Article

http://justc.ustc.edu.cn

Received: June 29, 2021; Accepted: November 26, 2021

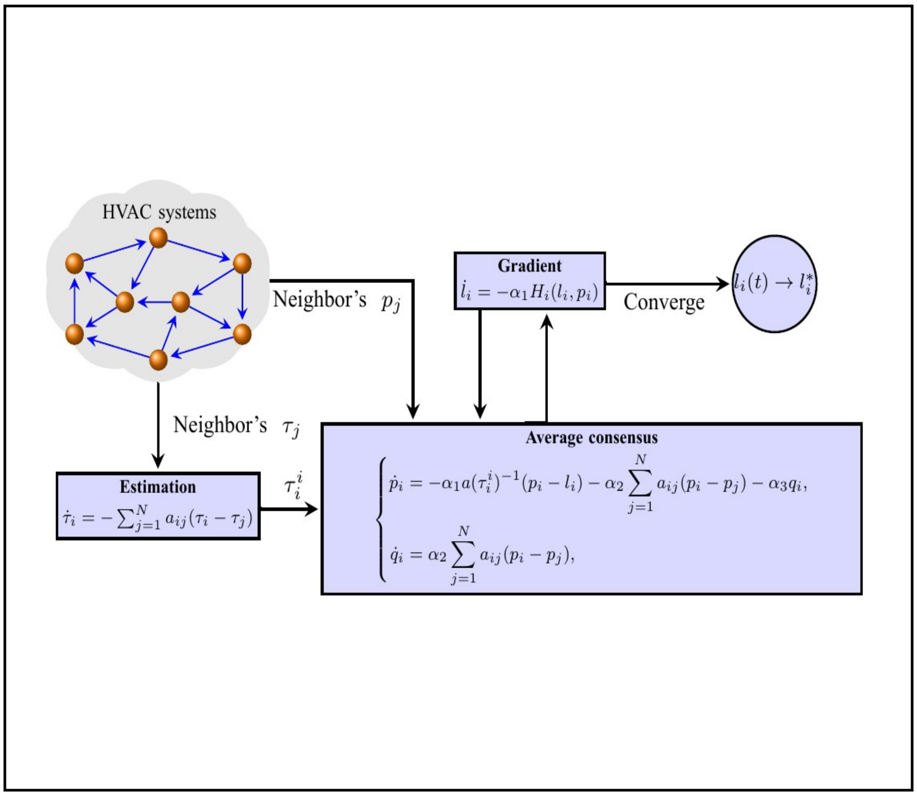
Distributed Nash equilibrium seeking design for energy consumption games of HVAC systems over digraphs

Xinyu Liu, Jin Zhang, Haibo Ji, and Xinghu Wang ✉

Department of Automation, University of Science and Technology of China, Hefei 230027, China

✉Correspondence: Xinghu Wang, E-mail: xinghuw@ustc.edu.cn

Graphical abstract



A novel distributed Nash equilibrium seeking algorithm is proposed to solve the energy consumption problem of heating, ventilation, and air conditioning systems over directed graphs

Public summary

- The energy consumption problem of heating, ventilation, and air conditioning systems over general directed graphs is investigated and a distributed consensus-based algorithm is proposed.
- To address the challenge arising from general directed graphs, a distributed estimation algorithm is embedded such that the explicit dependence on the left eigenvector associated with the eigenvalue zero of the Laplacian matrix can be avoided.
- The exponential convergence of the proposed distributed Nash equilibrium seeking algorithm is established under a standing assumption.

Citation: Liu X Y, Zhang J, Ji H B, et al. Distributed Nash equilibrium seeking design for energy consumption games of HVAC systems over digraphs. *JUSTC*, **2022**, 52(1): 5. DOI: 10.52396/JUSTC-2021-0153

JUSTC Article
http://justc.ustc.edu.cn Received: June 29, 2021; Accepted: November 26, 2021

Distributed Nash equilibrium seeking design for energy consumption games of HVAC systems over digraphs

Xinyu Liu, Jin Zhang, Haibo Ji, and Xinghu Wang

Department of Automation, University of Science and Technology of China, Hefei 230027, China

Correspondence: Xinghu Wang, E-mail: xinghuw@ustc.edu.cn

Cite This: *JUSTC*, **2022**, 52(1): 5 (7pp) Read Online Supporting Information

Abstract: The energy consumption problem of heating, ventilation, and air conditioning systems over general directed graphs is investigated. The considered problem is firstly reformulated as a Nash equilibrium seeking problem, and a distributed consensus-based algorithm is then proposed to solve it. To address the challenge arising from general directed graphs, a distributed estimation algorithm is embedded such that the explicit dependence on the left eigenvector associated with the eigenvalue zero of the Laplacian matrix can be avoided. Then, the exponential convergence of the proposed distributed Nash equilibrium seeking algorithm is established under a standing assumption. A numerical example is finally provided to verify the effectiveness of the proposed algorithm.

Keywords: energy consumption game; HVAC; Nash equilibrium; directed graph

CLC number: TP274 **Document code:** A

1 Introduction

In the past few years, the energy consumption problem has attracted considerable attention due to the growing demand for energy[1−4]. It is worth noting that heating, ventilation, and air conditioning (HVAC) systems are responsible for a large proportion of energy consumption in practice[5,6]. A few centralized approaches have been developed to solve the energy consumption problem of HVAC systems, see, for example, Refs.[7−9]. A limitation of such centralized scheme is its resulting heavy computational burden on the centralized location[10].

More recently, motivated by the investigation of the distributed control and optimization (see, for instance, Refs. [11−14] and references therein), several distributed algorithms have been proposed to solve the energy consumption problem of HVAC systems. In particular, Refs. [15−18] proposed distributed optimization schemes for HVAC systems to minimize the total energy consumption cost while to maintain the zone thermal comfort. Ref. [19] designed a distributed real-time pricing feedback algorithm for HVAC systems in a noncooperative game scheme. Further, Ref. [20] developed a distributed Nash equilibrium seeking strategy based on average consensus protocol for the energy consumption game of HVAC systems over undirected graphs.

It should be pointed out that the existing works[16,17,20] only considered the energy consumption problem of HVAC systems for undirected graphs. Nevertheless, it is of much more theoretical and practical significance to consider directed graphs because the information exchange between agents may be unidirectional due to limited bandwidth or other constraints. Recently, by utilizing the left eigenvector associated with the eigenvalue zero of the Laplacian matrix, several distributed algorithms were developed in Refs. [21−23] to cope with directed graphs from various aspects. However, since the left eigenvector is some kind of global information, the above-mentioned algorithms may not be applicable when the left eigenvector is not known *a priori*. This brings difficulties in both design and convergence analysis of the distributed algorithms.

In this paper, the energy consumption problem of HVAC systems over directed graphs is considered. The problem is firstly reformulated by a game model similar to that in Ref. [20]. Then, a novel distributed Nash equilibrium seeking algorithm is proposed in case that the left eigenvector associated with the eigenvalue zero of the Laplacian matrix is unavailable. Exponential convergence of the proposed algorithm is also established under a standing assumption. Main contributions of this paper compared with relevant literatures can be summarized as follows.

(Ⅰ) In contrast with the existing work[20] for undirected graphs, this work considers more general and thus more challenging directed graphs. A new distributed Nash equilibrium seeking algorithm is developed to address such issue of general directed graphs.

(Ⅱ) Different from the distributed design in Refs. [21−23] requiring the exact value of the left eigenvector associated with the eigenvalue zero of Laplacian matrix, our result does not rely on the exact value of the left eigenvector. In fact, it is a challenge to obtain the left eigenvector, especially in large-scale networks. Thus, our result gives rise to a promising design in implementation.

The remainder of this paper is organized as follows. Some preliminaries are presented and the energy consumption game is formulated in Section Ⅱ, which is followed by designing a distributed Nash equilibrium seeking algorithm in Section Ⅲ. Then, the effectiveness of the proposed algorithm is validated via a numerical example in Section Ⅳ. Finally, the conclu-

DOI: 10.52396/JUSTC-2021-0153
JUSTC, **2022**, 52(1): 5

sion is stated in Section Ⅴ.

Notations: $\|\cdot\|$ denotes the Euclidean norm. Let $\mathbb{R}$, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the sets of real numbers, n-dimensional real vectors and $n\times m$-dimensional real matrices, respectively. x^{T} and A^{T} refer to the transpose of vector x and matrix A, respectively. Let $1_n=[1, 1, \ldots, 1]^{\mathrm{T}}\in\mathbb{R}^n$ and $0_n=[0, 0, \ldots, 0]^{\mathrm{T}}\in\mathbb{R}^n$ denote the all-zeros vector and the all-ones vector, respectively. $I_n\in\mathbb{R}^{n\times n}$ refers to the identity matrix. The matrix $\mathrm{diag}(x_1, x_2, \ldots, x_n)$ denotes the diagonal matrix with the ith diagonal entry being x_i. $\lambda_{\min}(B)$ and $\lambda_{\max}(B)$ represent the smallest eigenvalue and the largest eigenvalue of matrix B, respectively. $\otimes$ represents the Kronecker product. $\nabla f(x)$ represents the gradient vector of $f(x)$.

2 Preliminary and formulation

In this section, some preliminaries on convex analysis and graph theory are presented, and the energy consumption game of HVAC systems is given.

2.1 Preliminary

A function g: $\mathbb{R}^n\to\mathbb{R}$ is said to be convex if $g(\lambda x+(1-\lambda)y)\le\lambda g(x)+(1-\lambda)g(y)$ for all x, $y\in\mathbb{R}^n$ and $0\le\lambda\le1$[24]. A function f: $\mathbb{R}^n\to\mathbb{R}^n$ is globally Lipschitz on $\mathbb{R}^n$ if there exists a positive constant L such that $\|f(x)-f(y)\|\le L\|x-y\|$ for all x, $y\in\mathbb{R}^n$. A function f: $\mathbb{R}^n\to\mathbb{R}^n$ is u-strongly monotone if there exists a positive constant u such that $(x-y)^{\mathrm{T}}(f(x)-f(y))\ge u\|x-y\|^2$ for all x, $y\in\mathbb{R}^n$.

We use a directed graph to describe the information communication among agents. A directed graph is expressed as $G=\{V, E, \mathcal{A}\}$, where $V=\{1, 2, \ldots, N\}$ is the node set, $E\subseteq V\times V$ is the edge set and $\mathcal{A}$ is the adjacency matrix. If the ith agent can receive information from the jth agent, there is an edge $(j, i)\in E$. $\mathcal{A}=[a_{ij}]_{N\times N}$ represents the adjacency matrix, where $a_{ij}=1$ if $(j, i)\in E$, otherwise $a_{ij}=0$.

A path from node x_0 to node x_r in graph G is a sequence of edges with the form (x_0, x_1), (x_1, x_2), $\ldots$, $(x_{r-1}, x_r)\in E$. If there exists at least a directed path between any two nodes in graph G, the directed graph is strongly connected. $L=D_{\mathrm{in}}-\mathcal{A}$ is the Laplacian matrix, where $D_{\mathrm{in}}=\mathrm{diag}\left(\sum_{j=1}^{N}a_{1j},\sum_{j=1}^{N}a_{2j},\cdots,\sum_{j=1}^{N}a_{Nj}\right)$. It can be verified that $L1_N=0_N$. For more details of graph theory, please see Ref. [25].

2.2 Problem formulation

Consider the energy consumption game for a network of N ($N>3$) electricity users been equipped with HVAC systems. Three elements including player, strategy and cost function of the energy consumption game are described as follows. The electricity users are the players, the strategy of the ith player is its energy consumption, and the cost function of the ith player is defined as follows[19]:

$$C_i(l_i,\sigma(l))=v_i\xi_i^2\left(l_i-\widehat{l_i}\right)^2+P(\sigma(l))l_i, i\in V \tag{1}$$

where $v_i\xi_i^2\left(l_i-\widehat{l_i}\right)^2$ represents the load curtailment cost, v_i and ξ_i are thermal coefficients satisfying $v_i\xi^2_i>0$. l_i is the strategy of the ith player. $\widehat{l_i}$ is the energy needed for the ith user to maintain the indoor temperature. $P(\sigma(l))l_i$ denotes the billing payment for the energy consumption l_i.

The pricing function is defined as

$$P(\sigma(l))=a\sigma(l)+p_0,$$

where $\sigma(l)=\dfrac{1}{N}\sum_{j=1}^{N}l_j$ is the aggregate strategy of all the players, p_0 is a basic price for energy consumption and a is a positive parameter to implement elastic pricing with $0<a\le\min_{1\le i\le N}2v_i\xi^2_i$ being satisfied[20].

It can be verified that the cost function $C_i(l_i, \sigma(l))$ is continuously differentiable in l and convex in l_i for every fixed $L_{-i}=[l_1, \ldots, l_{i-1}, l_{i+1}, \ldots, l_N]^{\mathrm{T}}$. Moreover, $[\nabla_{l_1}C_1(l_1,\sigma(l)),\cdots,\nabla_{lN}C_N(l_N,\sigma(l))]^{\mathrm{T}}$ is strongly monotone. Therefore, the existence and uniqueness of the NE can be guaranteed by Theorem 3 in Ref. [26].

The Nash equilibrium of the energy consumption game is defined as follows.

Definition 2.1 A strategy profile $l^*=[l^*_1, l^*_2, \ldots, l^*_N]^{\mathrm{T}}$ is a Nash equilibrium (NE) if for all $i\in V$, the following inequality is satisfied.

$$C_i\left(l_i^*,l_i^*+\sum_{j=1,j\ne i}^{N}l_j^*\right)\leqslant C_i\left(l_i,l_i+\sum_{j=1,j\ne i}^{N}l_j^*\right).$$

Remark 2.1 For the ith player, the objective is to minimize its cost function[27]. If the strategy of the ith player is the ith element of the NE solution l^*, no player can reduce its cost function value by unilaterally changing its individual strategy. Thus, all players have no motivation to change their strategies at the NE.

In this paper, we aim to solve the following defined problem.

Problem 2.1 For the energy consumption game over directed graphs with cost functions described in Eq. (1), design a distributed algorithm for each player so that the strategy $l(t)=[l_1(t), l_2(t), \ldots, l_N(t)]^{\mathrm{T}}$ converges to the NE l^*.

An assumption is needed for designing the distributed Nash equilibrium seeking algorithm.

Assumption 2.1 The directed graph G is strongly connected.

Assumption 2.1 depicts the connectivity condition of the directed graph G. Under Assumption 2.1, there exists a positive left eigenvector $\upsilon=[\upsilon_1, \upsilon_2, \ldots, \upsilon_N]^{\mathrm{T}}$ of L associated with the zero eigenvalue, such that $\sum_{i=1}^{N}{}_i$ is satisfied[28].

3 Distributed Nash equilibrium seeking design for HVAC systems

In this section, we propose a new distributed NE seeking algorithm to solve Problem 2.1. Its design diagram is shown in Fig. 1. The proposed algorithm includes three parts: gradient, average consensus and the distributed estimation of a positive left eigenvector of L associated with the zero eigenvalue. We analyse the convergence of the proposed algorithm by two steps. First, some preliminary results are given. Then, the convergence analysis of the proposed algorithm is presented.

3.1 Algorithm design

DOI: 10.52396/JUSTC-2021-0153
JUSTC, 2022, 52(1): 5

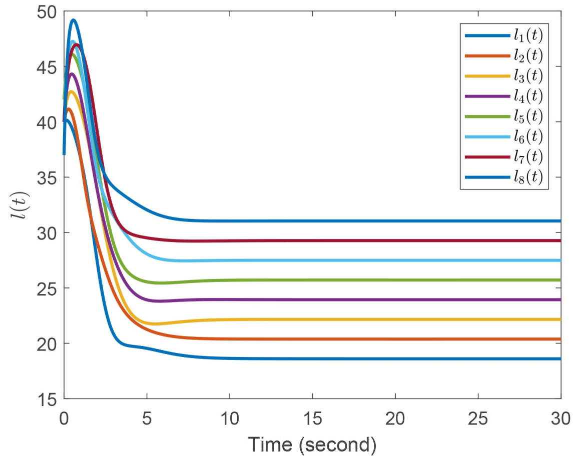


Fig. 1. Design diagram for distributed NE seeking algorithm.

In this subsection, a distributed NE seeking algorithm is designed. To proceed, define

$$F_i(l_i,\sigma(l)) = \nabla_{l_i} C_i(l_i,\sigma(l)),$$

$$H_i(l_i,p_i) = F_i(l_i,\sigma(l))|_{\sigma(l)=p_i},$$

$$F(l,\sigma(l)) = [F_1(l_1,\sigma(l)),\cdots,F_N(l_N,\sigma(l))]^{\mathrm{T}},$$

$$H(l,p) = [H_1(l_1,p_1),\cdots,H_N(l_N,p_N)]^{\mathrm{T}}.$$

Then we have

$$F_i(l_i,\sigma(l)) = \left(2v_i\xi_i^2+\frac{a}{N}\right)l_i + a\sigma(l) + \left(p_0 - 2v_i\xi_i^2\widehat{l_i}\right) \tag{2}$$

$$H_i(l_i,p_i) = \left(2v_i\xi_i^2+\frac{a}{N}\right)l_i + ap_i + \left(p_0 - 2v_i\xi_i^2\widehat{l_i}\right) \tag{3}$$

The distributed NE seeking algorithm is designed as

$$\left.\begin{aligned} &\dot{l}_i = -\alpha_1 H_i(l_i,p_i)\\ &\dot{p}_i = -\alpha_1 a(\tau_i^i)^{-1}(p_i-l_i) - \alpha_2\sum_{j=1}^{N} a_{ij}(p_i-p_j) - \alpha_3 q_i\\ &\dot{q}_i = \alpha_2 \sum_{j=1}^{N} a_{ij}(p_i-p_j)\\ &\dot{\tau}_i = -\sum_{j=1}^{N} a_{ij}(\tau_i-\tau_j), i\in V \end{aligned}\right\} \tag{4}$$

where α_1, α_2 and α_3 are positive constants to be determined later, $\tau_i=[\tau^1{}_i, \tau^2{}_i, \ldots, \tau^N{}_i]^{\mathrm{T}}\in\mathbb{R}^N$ with τ^j_i being the jth component of τ_i, for each $i\in V$. The initial value $q_i(0)=0$, while $\tau_i(0)$ satisfies

$$\tau_i^j(0) = \begin{cases} 1, j=i\\ 0, j\neq i \end{cases} \tag{5}$$

Define $l=[l_1, l_2, \ldots, l_N]^{\mathrm{T}}$, $p=[p_1, p_2, \ldots, p_N]^{\mathrm{T}}$, $q=[q_1, q_2, \ldots, q_N]^{\mathrm{T}}$, $\tau=[\tau_1, \tau_2, \ldots, \tau_N]^{\mathrm{T}}$, $\tilde{\tau}$ = diag $(\tau^1{}_1, \tau_2{}^2, \ldots, \tau^N{}_N)$ and $\varXi$ = diag $(\upsilon_1, \upsilon_2, \ldots, \upsilon_N)$. Then the distributed algorithm (4) can be rewritten in the following compact form,

$$\left.\begin{aligned} &\dot{l} = -\alpha_1 H(l,p)\\ &\dot{p} = -\alpha_1 a\tilde{\tau}^{-1}(p-l) - \alpha_2 Lp - \alpha_3 q_i\\ &\dot{q} = \alpha_2 Lp\\ &\dot{\tau} = -(L\otimes I_N)\tau \end{aligned}\right\} \tag{6}$$

Under Assumption 2.1, we obtain that $\exp(-Lt)(t>0)$ is a nonnegative matrix with positive diagonal entries and $\lim\limits_{t\to+\infty}\exp(-Lt) = 1_N{}^{\mathrm{T}}$ [28]. By referring to the initial condition (5), it can be deduced that

$$\begin{aligned} &\lim_{t\to+\infty}\tau(t) = \lim_{t\to+\infty}\exp(-(L\otimes I_N)t)\tau(0) = \\ &(1_N\upsilon^{\mathrm{T}}\otimes I_N)\tau(0) = 1_N\otimes\upsilon \end{aligned} \tag{7}$$

Remark 3.1 In algorithm (4), $H_i(l_i, p_i)$ is the gradient information for seeking the NE. The dynamics of p_i are an average consensus for estimating the aggregate variable $\sigma(l)$. The q_i dynamics are designed for eliminating the average consensus error. The τ_i dynamics are needed to cancel the imbalance arising from the directed graphs, and the requirement on the left eigenvector of the Laplacian matrix associated with the zero eigenvalue thus can be removed.

Remark 3.2 Since $\exp(-Lt)(t>0)$ is a nonnegative matrix with positive diagonal entries, it can be proved that $\tau^i_i(t)>0$ for $t\geq 0$ by noting the initial condition (5). Thus, $\tilde{\tau}(t)$ is invertible. The Eq. (7) implies that $\lim\limits_{t\to+\infty}\tau_i^i(t) = {}_i$ and $\lim\limits_{t\to+\infty}\tilde{\tau}^{-1}(t) = \varXi^{-1}$.

3.2 Preliminary result

To facilitate the convergence analysis, some lemmas are presented in this subsection. Define $\bar{\omega} = [l^{\mathrm{T}}, p^{\mathrm{T}}, q^{\mathrm{T}}]^{\mathrm{T}}$. Then the dynamics of can be described as follows,

$$\dot{\bar{\omega}} = G_1(t,\bar{\omega}) + G_2(t,\bar{\omega}),$$

where

$$G_1(t,\bar{\omega}) = \begin{bmatrix} -\alpha_1 H(l,p)\\ -\alpha_1 a\varXi^{-1}(p-l) - \alpha_2 Lp - \alpha_3 q\\ \alpha_2 Lp \end{bmatrix},$$

$$G_2(t,\bar{\omega}) = \begin{bmatrix} 0\\ \alpha_1 a(\varXi^{-1}-\tilde{\tau}^{-1})(p-l)\\ 0 \end{bmatrix}.$$

The following lemma shows the optimality analysis of the concerned system.

Lemma 3.1 Under Assumption 2.1, let ${}^* = \left(\tilde{l}^*, \tilde{p}^*, \tilde{q}^*\right)$ be the equilibrium point of $\dot{\bar{\omega}} = G_1(t,\bar{\omega})$, then $\tilde{l}^*$ is the Nash equilibrium of the considered aggregative game.

Proof Let $\bar{\omega}^* = \left(\tilde{l}^*, \widetilde{p}^*, \widetilde{q}^*\right)$ be the equilibrium point of the system $\dot{\bar{\omega}} = G_1(t,\bar{w})$. Then, it is satisfied that

$$0_N = -\alpha_1 H\left(\widetilde{l}^*, \widetilde{p}^*\right) \tag{8a}$$

$$0_N = -\alpha_1 a\varXi^{-1}\left(\widetilde{p}^* - \widetilde{l}^*\right) - \alpha_2 L\widetilde{p}^* - \alpha_3\widetilde{q}^* \tag{8b}$$

$$0_N = \alpha_2 L\widetilde{p}^* \tag{8c}$$

According to Eq.(8c), we obtain that $\widetilde{p}_1^* = \widetilde{p}_2^* = \cdots = \widetilde{p}_N^*$. Note that υ is the positive left eigenvector of L associated with the zero eigenvalue. Left multiplying $\dot{q}$ by υ^{T} yields $\upsilon^{\mathrm{T}}\dot{q}=0$. This implies that

$$\upsilon^{\mathrm{T}}q(t)=\upsilon^{\mathrm{T}}q(0)=0 \tag{9}$$

Then multiplying both sides of Eq.(8b) by υ^{T}, we have $\upsilon^{\mathrm{T}}L=0^{\mathrm{T}}_N$ and $\upsilon^{\mathrm{T}}\Xi^{-1}=1^{\mathrm{T}}_N$. One then has $\sum_{i=1}^{N}\tilde{p}_i^*=\sum_{i=1}^{N}\tilde{l}_i^*$, $\tilde{p}^*=\frac{1}{N}1_N\sum_{i=1}^{N}\tilde{l}_i^*$. Thus, $F\left(\tilde{l}^*,\frac{1}{N}\sum_{i=1}^{N}\tilde{l}_i^*\right)=H\left(\tilde{l}^*\ \tilde{p}^*\right)=0_N$ is satisfied by using Eq.(8a). This implies that $\tilde{l}^*=l^*$, in other words, $\tilde{l}^*$ is the NE of the considered game by Lemma 2 in Ref. [29]. This completes the proof.

To proceed, introduce the coordinate transformations $\bar{l}=l-l^*,\bar{p}=p-\widetilde{p}^*,\bar{q}=q-\widetilde{q}^*$ and $\widetilde{\omega}=\bar{\omega}-\bar{\omega}^*=\left[\bar{l}^{\mathrm{T}},\bar{p}^{\mathrm{T}},\bar{q}^{\mathrm{T}}\right]^{\mathrm{T}}$. Then referring to Eq. (6), we can obtain the following system,

$$\dot{\widetilde{\omega}}=D_1(t,\widetilde{\omega})+D_2(t,\widetilde{\omega})+D_3(t) \tag{10}$$

where

$$D_1(t,\widetilde{\omega})=\begin{bmatrix}-\alpha_1 H(l,p)+\alpha_1 H(l^*,\widetilde{p}^*)\\ -\alpha_1 a\Xi^{-1}(\bar{p}-\bar{l})-\alpha_2 L\bar{p}-\alpha_3\bar{q}\\ \alpha_2 L\bar{p}\end{bmatrix},$$

$$D_2(t,\widetilde{\omega})=\begin{bmatrix}0\\ \alpha_1 a(\Xi^{-1}-\widetilde{\tau}^{-1})(\bar{p}-\bar{l})\\ 0\end{bmatrix},$$

$$D_3(t)=\begin{bmatrix}0\\ \alpha_1 a(\Xi^{-1}-\tilde{\tau}^{-1})(\widetilde{p}^*-l^*)\\ 0\end{bmatrix}.$$

To show the stability of Eq.(10), we establish an auxillary stability result on the truncated system

$$\dot{\widetilde{\omega}}=D_1(t,\widetilde{\omega}) \tag{11}$$

which is summarized in the following lemma.

Lemma 3.2 Under Assumption 2.1, there exist positive constants α_1, α_2 and α_3 such that the origin is a globally exponentially stable equilibrium point of the system (11).

Proof The dynamics of , and satisfy

$$\left.\begin{aligned}\dot{\bar{l}}&=-\alpha_1 H(l,p)+\alpha_1 H(l^*,\tilde{p}^*)\\ \dot{\bar{p}}&=-\alpha_1 a\Xi^{-1}(\bar{p}-\bar{l})-\alpha_2 L\bar{p}-\alpha_3\bar{q}\\ \dot{\bar{q}}&=\alpha_2 L\bar{p}\end{aligned}\right\} \tag{12}$$

Consider the following Lyapunov function candidate,

$$V(\bar{l},\bar{p},\bar{q})=V_1(\bar{l})+V_2(\bar{p}+\bar{q})+V_3(\bar{p}),$$

where $V_1(\bar{l})=\|\bar{l}\|^2, V_2(\bar{p}+\bar{q})=\frac{1}{2}(\bar{p}+\bar{q})^{\mathrm{T}}\Xi(\bar{p}+\bar{q})$ and $V_3(\bar{p})=\frac{1}{2}\bar{p}^{\mathrm{T}}\Xi\bar{p}$. It can be verified that

$$\delta_1\left(\|\bar{l}\|^2+\|\bar{p}\|^2+\|\bar{q}\|^2\right)\leqslant V(\bar{l},\bar{p},\bar{q})\leqslant \delta_2\left(\|\bar{l}\|^2+\|\bar{p}\|^2+\|\bar{q}\|^2\right),$$

where $\delta_1=\min\left\{1,\frac{1}{6}\lambda_{\min}(\Xi)\right\}$ and $\delta_2=\max\left\{1,\frac{3}{2}\lambda_{\max}(\Xi)\right\}$.

By referring to Eq. (3), we obtain that

$$\dot{V}_1(\bar{l})\leqslant -2\alpha_1\bar{l}^{\mathrm{T}}\left[\left(\tilde{v}+\frac{a}{N}\right)\bar{l}+a\bar{p}\right],$$

where $\widetilde{v}=\min_{i\in V}\{2v_i\xi^2_i\}$. Moreover, we have

$$\begin{aligned}\dot{V}_2(\bar{p}+\bar{q})=&-\alpha_1 a\|\bar{p}\|^2+\alpha_1 a\bar{p}^{\mathrm{T}}\bar{l}-\alpha_1 a\bar{q}^{\mathrm{T}}(\bar{p}-\bar{l})-\\ &\alpha_3\bar{p}^{\mathrm{T}}\Xi\bar{q}-\alpha_3\bar{q}^{\mathrm{T}}\Xi\bar{q},\end{aligned}$$

$$\dot{V}_3(\bar{p})=-\alpha_1 a\|\bar{p}\|^2+\alpha_1 a\bar{p}^{\mathrm{T}}\bar{l}-\alpha_2\bar{p}^{-\mathrm{T}}\Xi L\bar{p}-\alpha_3\bar{p}^{\mathrm{T}}\Xi\bar{q}.$$

With the obtained facts, one has

$$\begin{aligned}\dot{V}(\bar{l},\bar{p},\bar{q})\leqslant&-2\alpha_1\left(\tilde{v}+\frac{a}{N}\right)\|\bar{l}\|^2-\\ &2\alpha_1 a\bar{l}^{\mathrm{T}}\bar{p}-2\alpha_1 a\|\bar{p}\|^2+\\ &2\alpha_1 a\bar{p}^{\mathrm{T}}\bar{l}-\alpha_1 a\bar{q}^{\mathrm{T}}(\bar{p}-\bar{l})-2\alpha_3\bar{p}^{\mathrm{T}}\Xi\bar{q}-\\ &\alpha_3\lambda_{\min}(\Xi)\|\bar{q}\|^2-\alpha_2\bar{p}^{\mathrm{T}}\widehat{L}\bar{p},\end{aligned}$$

where $\widehat{L}=\frac{\Xi L+L^{\mathrm{T}}\Xi}{2}$.

By Lemma 2.2 in Ref. [11], the eigenvalues of $\widehat{L}$ can be ordered as $0=\lambda_1<\lambda_2\leq\lambda_3\leq\ldots\leq\lambda_N$. Then, there are orthonormal vectors $\frac{1}{\sqrt{N}}1_N$, ϕ_i, i=2, 3, …, N, such that $\widehat{L}1_N=0_N$ and $\widehat{L}\phi_i=\lambda_i\phi_i$. Since $\frac{1}{\sqrt{N}}1_N$, ϕ_2, ϕ_3, …, ϕ_N are a set of orthonormal bases, there exists a vector $d=[d_1, d_2, \ldots, d_N]^{\mathrm{T}}\in\mathbb{R}^N$, such that $=\Phi d$ with $\Phi=\left[\frac{1}{\sqrt{N}}1_N,\phi_2,\phi_3,\cdots,\phi_N\right]\in\mathbb{R}^{N\times N}$. Therefore, we have

$$\begin{aligned}\bar{p}^{\mathrm{T}}\widehat{L}\bar{p}=\mathrm{tr}\left(d^{\mathrm{T}}\Phi^{\mathrm{T}}\widehat{L}\Phi d\right)=\mathrm{tr}\left(\Phi^{\mathrm{T}}\widehat{L}\Phi dd^{\mathrm{T}}\right)=\\ \sum_{i=2}^{N}\lambda_i\phi_i^{\mathrm{T}}\phi_i d_i^2\geqslant\sum_{i=2}^{N}\lambda_2 d_i^2=\lambda_2\|\rho\|^2,\end{aligned}$$

where $\rho=[d_2, d_3, \ldots, d_N]^{\mathrm{T}}\in\mathbb{R}^{N-1}$.

It follows from the Eq. (9) that $^{\mathrm{T}}\Xi 1_N=0$. Then, we have

$$\begin{aligned}\bar{p}^{\mathrm{T}}\Xi\bar{q}=\bar{q}^{\mathrm{T}}\Xi\bar{p}=\bar{q}^{\mathrm{T}}\Xi\Phi d=\\ \frac{1}{\sqrt{N}}\bar{q}^{\mathrm{T}}\Xi 1_N d_1+\bar{q}^{\mathrm{T}}\Xi\Delta=\\ \bar{q}^{\mathrm{T}}\Xi\Delta\end{aligned} \tag{13}$$

where $\Delta=\sum_{i=2}^{N}d_i\phi_i$. Since ϕ_i, i=2,3,…,N are orthonormal vectors, we have $\|\Delta\|^2=\|\rho\|^2$. Note that $\upsilon_i<1$, $i=1, 2, \ldots, N$ and the Eq. (13) holds. By completing the squares, we obtain that

$$\begin{aligned}-2\alpha_3\bar{p}^{\mathrm{T}}\Xi\bar{q}\leqslant\frac{\alpha_3^2}{\delta_1}\|\Delta\|^2+\delta_1\|\bar{q}\|^2=\\ \frac{\alpha_3^2}{\delta_1}\|\rho\|^2+\delta_1\|\bar{q}\|^2,\end{aligned}$$

$$\begin{aligned}-\alpha_1 a\bar{q}^{\mathrm{T}}(\bar{p}-\bar{l})\leqslant\frac{a^2}{2\delta_2}\|\bar{q}\|^2+\\ \alpha_1^2\delta_2\|\bar{p}\|^2+\alpha_1^2\delta_2\|\bar{l}\|^2,\end{aligned}$$

where δ_1 and δ_2 are positive constants.

Then, it can be obtained that

DOI: 10.52396/JUSTC-2021-0153
JUSTC, 2022, 52(1): 5

$$\dot{V}(\bar{l},\bar{p},\bar{q}) \leqslant -\left[2\alpha_1\left(\tilde{v}+\frac{a}{N}\right)-\alpha_1^2\delta_2\right]\|\bar{l}\|^2- (2\alpha_1 a-\alpha_1^2\delta_2)\|\bar{p}\|^2- \left[\alpha_3\lambda_{\min}(\varXi)-\delta_1-\frac{a^2}{2\delta_2}\right]\|\bar{q}\|^2- \left(\alpha_2\lambda_2-\frac{\alpha_3^2}{\delta_1}\right)\|\rho\|^2.$$

Choose the positive constants α_1, α_2, α_3, δ_1 and δ_2 such that the following inequalities are satisfied,

$$\alpha_1\delta_2 < \min\left\{2\left(\tilde{v}+\frac{a}{N}\right),2a\right\},$$

$$\alpha_3 > \frac{1}{\lambda_{\min}(\varXi)}\left(\delta_1+\frac{a^2}{2\delta_2}\right),$$

$$\alpha_2 > \frac{\alpha_3^2}{\lambda_2\delta_1}.$$

One then has

$$\dot{V}(\bar{l},\bar{p},\bar{q}) \leqslant -\widehat{\alpha}\left(\|\bar{l}\|^2+\|\bar{p}\|^2+\|\bar{q}\|^2\right),$$

where

$$\widehat{\alpha} = \min\left\{2\alpha_1\left(\tilde{v}+\frac{a}{N}\right)-\alpha_1^2\delta_2, 2\alpha_1 a-\alpha_1^2\delta_2, \alpha_3\lambda_{\min}(\varXi)-\delta_1-\frac{a^2}{2\delta_2}\right\}.$$

This reveals that the origin is a globally exponentially stable equilibrium point of the system $\dot{\widetilde{\omega}} = D_1(t,\widetilde{\omega})$ by applying Theorem 4.10 in Ref. [30]. The proof is thus completed.

Remark 3.3 Lemma 3.2 indicates that if υ is available in advance, we can design a distributed NE seeking algorithm of the form $\dot{\bar{w}} = G_1(t,\bar{w})$ such that $l(t)$ can exponentially converge to the desired NE l^*.

3.3 Main result

We show the convergence of algorithm (6) in this subsection. The main result of this paper is given below.

Theorem 3.1 Under Assumption 2.1, there exist positive constants α_1, α_2 and α_3 such that the trajectory of $l(t)$ generated by algorithm (6) exponentially converges to the NE l^* of the considered aggregative game.

Proof First, recall the dynamics of $\widetilde{\omega}$ in Eq.(10). According to Lemma 3.2, there exist positive constants α_1, α_2 and α_3 such that the origin is a globally exponentially stable equilibrium point of the system (11).

Second, note that $D_2(t, 0_{3N})=0_{3N}$. Moreover, it is satisfied that

$$\|D_2(t,\widetilde{\omega})\| \leqslant 2\alpha_1 a\gamma(t)\|\widetilde{\omega}\|,$$

where $\gamma(t)=\max i\in V\{|\upsilon_i^{-1}-(\tau_i^i(t))^{-1}|\}$. Clearly, $\gamma(t)$ satisfies

$$\lim_{t\to+\infty}\gamma(t) = 0.$$

It then follows from Corollary 9.1 and Lemma 9.5 in Ref.[30] that the origin is a globally exponentially stable equilibrium point of the system $\dot{\widetilde{\omega}} = D_1(t,\widetilde{\omega})+D_2(t,\widetilde{\omega})$. Thus, according to Theorem 4.14 in Ref.[30], there is a function $V_0(t,\widetilde{\omega})$ satisfying the inequalities $d_1\|\widetilde{\omega}\|^2 \leqslant V_0(t,\widetilde{\omega}) \leq d_2\|\widetilde{\omega}\|^2$, $\dfrac{\partial V}{\partial t}+\dfrac{\partial V}{\partial\widetilde{\omega}}[D_1(t,\widetilde{\omega})+D_2(t,\widetilde{\omega})] \leqslant -d_3\|\widetilde{\omega}\|^2$, $\left\|\dfrac{\partial V}{\partial\widetilde{\omega}}\right\| \leqslant d_4\|\widetilde{\omega}\|$ for some positive constants d_1, d_2, d_3 and d_4.

Finally, exponential stability of system (10) is presented. It then follows from the proof of Theorem 2 in Ref.[14] that there exist two positive constants c_1 and $c_2\left(c_2 \neq \dfrac{d_3}{2d_2}\right)$ such that $\gamma(t)\leq c_1 e^{-c_2 t}$. Thus, it can be verified that $\| D_3(t) \| \leq c_3 e^{-c_2 t}$, where $c_3 = a\alpha_1 c_1\|\tilde{p}^*-l^*\|$. Therefore, the derivative of $V_0(t,\widetilde{\omega})$ along the trajectories of the system (10) can be described as

$$\dot{V}_0(t,\widetilde{\omega}) \leqslant -d_3\|\widetilde{\omega}\|^2+d_4c_3\|\widetilde{\omega}\|e^{-c_2 t}.$$

Taking $V_0^*(t,\widetilde{\omega}) = \sqrt{V_0(t,\widetilde{\omega})}$ leads to

$$\dot{V}_0^*(t,\widetilde{\omega}) \leqslant -d_5V_0^*(t,\widetilde{\omega})+d_6e^{-c_2 t},$$

where $d_5 = \dfrac{d_3}{2d_2}$ and $d_6 = \dfrac{d_4c_3}{2\sqrt{d_1}}$. By the comparison lemma in Ref.[30], it can be obtained that

$$V_0^*(t,\widetilde{\omega}) \leqslant V_0^*(0,\widetilde{\omega})e^{-d_5 t}+d_6\int_0^t e^{-d_5(t-\tau)}e^{-c_2\tau}\mathrm{d}\tau \leqslant V_0^*(0,\widetilde{\omega})e^{-c_4 t}+c_5e^{-c_4 t},$$

where $c_4=\min\{d_5, c_2\}$ and $c_5 = 2\left|\dfrac{d_6}{d_5-c_2}\right|$. This implies that $V_0^*(0,\widetilde{\omega})$ converges to zero exponentially. One can further claim that $\widetilde{\omega}(t)$ converges to zero exponentially. It then follows that $l(t)$ converges to l^* exponentially, and the proof is thus completed.

Remark 3.4 The related work[20] mainly focuses on the energy consumption problem of HVAC systems over undirected graphs. On the contrary, this paper considers the more general and also more challenging scenarios in directed graphs. It is worth noting that the selection of the control gains α_2 and α_3 in algorithm (6) relies on the second smallest eigenvalue of $\widehat{L}$ and the minimum value of the left eigenvector υ. Some distributed algorithms[31,32] can be utilized to estimate them in advance.

Remark 3.5 A relevant result can be found in Ref.[33], where a distributed algorithm was developed to address the Nash equilibrium seeking problem. However, the following two significant differences should be noted. Firstly, this work ensures that the trajectory of $l(t)$ generated by the designed algorithm converges to the desired NE exponentially, while the algorithm in Ref.[33] can only achieve asymptotic convergence according to Theorem 2 in Ref.[33]. Secondly, this work focuses on a more practical scenario in HVAC systems, while it is not considered in Ref.[33].

4 An example

In this section, a numerical example is given to illustrate the effectiveness of the proposed algorithm.

Consider an energy consumption game of eight electricity users that are equipped with HVAC systems. The cost function for each electricity user i is given as follows.

$$C_i(l_i,\sigma(l)) = v_i\xi_i^2\left(l_i-\widehat{l_i}\right)^2+ [a\sigma(l)+p_0]l_i, 1\leqslant i\leqslant 8,$$

where $\sigma(l) = \dfrac{1}{8}\sum_{j=1}^{8} l_j$, $v_i\xi_i^2 = 2, a = 4, p_0 = 5$ and $\widehat{l_i} = 45+2i$.

The communication digraph among the electricity users is described by Fig. 2. It can be verified that Assumption 2.1 is satisfied. Moreover, it can be obtained that $\lambda_{\min}(\varXi) = \dfrac{1}{10}$,

DOI: 10.52396/JUSTC-2021-0153
JUSTC, **2022**, 52(1): 5

$\lambda_2 = \dfrac{234}{3691}$ and the NE solution l^* of the game is

$$\left[\frac{2846}{153}, \frac{3118}{153}, \frac{1130}{51}, \frac{3662}{153}, \frac{3934}{153}, \frac{1402}{51}, \frac{4478}{153}, \frac{4750}{153}\right]^{\mathrm{T}}.$$

In the simulation, choose $\alpha_1 = \alpha_2 = \alpha_3 = 10$. The simulation results are shown in Figs. 3 and 4. It is observed from the figures that the error between the players' strategy $l(t)$ and the NE l^* will converge to zero before 30s. In other words, the players' strategy $l(t)$ converges to the NE l^* eventually. This is consistent with the theoretical results.

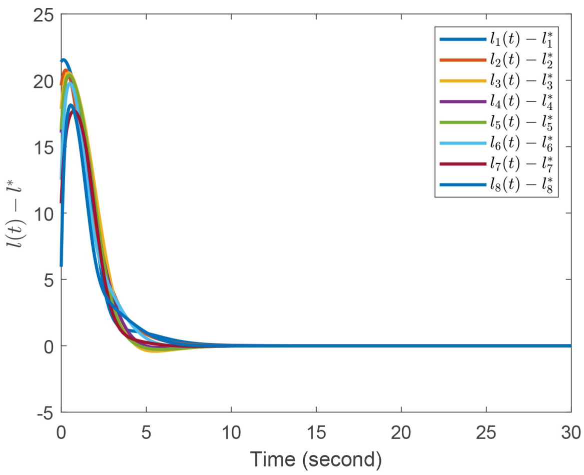


Fig. 2. A directed graph G of the energy consumption game.

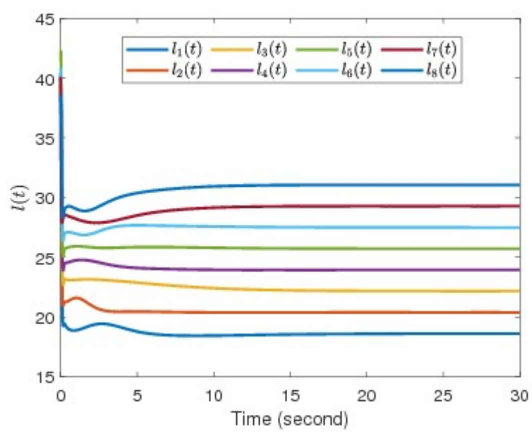


Fig. 3. Trajectories of the generated NE seeking signal: $l(t)$.

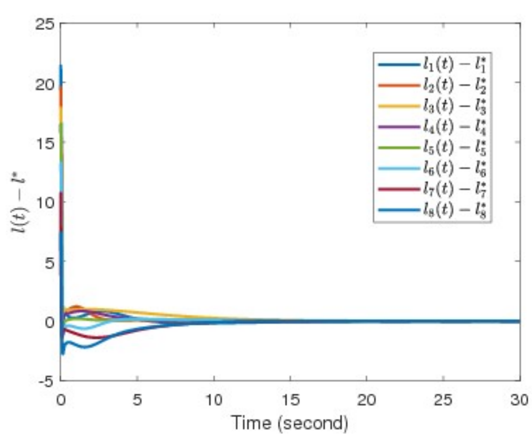


Fig. 4. Trajectories of $l(t)$-l^*.

5 Conclusions

In this paper, a novel distributed Nash equilibrium seeking algorithm is proposed to solve the energy consumption game of HVAC systems over directed graphs. The proposed algorithm is capable of addressing the challenge issue resulting from directed graphs without a prior knowledge of the left eigenvector associated with the eigenvalue zero of the Laplacian matrix. A numerical example is provided to demonstrate the effectiveness of the developed algorithm. Our future work will consider the energy consumption problem for more practical multi-agent systems.

Acknowledgements

This work is supported the Beijing Natural Science Foundation (4192068), USTC Research Funds of the Double First-Class Initiative (YD2100002002).

Conflict of interest

The authors declare that they have no conflict of interest.

Biographies

Xinyu Liu received the BS degree in statistics from Henan University, Kaifeng, China, in 2019. She is currently pursuing the MS degree with the Department of Automation, University of Science and Technology of China, Hefei, China. Her current research interests include distributed game and multi-agent systems.

Xinghu Wang received his BSc and PhD degrees from Shandong University, Weihai, and University of Science and Technology of China in 2007 and 2012, respectively. From March 2013 to January 2015, he was a postdoctoral fellow in Academy of Mathematics and Systems Science, Chinese Academy of Sciences. He is currently with Department of Automation, University of Science and Technology of China. His research interests include nonlinear control and multi-agent systems.

References

[1] Li L, Sun Z. Dynamic energy control for energy efficiency improvement of sustainable manufacturing systems using Markov decision process. *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* **2013**, *43* (5): 1195–1205.

[2] Deng R, Yang Z, Chen J, et al. Residential energy consumption scheduling: A coupled-constraint game approach. *IEEE Transactions on Smart Grid,* **2014**, *5* (3): 1340–1350.

[3] Xu Z, Liu S, Hu G, et al. Optimal coordination of air conditioning system and personal fans for building energy efficiency improvement. *Energy and Buildings,* **2017**, *141*: 308–320.

[4] Rao D M K K V, Ukil A. Modeling of room temperature dynamics for efficient building energy management. *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* **2020**, *50* (2): 717–725.

[5] Xu J, Peter B. Luh P, Blankson W E, et al. An optimization-based approach for facility energy management with uncertainties. *HVAC&R Research,* **2005**, *11* (2): 215–237.

[6] Sun B, Luh P B, Jia Q S, et al. Building energy management: Integrated control of active and passive heating, cooling, lighting, shading, and ventilation systems. *IEEE Transactions on Automation*

DOI: 10.52396/JUSTC-2021-0153
JUSTC, **2022**, 52(1): 5

Science and Engineering, **2013**, *10* (3): 588–602.
[7] Qu G, Zaheeruddin M. Real-time tuning of PI controllers in HVAC systems. *International Journal of Energy Research,* **2004**, *28* (15): 1313–1327.
[8] Bai J, Wang S, Zhang X. Development of an adaptive smith predictor-based self-tuning PI controller for an HVAC system in a test room. *Energy and Buildings,* **2008**, *40* (12): 2244–2252.
[9] Pinzon J A, Vergara P P, Da Silva L C P, et al. Optimal management of energy consumption and comfort for smart buildings operating in a microgrid. *IEEE Transactions on Smart Grid,* **2019**, *10* (3): 3236–3247.
[10] Ren W, Beard R W. Distributed consensus in multivehicle cooperative control: Theory and applications. Springer Publishing Company, Incorporated, 2007. https://linkspringer.53yu.com/book/10.1007%2F978-1-84800-015-5.
[11] Mei J, Ren W, Chen J. Distributed consensus of second-order multi-agent systems with heterogeneous unknown inertias and control gains under a directed graph. *IEEE Transactions on Automatic Control,* **2016**, *61* (8): 2019–2034.
[12] Wang Z, Wang D, Gu D. Distributed optimal state consensus for multiple circuit systems with disturbance rejection. *IEEE Transactions on Network Science and Engineering,* **2020**, *7* (4): 2926–2939.
[13] Wang D, Wang Z, Wen C. Distributed optimal consensus control for a class of uncertain nonlinear multiagent networks with disturbance rejection using adaptive technique. *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* **2021**, *51* (7): 4389–4399.
[14] Zhu Y, Ren W, Yu W, et al. Distributed resource allocation over directed graphs via continuous-time algorithms. *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* **2021**, *51* (2): 1097–1106.
[15] Yu L, Xie D, Jiang T, et al. Distributed real-time HVAC control for cost-efficient commercial buildings under smart grid environment. *IEEE Internet of Things Journal,* **2018**, *5* (1): 44–55.
[16] Zhang X, Shi W, Yan B, et al. Decentralized and distributed temperature control via HVAC systems in energy efficient buildings. 2017, arXiv: 1702.03308. https://arxiv.53yu.com/abs/1702.03308.
[17] He X, Fang X, Yu J. Distributed energy management strategy for reaching cost-driven optimal operation integrated with wind forecasting in multimicrogrids system. *IEEE Transactions on Systems, Man, and Cybernetics. Systems,* **2019**, *49* (8): 1643–1651.
[18] Yang Y, Hu G, Spanos C J. HVAC energy cost optimization for a multizone building via a decentralized approach. *IEEE Transactions on Automation Science and Engineering,* **2020**, *17* (4): 1950–1960.
[19] Ma K, Hu G, Spanos C J. Distributed energy consumption control via real-time pricing feedback in smart grid. *IEEE Transactions on Control Systems Technology,* **2014**, *22* (5): 1907–1914.
[20] Ye M, Hu G. Game design and analysis for price-based demand response: An aggregate game approach. *IEEE Transactions on Cybernetics,* **2017**, *47* (3): 720–730.
[21] Li Z, Ding Z, Sun J, et al. Distributed adaptive convex optimization on directed graphs via continuous time algorithms. *IEEE Transactions on Automatic Control,* **2018**, *63* (5): 1434–1441.
[22] Tang Y, Yi P. Nash equilibrium seeking under directed graphs. 2020, arXiv: 2005.10495. https://arxiv.53yu.com/abs/2005.10495.
[23] Zhang J, Liu L, Ji H. Exponential convergence of distributed optimal coordination for linear multi-agent systems over general digraphs. 39th Chinese Control Conference. Shenyang, China: IEEE, 2020: 5047-5051. https://ieeexploreieee.53yu.com/abstract/document/9189181.
[24] Hiriart-Urruty J B, Lemaréchal C. Convex Analysis and Minimization Algorithms I: Fundamentals. Berlin Heidelberg: Springer-Verlag,1993.https://xs.dailyheadlines.cc/books?hl=zh-CN&lr&idBen6nm_yapMC&oifnd&pgPR5&dq+Hiriart-Urruty+J+B,+Lemar%C3%A9chal+C.+Convex+Analysis+and+Minimization+Algorithms+I:+Fundamentals.+Berlin+Heidelberg:+Springer-Verlag,+1993.&otsF-CuS39IuZ&sig2m4HG9SrFhw2YJtkdGKotUZ3AAE.
[25] Godsil C, Royle G. Algebraic Graph Theory. New York: Springer-Verlag, 2001. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.479.536&reprep1&typepdf.
[26] Scutari G, Facchinei F, Pang J S, et al. Real and complex monotone communication games. *IEEE Transactions on Information Theory,* **2014**, *60* (7): 4197–4231.
[27] Afram A, Janabi-Sharifi F. Theory and applications of HVAC control systems-A review of model predictive control (MPC). *Building and Environment,* **2014**, *72*: 343–355.
[28] Zhu Y, Yu W, Wen G, et al. Continuous-time coordination algorithm for distributed convex optimization over weight-unbalanced directed networks. *IEEE Transactions on Circuits and Systems Ⅱ: Express Briefs,* **2019**, *66* (7): 1202–1206.
[29] Deng Z, Liang S. Distributed algorithms for aggregative games of multiple heterogeneous Euler-Lagrange systems. *Automatica,* **2019**, *99*: 246–252.
[30] Khalil H K. Nonlinear Systems. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.
[31] Charalambous T, Rabbat M G, Johansson M, et al. Distributed finite-time computation of digraph parameters: Left-eigenvector, out-degree and spectrum. *IEEE Transactions on Control of Network Systems,* **2016**, *3* (2): 137–148.
[32] Aragues R, Shi G, Dimarogonas D V, et al. Distributed algebraic connectivity estimation for adaptive event-triggered consensus. American Control Conference. Montreal, QC, Canada: IEEE, 2012: 32-37. https://ieeexploreieee.53yu.com/abstract/document/6315110.
[33] Zhu Y, Yu W, Wen G, et al. Distributed Nash equilibrium seeking in an aggregative game on a directed graph. *IEEE Transactions on Automatic Control,* **2021**, *66* (6): 2746–2753.

DOI: 10.52396/JUSTC-2021-0153
JUSTC, **2022**, 52(1): 5