2}^{\infty}|\mathrm{Cov}(X_1, X_j)| < \infty \tag{10}$$

Then,

$$\frac{\sqrt{n}}{\sigma}(\hat{\theta}_n - \theta) \xrightarrow{D} N(0,1).$$

Remark 1 Under certain mixing coefficients and moment conditions, Theorems 1 and 2 are also true for α-mixing and ρ-mixing random errors. For example, if $\{(\varepsilon_i, \delta_i), i\geqslant 1\}$ are identically distributed sequences of the α-mixing random variables with $\alpha(n) =$

$O\left(\frac{1}{n^{2(2+p)/(p-1)}\log^t n}\right)$, $t > \frac{3(2+p)}{p-1}$, $E|\varepsilon_1|^{2+p} < \infty$, $E|\delta_1|^{2+p}<\infty$ for some $p>1$, Theorems 1 and 2 will be held by Theorem 2.2 of Ref. [23] and Corollary 2.5 of Ref. [24]. Note that the moment conditions here are stronger than Theorems 1 and 2 in this paper.

Theorem 3 Under the model (1), assume that $E|\varepsilon_1|^p<\infty$, $E|\delta_1|^p<\infty$ for some $p>1/\delta, 0<\delta\leqslant 1/2$. Let $\tau>0$. If

$$\max_{1\leqslant i\leqslant n}\frac{|x_i-\bar{x}_n|}{n^{\tau-\delta}\sqrt{S_n}} = O(1), \frac{n^\tau}{\sqrt{S_n}} = O(1), \frac{n^{1-\tau}}{\sqrt{S_n}}\to 0 \quad (11)$$

Then,

$$\frac{\sqrt{S_n}}{n^\tau}(\hat{\beta}_n-\beta)\to 0 \text{ a. s. }.$$

Theorem 4 Under the assumptions of Theorem 3, if

$$\frac{n^{\tau+\nu}}{\sqrt{S_n}}|\bar{x}_n| = O(1) \quad (12)$$

for some $\nu\in(0,1/2)$, then,

$$n^\nu(\hat{\theta}_n-\theta)\to 0 \text{ a. s. }.$$

2 Simulation

In the subsection, we will carry out simulations to study the numerical performance of the asymptotic normality results and the strong consistency results.

The data are generated from model (1). For the fixed positive integer m, let $e_i \overset{\text{i. i. d.}}{\sim} N(0,\sigma_0^2)$, where $\sigma_0^2 = 1/(m+1)$. Let $\varepsilon_i = \sum_{k=0}^{m} e_{i+k}$ and $\delta_i = \sum_{k=0}^{m} e_{i+k}$ for each $i\geqslant 1$, then $\{\varepsilon_i, i\geqslant 1\}$ and $\{\delta_i, i\geqslant 1\}$ are sequences of the m-dependent random variables, thus they are also φ-mixing random variables with $\varepsilon_i\sim N(0,1)$ and $\delta_i\sim N(0,1)$. Set $\beta=2, \theta=4, m=4$ and $x_i=(-1)^i\frac{i}{n^{0.3}}$ for all $1\leqslant i\leqslant n$. We can calculate $\sigma=\sqrt{5}$. By taking the sample size n as $n=300$, 600, 900, 1200 respectively, we compute $\frac{\sqrt{S_n}}{\sigma}(\hat{\beta}_n-\beta)$ and $\frac{\sqrt{n}}{\sigma}(\hat{\theta}_n-\theta)$ for 1000 times and present the Q-Q plots of them in Figures 1–4. It is easily seen that the Q-Q plots show a good fit of them to normal distribution.

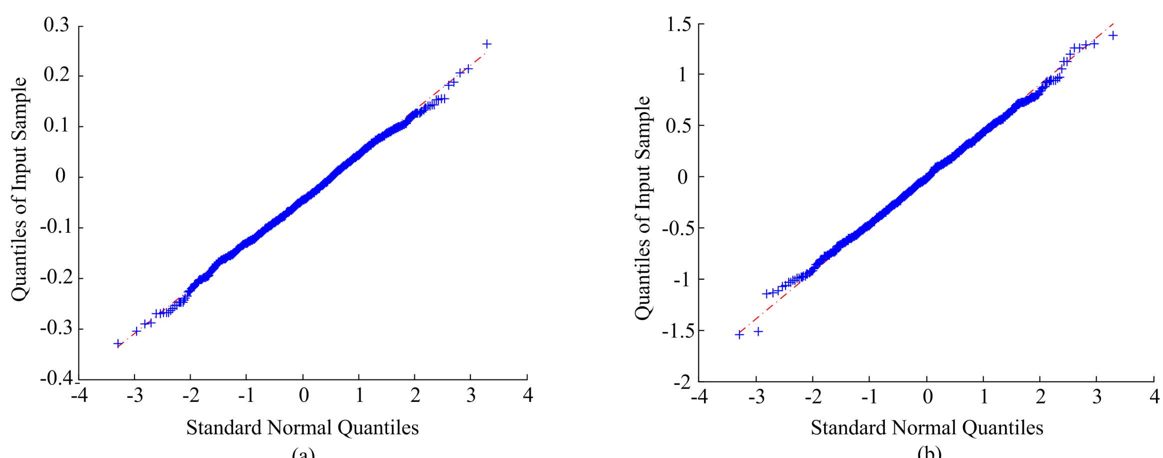

Figure 1. Q-Q plots of (a) $\frac{\sqrt{S_n}}{\sigma}(\hat{\beta}_n-\beta)$ and (b) $\frac{\sqrt{n}}{\sigma}(\hat{\theta}_n-\theta)$ with $n=300$.

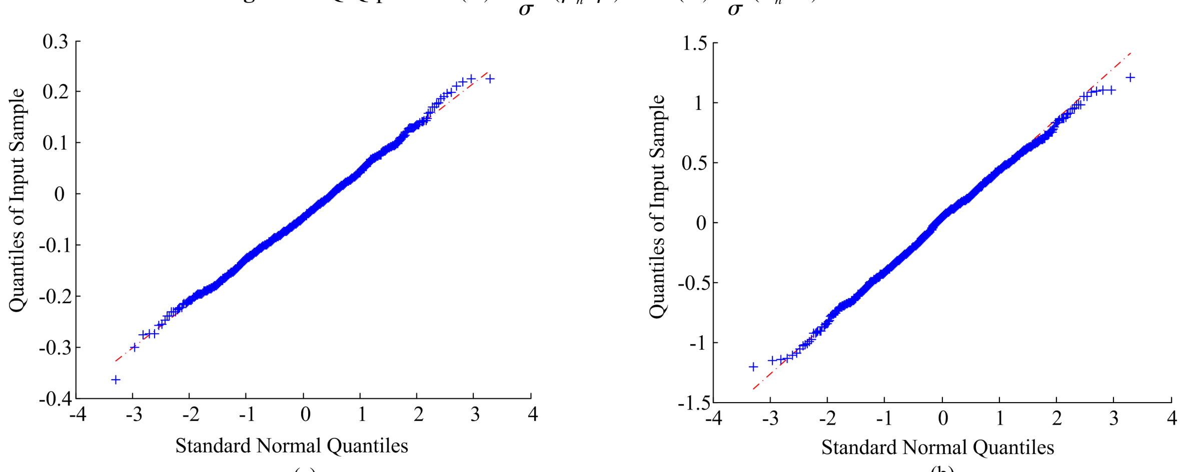

Figure 2. Q-Q plots of (a) $\frac{\sqrt{S_n}}{\sigma}(\hat{\beta}_n-\beta)$ and (b) $\frac{\sqrt{n}}{\sigma}(\hat{\theta}_n-\theta)$ with $n=600$.

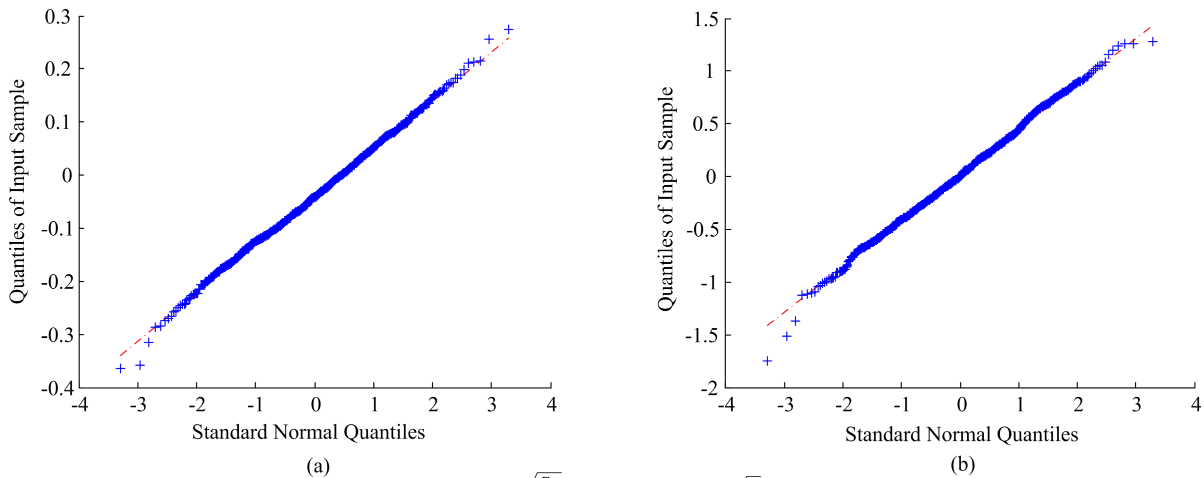

Figure 3. Q-Q plots of (a) $\frac{\sqrt{S_n}}{\sigma}(\widehat{\beta}_n-\beta)$ and (b) $\frac{\sqrt{n}}{\sigma}(\widehat{\theta}_n-\theta)$ with $n=900$.

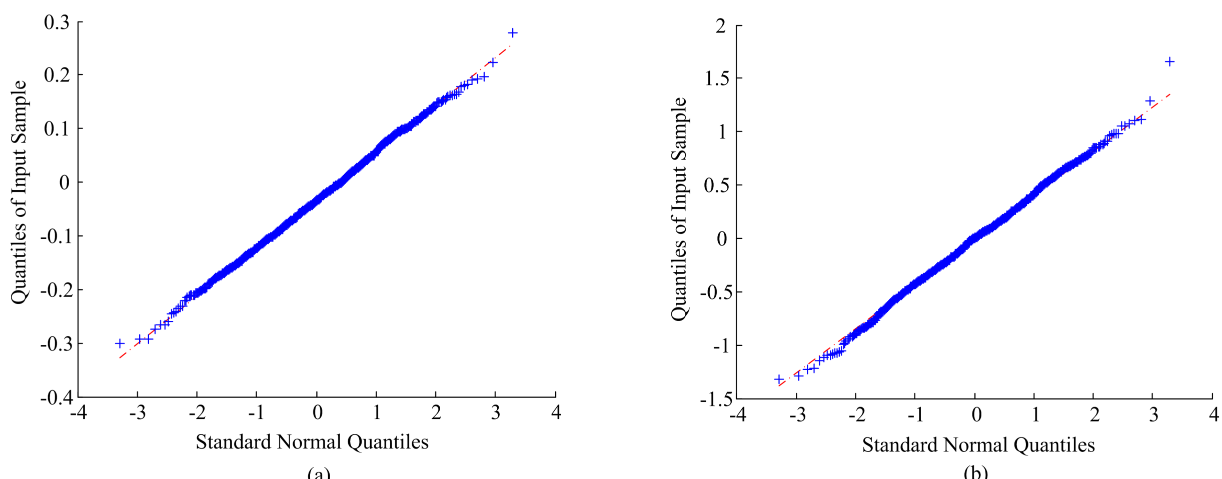

Figure 4. Q-Q plots of (a) $\frac{\sqrt{S_n}}{\sigma}(\widehat{\beta}_n-\beta)$ and (b) $\frac{\sqrt{n}}{\sigma}(\widehat{\theta}_n-\theta)$ with $n=1200$.

In addition, we also compute $\frac{\sqrt{S_n}}{n^{\tau}}(\widehat{\beta}_n-\beta)$ and $n^{\nu}(\widehat{\theta}_n-\theta)$ for 1000 times and depict the boxplots of them in Figures 5 and 6. Here taking $\tau=0.3$ and $\nu=0.2$. We can see clearly that they approach to the zero line and the ranges of them decrease as n increases.

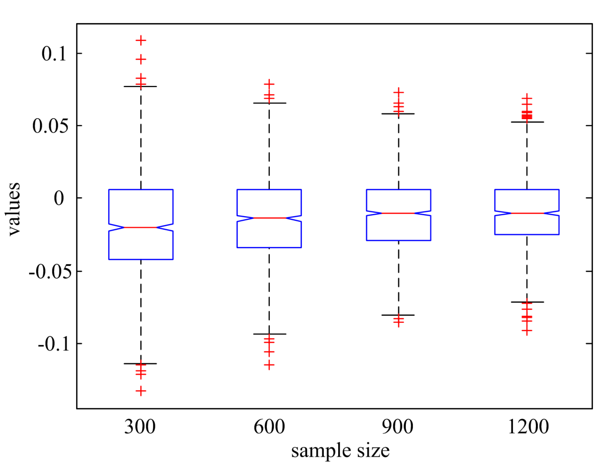

Figure 5. Boxplots of $\frac{\sqrt{S_n}}{n^{\tau}}(\widehat{\beta}_n-\beta)$.

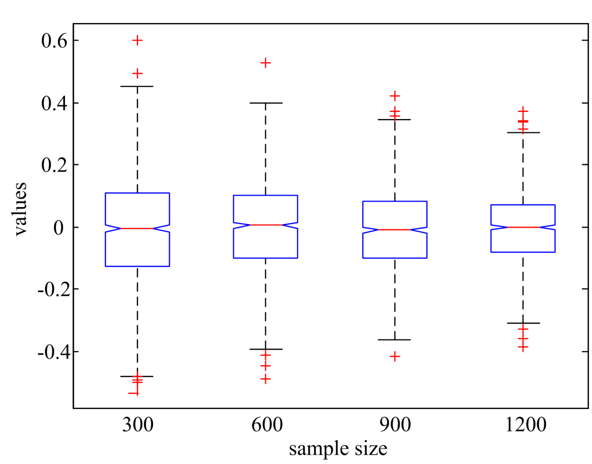

Figure 6. Boxplots of $n^{\nu}(\widehat{\theta}_n-\theta)$.

3 The proof of main results

By simple calculation, we have

$$\widehat{\beta}_n - \beta = \frac{\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)\varepsilon_i - \sum_{i=1}^{n}(x_i - \bar{x}_n)(\varepsilon_i - \beta\delta_i) - \beta\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)^2}{\sum_{i=1}^{n}(\xi_i - \bar{\xi}_n)^2} \tag{13}$$

and

$$\widehat{\theta}_n - \theta = (\beta - \widehat{\beta}_n)\bar{x}_n + (\beta - \widehat{\beta}_n)\bar{\delta}_n - \beta\bar{\delta}_n + \bar{\varepsilon}_n \tag{14}$$

In order to prove the main results of this paper, we need the following lemmas, which are the central limit theorem, the Marcinkiewicz-type strong law of large numbers for φ-mixing sequence and the strong convergence for the weighted sums of the φ-mixing random variables, respectively.

Lemma 1[23] Let $\{X_n, n \geqslant 1\}$ be a centered stochastic sequence of φ-mixing random variables and $\{a_{ni}, 1 \leqslant i \leqslant n, n \geqslant 1\}$ be a triangular array of real numbers such that:

$$\sup_n \sum_{i=1}^{n} a_{ni}^2 < \infty, \ \max_{1\leqslant i\leqslant n}|a_{ni}| \to 0, \ n \to \infty.$$

Assume that $\{X_n^2, n\geqslant 1\}$ is a uniformly integrable family, and $\mathrm{Var}\left(\sum_{i=1}^{n} a_{ni}X_i\right) = 1$. Then:

$$\sum_{i=1}^{n} a_{ni}X_i \xrightarrow{D} N(0,1).$$

Remark 2 According to Ref. [23], the result still holds if we replace $\mathrm{Var}\left(\sum_{i=1}^{n} a_{ni}X_i\right) = 1$ in Lemma 1 by $\mathrm{Var}\left(\sum_{i=1}^{n} a_{ni}X_i\right) \to 1$.

Lemma 2[25] Let $\{X_n, n\geqslant 1\}$ be a sequence of φ-mixing random variables which is stochastically dominated by a random variable X with $E|X|^p<\infty$ for some $0<p<2$ and $\sum_{n=1}^{\infty}\varphi^{1/2}(n) < \infty$. Assume that $EX_k=0$ for each $k\geqslant 1$ if $1\leqslant p<2$. Then,

$$\frac{1}{n^{1/p}}\sum_{i=1}^{n} X_i \to 0 \text{ a. s.}.$$

Lemma 3 Let $\{X_n, n \geqslant 1\}$ be a sequence of φ-mixing random variables with $\sum_{n=1}^{\infty}\varphi^{1/2}(n) < \infty$, $EX_n=0$ and for some $0<\delta\leqslant 1, q>1/\delta, \sup_{n\geqslant 1}E|X_n|^q<\infty$. Assume that $\{a_{ni}, 1\leqslant i\leqslant n, n\geqslant 1\}$ is an array of real numbers, and satisfies

$$|a_{ni}| = O(n^{-\delta}) \text{ for } 1 \leqslant i \leqslant n,$$

$$\sum_{i=1}^{n}|a_{ni}|^t = O(n^{-\alpha}) \text{ for some } \alpha > 0,$$

where $t=\min(q,2)$. Then,

$$\sum_{i=1}^{n} a_{ni}X_i \to 0 \text{ a. s.}.$$

Proof The proof of the lemma can be referred to Lemma 2 of Wu[26].

The proof of Theorem 1

Firstly, by Markov's inequality and condition (4), we can get that

$$P\left(\frac{1}{\sqrt{S_n}}\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)^2 > \varepsilon\right) \leqslant \frac{1}{\varepsilon}\cdot\frac{1}{\sqrt{S_n}}E\left(\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)^2\right) = \frac{1}{\varepsilon}\cdot\frac{1}{\sqrt{S_n}}E\left(\sum_{i=1}^{n}\delta_i^2 - n\bar{\delta}_n^2\right) \leqslant \frac{1}{\varepsilon}\cdot\frac{1}{\sqrt{S_n}}\sum_{i=1}^{n}E\delta_i^2 = \frac{1}{\varepsilon}\cdot\frac{n}{\sqrt{S_n}}E\delta_1^2 \to 0,$$

Hence,

$$\frac{1}{\sqrt{S_n}}\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)^2 \xrightarrow{P} 0 \tag{15}$$

By

$$\left|\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)\varepsilon_i\right| = \left|\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)(\varepsilon_i - \bar{\varepsilon}_n)\right| \leqslant \frac{1}{2}\left(\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)^2 + \sum_{i=1}^{n}(\varepsilon_i - \bar{\varepsilon}_n)^2\right) \tag{16}$$

and the proof of (15), we have that

$$\frac{1}{\sqrt{S_n}}\sum_{i=1}^{n}(\varepsilon_i - \bar{\varepsilon}_n)^2 \xrightarrow{P} 0,$$

Thus,

$$\frac{1}{\sqrt{S_n}}\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)\varepsilon_i \xrightarrow{P} 0 \tag{17}$$

Secondly, note that

$$\sum_{i=1}^{n}(\xi_i - \bar{\xi}_n)^2 = \sum_{i=1}^{n}[x_i + \delta_i - (\bar{x}_n + \bar{\delta}_n)]^2 = \sum_{i=1}^{n}(x_i - \bar{x}_n)^2 + 2\sum_{i=1}^{n}(x_i - \bar{x}_n)(\delta_i - \bar{\delta}_n) + \sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)^2.$$

For any $\gamma>0$, we have that

$$\left|\sum_{i=1}^{n}(\xi_i - \bar{\xi}_n)^2 - S_n\right| = \left|2\sum_{i=1}^{n}(x_i - \bar{x}_n)(\delta_i - \bar{\delta}_n) + \sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)^2\right| \leqslant \frac{\gamma}{2}\sum_{i=1}^{n}(x_i - \bar{x}_n)^2 + \frac{2}{\gamma}\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)^2 + \sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)^2 =$$

$$\frac{\gamma}{2}S_n + \frac{\gamma+2}{\gamma}\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)^2 \quad (18)$$

Thereby,

$$P\left(\left|\frac{\sum_{i=1}^{n}(\xi_i-\bar{\xi}_n)^2}{S_n}-1\right|\geqslant\gamma\right)\leqslant$$

$$P\left(\frac{\gamma}{2}+\frac{\gamma+2}{\gamma}\cdot\frac{\sum_{i=1}^{n}(\delta_i-\bar{\delta}_n)^2}{S_n}\geqslant\gamma\right)=$$

$$P\left(\frac{\sum_{i=1}^{n}(\delta_i-\bar{\delta}_n)^2}{S_n}\geqslant\frac{\gamma^2}{4+2\gamma}\right),$$

which implies by (15) that:

$$\frac{1}{S_n}\sum_{i=1}^{n}(\xi_i-\bar{\xi}_n)^2\xrightarrow{P}1 \quad (19)$$

From (13), we have that

$$\frac{\sqrt{S_n}(\hat{\beta}_n-\beta)}{\sigma}=\frac{\frac{1}{\sigma}\cdot\frac{1}{\sqrt{S_n}}\sum_{i=1}^{n}(\delta_i-\bar{\delta}_n)\varepsilon_i+\frac{1}{\sigma\sqrt{S_n}}\sum_{i=1}^{n}(x_i-\bar{x}_n)(\varepsilon_i-\beta\delta_i)-\frac{\beta}{\sigma}\cdot\frac{1}{\sqrt{S_n}}\sum_{i=1}^{n}(\delta_i-\bar{\delta}_n)^2}{\frac{1}{S_n}\sum_{i=1}^{n}(\xi_i-\bar{\xi}_n)^2}.$$

Combined with (15), (17) and (19), it is sufficient to prove

$$\frac{1}{\sigma\sqrt{S_n}}\sum_{i=1}^{n}(x_i-\bar{x}_n)(\varepsilon_i-\beta\delta_i)\xrightarrow{D}N(0,1).$$

Let $X_i=\varepsilon_i-\beta\delta_i$, $a_{n,i}^*=\frac{x_i-\bar{x}_n}{\sqrt{S_n}}$, then,

$$\mathrm{Var}\left(\frac{1}{\sigma\sqrt{S_n}}\sum_{i=1}^{n}(x_i-\bar{x}_n)(\varepsilon_i-\beta\delta_i)\right)=$$

$$\frac{1}{\sigma^2}E\Big(\sum_{i=1}^{n}a_{n,i}^*X_i\Big)^2=$$

$$\frac{1}{\sigma^2}\Big(EX_1^2+2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}a_{n,i}^*a_{n,j}^*EX_iX_j\Big)=$$

$$\frac{1}{\sigma^2}\Big(EX_1^2+2\sum_{j=2}^{n}\sum_{i=1}^{n+1-j}a_{n,i}^*a_{n,i+j-1}^*EX_1X_j\Big)=$$

$$\frac{1}{\sigma^2}(EX_1^2+2\sum_{j=2}^{n}\sum_{i=1}^{n+1-j}a_{n,i}^*(a_{n,i+j-1}^*-a_{n,i}^*)EX_1X_j+$$

$$\sum_{j=2}^{n}\sum_{i=1}^{n+1-j}(a_{n,i}^*)^2EX_1X_j)=$$

$$\frac{1}{\sigma^2}(EX_1^2+2\sum_{j=2}^{n}EX_1X_j+$$

$$2\sum_{j=2}^{n}\sum_{i=1}^{n+1-j}a_{n,i}^*(a_{n,i+j-1}^*-a_{n,i}^*)EX_1X_j-$$

$$2\sum_{j=2}^{n}\sum_{i=n+2-j}^{n}(a_{n,i}^*)^2EX_1X_j).$$

Next, we prove:

$$\sum_{j=2}^{n}\sum_{i=1}^{n+1-j}a_{n,i}^*(a_{n,i+j-1}^*-a_{n,i}^*)EX_1X_j\to0 \quad (20)$$

and

$$\sum_{j=2}^{n}\sum_{i=n+2-j}^{n}(a_{n,i}^*)^2EX_1X_j\to0 \quad (21)$$

For (20), by conditions (6) and (8), we can obtain that

$$\Big|\sum_{j=2}^{n}\sum_{i=1}^{n+1-j}a_{n,i}^*(a_{n,i+j-1}^*-a_{n,i}^*)EX_1X_j\Big|\leqslant$$

$$\sum_{j=2}^{n}\sum_{i=1}^{n+1-j}|a_{n,i}^*|\frac{|x_{i+j-1}-x_i|}{\sqrt{S_n}}|EX_1X_j|\leqslant$$

$$\sum_{j=2}^{n}\sum_{i=1}^{n+1-j}|a_{n,i}^*|\frac{c(j-1)}{\sqrt{S_n}}|EX_1X_j|\leqslant$$

$$\frac{cn}{\sqrt{S_n}}\max_{1\leqslant i\leqslant n}\frac{|x_i-\bar{x}_n|}{\sqrt{S_n}}\sum_{j=2}^{n}(j-1)|EX_1X_j|=$$

$$cr_n\frac{n}{\sqrt{S_n}}\sum_{j=2}^{n}(j-1)|EX_1X_j|\to0.$$

For (21), by condition (8), we can get that

$$\sum_{j=2}^{n}\sum_{i=n+2-j}^{n}(a_{n,i}^*)^2|EX_1X_j|\leqslant$$

$$\max_{1\leqslant i\leqslant n}(a_{n,i}^*)^2\sum_{j=2}^{n}(j-1)|EX_1X_j|=$$

$$r_n^2\sum_{j=2}^{n}(j-1)|EX_1X_j|\to0.$$

Hence, $\mathrm{Var}\Big(\sum_{i=1}^{n}a_{n,i}X_i\Big)\to\frac{1}{\sigma^2}\Big(EX_1+2\sum_{j=2}^{\infty}EX_1X_j\Big)=1.$

Therefore, the theorem is proved by using Lemma 1.

The proof of Theorem 2

According to (14), we have that

$$\frac{\sqrt{n}}{\sigma}(\hat{\theta}_n-\theta)=\frac{\sqrt{S_n}}{\sigma}(\beta-\hat{\beta}_n)\cdot$$

$$\frac{\sqrt{n}}{\sqrt{S_n}}(\bar{x}_n+\bar{\delta}_n)+\frac{\sqrt{n}}{\sigma}(\bar{\varepsilon}_n-\beta\bar{\delta}_n) \quad (22)$$

Note by condition (9) that

$$\frac{\sqrt{n}}{\sqrt{S_n}}\bar{x}_n=\frac{1}{\frac{\sqrt{S_n}}{\sqrt{n}}\cdot\frac{1}{\bar{x}_n}}=\frac{1}{\sqrt{\frac{S_n}{n(\bar{x}_n)^2}}}\to0.$$

From Lemma 2, it is followed by

$$\frac{\sqrt{n}}{\sqrt{S_n}}\bar{\delta}_n = \frac{\sqrt{n}}{\sqrt{S_n}} \cdot \frac{1}{n}\sum_{i=1}^{n}\delta_i = \frac{n}{\sqrt{S_n}} \cdot \frac{1}{n^{\frac{3}{2}}}\sum_{i=1}^{n}\delta_i \to 0 \text{ a. s.}.$$

It is observed that

$$\frac{\sqrt{n}}{\sigma}(\bar{\varepsilon}_n - \beta\bar{\delta}_n) = \frac{\sqrt{n}}{\sigma}\left(\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i - \beta \cdot \frac{1}{n}\sum_{i=1}^{n}\delta_i\right) = \frac{1}{\sigma} \cdot \frac{1}{\sqrt{n}}\sum_{i=1}^{n}(\varepsilon_i - \beta\delta_i) = \frac{1}{\sigma} \cdot \frac{1}{\sqrt{n}}\sum_{i=1}^{n}X_i$$

and

$$\begin{aligned}\operatorname{Var}\left(\frac{1}{\sigma} \cdot \frac{1}{\sqrt{n}}\sum_{i=1}^{n}X_i\right) &= \frac{1}{\sigma^2} \cdot \frac{1}{n}\operatorname{Var}\left(\sum_{i=1}^{n}X_i\right) = \\ &\frac{1}{\sigma^2} \cdot \frac{1}{n}\left(nEX_1^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}EX_iX_j\right) = \\ &\frac{1}{\sigma^2} \cdot \frac{1}{n}\left(nEX_1^2 + 2\sum_{k=1}^{n-1}\sum_{i=1}^{n-k}EX_iX_{k+i}\right) = \\ &\frac{1}{\sigma^2} \cdot \frac{1}{n}\left(nEX_1^2 + 2\sum_{k=1}^{n-1}(n-k)EX_1X_{k+1}\right) = \\ &\frac{1}{\sigma^2}\left(EX_1^2 + 2\sum_{k=1}^{n-1}EX_1X_{k+1} - 2\sum_{k=1}^{n-1}\frac{k}{n}EX_1X_{k+1}\right) = \\ &\frac{1}{\sigma^2}\left(EX_1^2 + 2\sum_{j=2}^{n}EX_1X_j - 2\sum_{j=2}^{n}\frac{j}{n}\operatorname{Cov}(X_1,X_j) + \frac{2}{n}\sum_{j=2}^{n}\operatorname{Cov}(X_1,X_j)\right).\end{aligned}$$

By Kronecker's Lemma and condition (10), it is easy to obtain that:

$$\sum_{j=2}^{n}\frac{j}{n}\operatorname{Cov}(X_1,X_j) \to 0.$$

It is followed by conditions (7) and (10) that:

$$\operatorname{Var}\left(\frac{1}{\sigma} \cdot \frac{1}{\sqrt{n}}\sum_{i=1}^{n}X_i\right) \to 1,$$

which is implied in Lemma 1 that

$$\frac{\sqrt{n}}{\sigma}(\bar{\varepsilon}_n - \beta\bar{\delta}_n) \xrightarrow{D} N(0,1).$$

Finally, the desired result follows from the result of Theorem 1 and (22).

The proof of Theorem 3

(Ⅰ) If $2<p\leqslant 4$, by taking $q=\frac{p}{2}\in(1,2]$ in Lemma 3 and (11), we can get:

$$\frac{1}{n^{\tau}\sqrt{S_n}}\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)^2 \leqslant \frac{1}{n^{\tau}\sqrt{S_n}}\sum_{i=1}^{n}(\delta_i^2 - E\delta_i^2) + \frac{n^{1-\tau}}{\sqrt{S_n}}E\delta_1^2 \to 0 \text{ a. s.} \quad (23)$$

Similar to the proof of (23), we can obtain $\frac{1}{n^{\tau}\sqrt{S_n}}\sum_{i=1}^{n}(\varepsilon_i - \bar{\varepsilon}_n)^2 \to 0$ a. s.. Hence,

$$\frac{1}{n^{\tau}\sqrt{S_n}}\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)\varepsilon_i \to 0 \text{ a. s.} \quad (24)$$

by (16).

According to (11) and Lemma 3 (taking $q=p>2$), we can obtain:

$$\frac{1}{n^{\tau}\sqrt{S_n}}\sum_{i=1}^{n}(x_i - \bar{x}_n)(\varepsilon_i - \beta\delta_i) \to 0 \text{ a. s.} \quad (25)$$

From (18), (23) and (11), we have that

$$\left|\frac{1}{S_n}\sum_{i=1}^{n}(\xi_i - \bar{\xi}_n)^2 - 1\right| \leqslant \frac{\gamma}{2} + \frac{2+\gamma}{\gamma}\frac{1}{S_n}\sum_{i=1}^{n}(\delta_i - \bar{\delta}_n)^2 \to \frac{\gamma}{2} \text{ a. s.},$$

which implies in the arbitrariness of γ that:

$$\frac{1}{S_n}\sum_{i=1}^{n}(\xi_i - \bar{\xi}_n)^2 \to 1 \text{ a. s.}.$$

Combining (23)−(25), we get $\frac{\sqrt{S_n}}{n^{\tau}}(\hat{\beta}_n - \beta) \to 0$ a. s..

(Ⅱ) If $p>4$, with the similar proofs as the case $2<p\leqslant 4$, the desired result can be obtained easily. This completes the proof of the theorem.

The proof of Theorem 4

Applying Lemma 3 (taking $q=p>2$), it is not difficult to show that

$$n^{\nu}(\bar{\varepsilon}_n - \beta\bar{\delta}_n) = n^{\nu-1}\sum_{i=1}^{n}(\varepsilon_i - \beta\delta_i) \to 0 \text{ a. s.}.$$

Furthermore, from condition (12), Theorem 3 and Lemma 3 (taking $q=p>2$), we have

$$n^{\nu}(\beta - \hat{\beta}_n)(\bar{x}_n + \bar{\delta}_n) = \frac{n^{\tau+\nu}}{\sqrt{S_n}}\bar{x}_n \cdot \frac{\sqrt{S_n}}{n^{\tau}}(\beta - \hat{\beta}_n) + \frac{n^{\tau+\nu-1}}{\sqrt{S_n}}\sum_{i=1}^{n}\delta_i \cdot \frac{\sqrt{S_n}}{n^{\tau}}(\beta - \hat{\beta}_n) \to 0 \text{ a. s.}.$$

Therefore, the desired result can be obtained from (14).

Acknowledgments

This work is supported by the Scientific Research Foundation Funded Project of Chuzhou University (2018qd01) and the Natural Science Foundation of Anhui Province (1908085QA01).

Conflict of interest

The authors declare no conflict of interest.

Author information

DENG Xin received her PhD degree in Statistics from Anhui University. She is currently a lecturer at the School of Mathematical and Finance of Chuzhou University. Her research mainly focuses on probability limit theory, parametric and non-parametric statistics.

WU Yi (corresponding author) received his PhD degree in Statistics from Anhui University. He is currently a lecturer at the

School of Big Data and Artificial Intelligence of Chizhou University. His research interests include probability limit theory, statistical models and sub-linear expectations.

References

[1] Mittag H J. Estimating parameters in a simple errors-in-variables model: A new approach based on finite sample distribution theory. Statistical Papers, 1989, 30: 133-140.
[2] Fuller W A. Measurement Error Models. New York: Wiley, 1987.
[3] Liu J X, Chen X R. Consistency of LS estimator in simple linear EV regression models. Acta Mathematica Scientia, Series B, 2005, 25(1): 50-58.
[4] Miao Y, Yang G Y, Shen L M. The central limit theorem for LS estimator in simple linear EV regression models. Communications in Statistics-Theory and Methods, 2007, 36(12): 2263-2272.
[5] Miao Y, Yang G Y. The loglog law for LS estimator in simple linear EV regression models. Statistics, 2011, 45(2): 155-162.
[6] Miao Y, Wang K, Zhao F F. Some limit behaviors for the LS estimator in simple linear EV regression models. Statistics and Probability Letters, 2011, 81(1): 92-102.
[7] Fan G L, Liang H Y, Wang J F, et al. Asymptotic properties for LS estimators in EV regression model with dependent errors. AStA Advances in Statistical Analysis, 2010, 94: 89-103.
[8] Yang Q L. Asymptotic normality of LS estimators in the simple linear EV regression model with PA errors. Communications in Statistics-Theory and Methods, 2012, 41(23): 4276-4284.
[9] Miao Y, Zhao F F, Wang K, et al. Asymptotic normality and strong consistency of LS estimators in the EV regression model with NA errors. Statistical Papers, 2013, 54: 193-206.
[10] Wang X J, Shen A T, Chen Z Y, et al. Complete convergence for weighted sums of NSD random variables and its application in the EV regression model. Test, 2015, 24: 166-184.
[11] Wang X J, Wu Y, Hu S H. Strong and weak consistency of LS estimators in the EV regression model with negatively superadditive-dependent errors. AStA Advances in Statistical Analysis, 2018, 102: 41-65.
[12] Wang X J, Xi M M, Wang H X, et al. On consistency of LS estimators in the errors-in-variable regression model. Probability in the Engineering and Informational Sciences, 2018, 32: 144-162.
[13] Shen A T. Asymptotic properties of LS estimators in the errors-in-variables model with MD errors. Statistical Papers, 2019, 60(4): 1193-1206.
[14] Dobrushin R L. The central limit theorem for non-stationary Markov chain. Theory of Probability and Its Applications, 1956, 1: 72-88.
[15] Babu G J, Ghosh M, Singh K. On rates of convergence to normality for φ-mixing processes. Sankhyā, Series A, 1978, 40(3): 278-293.
[16] Utev S A. The central limit theorem for φ-mixing arrays of random variables. Theory of Probability and Its Applications, 1990, 35(1): 131-139.
[17] Kiesel R. Strong laws and summability for φ-mixing sequences of random variables. Journal of Theoretical Probability, 1998, 11(1): 209-224.
[18] Hu S H, Wang X J. Large deviations for some dependent sequences. Acta Mathematica Scientic, Series B, 2008, 28(2): 295-300.
[19] Yang W Z, Wang X J, Li X Q, et al. Berry-Esseen bound of sample quantiles for φ-mixing random variables. Journal of Mathematical Analysis and Applications, 2012, 388: 451-462.
[20] Shen A T, Wang X H, Ling J M. On complete convergence for non-stationary φ-mixing random variables. Communications in Statistics-Theory and Methods, 2014, 43(22): 4856-4866.
[21] Billingsley P. Convergence of Probability Measures. New York: Wiley, 1968.
[22] Lu C R, Lin Z Y. Limit Theory for Mixing Dependent Sequences. Beijing: Science Press of China, 1997.
[23] Peligrad M, Utev S. Central limit theorem for linear processes. The Annals of Probability, 1997, 25(1): 443-456.
[24] Thanh Lv, Yin G. Weighted sums of strongly mixing random variables with an application to nonparametric regression. Statistics and Probability Letters, 2015, 105: 195-202.
[25] Wang X J, Hu S H. Some Baum-Katz type results for φ-mixing random variables with different distributions. RACSAM, 2012, 106: 321-331.
[26] Wu Q Y. Further study strong consistency of M estimator in linear model for $\widetilde{\rho}$-mixing random samples. Journal of Systems Science and Complexity, 2011, 24: 969-980.

φ-混合误差下线性 EV 模型中最小二乘估计的渐近性质

邓新[1],田春雨[2],葛梅梅[1],叶静[1],丁洋[1],吴燚[3]
1. 滁州学院数学与金融学院,安徽滁州 239000;
2. 中国电子科技集团公司第五十八研究所,江苏南京 210000;
3. 池州学院大数据与人工智能学院,安徽池州 247000
* 通讯作者. E-mail: wuyi8702@163.com

摘要:本文主要研究 φ-混合随机误差下的简单线性 EV 模型. 借助于 φ-混合序列的中心极限定理和 Marcinkiewicz 型强大数定律, 在较弱的假设条件下,建立了未知参数最小二乘估计的渐近正态性. 另外,利用 φ-混合随机变量加权和的强收敛性,得到了该最小二乘估计的强相合性. 最后,给出了相关理论结果的数值模拟.

关键词: EV 模型;渐近正态性;强相合性;最小二乘估计;φ-混合序列