

$\gamma\gamma$ physics analyses at BES III

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Abstract: The BES III Collaboration has recently embarked on a two-photon physics program. Its main motivation is given by the large uncertainty of the contribution of hadronic light-by-light scattering to the anomalous magnetic moment of the muon α_μ , and the need of electromagnetic transition form factors (TFF) as experimental input to improve the calculations. Data acquired with the BES III detector at center-of-mass energies from 3.77 to 4.6 GeV allow for the determination of TFFs of light pseudoscalar mesons. The measurements are performed with a single-tag technique and result in unprecedented accuracy at momentum transfers below 2 GeV^2 , the region of highest importance for the calculations of α_μ . Employing the same approach, the first double-tagged measurement of the pion transition form factor has been started. It is the first step towards a model independent parameterization of the TFF of the π^0 . Additionally, measurements of multi-meson final states have been engaged.

Key words: anomalous magnetic moment of the muon; hadronic light-by-light scattering; pseudoscalar mesons; transition form factor; single-tag measurement
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BES III 上双光子物理分析

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摘要: BES III 合作组最近开始双光子物理研究, 主要源于强子 light-by-light 散射对缪子反常磁矩的贡献有很大不确定性, 电磁跃迁形状因子作为实验输入是改善计算精度的需要。在 BES III 探测器上获得的质心能量 3.77 GeV 到 4.6 GeV 的数据使得我们能够测量轻赝标量介子的跃迁形状因子。在动量转移低于 $2 (\text{GeV}/c)^2$ 时单标记技术测量的结果达到前所未有的精确, 该区域对于 α_μ 的计算十分重要, 并且也首次开始了 π 介子跃迁形状因子的双标记测量。这是 π^0 跃迁形状因子模型无关参数化研究的第一步。另外, 多介子末态测量也在进行中。

关键词: 缪子反常磁矩; 强 light-by-light 散射; 赝标介子; 跃迁形状因子; 单标记测量

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0 Introduction

The anomalous magnetic moment of the muon is one of the most precisely known observables in particle physics. Over the recent decades enormous efforts have been made to reach a precision in the order of 10^{-10} in the direct experimental measurement, as well as in the Standard Model prediction of α_μ . However, there remains a discrepancy of 3 to 4 standard deviations between the measured and the calculated values of α_μ . The potential origin of this difference has triggered a lot of activity, since, even if not being significant enough to claim a discovery, it might be a hint for New Physics.

Newly proposed experiments aim at a fourfold improvement of the current precision of the direct measurements of α_μ ^[1-2]. These experimental endeavors have given rise to numerous theory efforts in order to improve the Standard Model prediction on the same level. While the dominating QED contribution has been calculated including corrections up to tenth order^[3], and the weak contribution also being well understood by means of perturbation theory^[4], the error of the prediction of α_μ is completely dominated by the hadronic contributions. The main challenge is that QCD cannot be handled by perturbative methods at the relevant energy regime.

The largest hadronic contribution α_μ^{hVP} is due to the hadronic vacuum polarization. It is handled within a dispersive framework, which requires $\sigma(e^+e^- \rightarrow \text{hadrons})$ as experimental input^[5]. Various laboratories world wide have measured hadronic cross section in scan experiments and exploiting the technique of Initial State Radiation (ISR), as has been reported in this workshop. These results can be used to significantly reduce the uncertainty of α_μ related to hadronic vacuum polarization in the near future.

Another hadronic contribution to the muon anomaly α_μ is due to the hadronic light-by-light scattering (HLbL), α_μ^{HLbL} . In contrast to α_μ^{hVP} , it cannot be directly related to experimentally measurable quantities. The HLbL involves the coupling of real and virtual photons to virtual hadronic states, described by

transition form factors (TFF), while only transitions of real mesons to real and virtual photons can be measured. Calculations of the HLbL process are possible in the low energy regime by means of chiral perturbation theory, or at high energies, using perturbative QCD (pQCD). The energy scale relevant for the magnetic moment of the muon is, however, the intermediate regime, which cannot be addressed by perturbative means. Thus, hadronic models have been developed and used for the calculations of α_μ^{HLbL} . The contributions of individual hadronic processes to the models is dominated by the transitions of the pseudoscalar mesons π^0 , η and η' , followed by multi-meson states and higher resonances^[6]. The validity of the models can be tested with measurements of the momentum transfer dependence of the respective TFF. The uncertainties of the resulting values of α_μ^{HLbL} depend strongly on the models. Two of the frequently quoted examples are the so called "Glasgow Consensus" by Prades, de Rafael and Vainshtain^[7] and the recent calculations by Jegerlehner and Nyffeler^[8]. The relative uncertainties of the respective results of α_μ^{HLbL} differs by approximately 30%.

In order to reduce the model dependency for the calculation of α_μ^{HLbL} , recently, the development of data driven approaches has been started. The goal is to provide a more reliable estimate of the uncertainty of α_μ^{HLbL} , based on dispersive analysis^[9-10]. Important inputs to these calculations are the TFF of π^0 , of two pions and the pion polarizabilities.

The need for additional data on TFF, in order to improve the calculations of HLbL and, thus, to improve the prediction of α_μ , has motivated the $\gamma\gamma$ physics program, recently started at BES III. Experiments at e^+e^- colliders can provide data on space-like TFF through the measurement of meson production in two-photon collision. The differential cross section of this production mechanism is directly proportional to the square of the TFF, but it is peaked towards small polar angles of the scattered leptons. Common detector setups cannot cover smallest scattering angles, due to the beam optics necessary to collide the beam at the desired interaction region.

Thus, the momentum dependence of the TFF is usually studied in single-tag analyses, where only one of the scattered leptons is reconstructed in the detector volume along with the produced hadronic system. The second lepton is expected to escape detection at smallest scattering angles, i. e. along the beam axis. The corresponding momentum transfer is small and the exchanged photon is quasi-real. Thus, the form factor $F(Q_1^2, Q_2^2)$ depends only on a single momentum transfer $F(Q^2)$.

Available measurements of the π^0 , η and η' TFF are dominated by the recent measurements at B-factories^[11-12]. The TFF has been determined for momentum transfers Q^2 from 4 GeV² to 40 GeV² and the results triggered a discussion on the applicability of pQCD in this energy range, by the so called Belle-BaBar-puzzle. For α_μ^{HLL} , however, these data are less influential. Here, the Q^2 -behavior of the TFF at approximately 1.5 GeV² is most important^[13-14]. At these energies only measurements from the CELLO and CLEO experiments^[15-16] are available, which suffer from comparatively low statistics.

The BES III experiment can contribute in exactly this region of low momentum transfer and can provide data on the TFF dependence with high statistical accuracy.

1 The BES III detector at BEPC II

The BES III experiment^[17] is located at the symmetric $e^+ e^-$ collider BEPC II, which is operated at the IHEP in Beijing (China). Data can be collected in a center-of-mass energy range from 2.0 GeV to 4.6 GeV. The BES III detector setup consists of a helium-based drift chamber surrounding the Beryllium beam pipe, a plastic scintillator time-of-flight system, and a CsI(Tl) electromagnetic calorimeter, which are placed inside the bore of a superconducting solenoidal magnet providing a 1.0 T magnetic field. The flux-return yoke is instrumented with resistive plate counters and serves as muon chamber. The detector covers 93% of the solid angle. Momenta of charged particles and photon energies are measured with a resolution of 0.5% and 2.5% at 1 GeV, respectively.

Data taking has been routinely performed since 2009. In the recent data taking campaigns BES III has collected $1.25 \times 10^9 e^+ e^- \rightarrow J/\psi$ events, more than $500 \times 10^6 e^+ e^- \rightarrow \psi(2S)$ events, 2.9 fb^{-1} at the $\psi(3770)$ peak, and more than 5 fb^{-1} in the center-of-mass region above 4 GeV, which are devoted to studies of the charmonia and charmonium-like states^[18-20]. This data set comprises the world's largest samples of $J/\psi, \psi(2S)$, and $\psi(3770)$ mesons. Additional data have been acquired for a τ -mass scan and a high statistics R -scan. Based on these data, the BES III collaboration pursues a physics program^[21], which focuses on charmonium spectroscopy, charm physics, light hadron spectroscopy, τ physics, and R -measurements.

The investigations in the field of $\gamma\gamma$ physics make use of all larger data sets, whereas the data taken at $\sqrt{s} = 3.77 \text{ GeV}$ constitute the largest individual sample. It has been used to study the TFF of π^0 , η and η' .

2 Transition form factors of π^0 , η and η'

The analysis of the TFF of π^0 , η and η' is based on a single-tag technique: Only the decay products of the produced mesons $P = \pi^0, \eta, \eta'$ and one of the two scattered leptons of the reaction $e^+ e^- \rightarrow Pe^+ e^-$ are measured in the detector. The other lepton is reconstructed from four-momentum conservation and it is required to have a scattering angle smaller than 8° , which corresponds to the radius of the beam pipe at the end of the detector.

The π^0 meson is reconstructed from its decay into two photons. The main source of background in the event selection are QED processes, such as radiative Bhabha scattering. Hard radiative photons in combination with a soft photon from any secondary process easily mimic the invariant mass of a pion. Three conditions are applied on the measured photon pairs to successfully reduce this background: A limit on the helicity angle of the photons, which is the angle between one of the photons in the rest frame of the pion and the direction of motion in the laboratory frame; A threshold on the scattering angle of the pion; A

minimum polar angle difference of the two photons in the laboratory frame. Hadronic background from decays of $\psi(3770)$ into pairs of D mesons, the radiative return to $\psi(2S)$ and J/ψ , and from the $q\bar{q}$ continuum is rejected by a condition, which has been introduced in the BaBar analysis^[11], in order to suppress initial state radiation in the signal channels. Since it is based on energy and momentum conservation, it is also efficiently suppressing background from incompletely reconstructed hadronic sources and other two-photon production channels. Remaining event candidates show clear signal peaks, not only of the π^0 , but also of the η meson in the invariant mass distribution of the two photons.

In order to extract the TFF of the π^0 , remaining background is subtracted bin-by-bin from the differential distribution of momentum transfer. For every bin in Q^2 the invariant mass spectrum of the two photon system is fitted and the number of signal events above the continuous background is counted. The background free Q^2 spectrum is converted to the TFF-distribution by normalizing to the reconstruction efficiency and luminosity and finally dividing out the point-like cross section, by means of a Monte Carlo simulation based on the Wess-Zumino-Witten term^[22-23].

The data taken at the $\psi(3770)$ peak statistically only allow for a measurement of the TFF of the π^0 at momentum transfers of $0.3 \leq Q^2 \text{ (GeV}^2\text{)} \leq 3.1$. This corresponds to the kinematical region, which is of importance as input for the calculations on hadronic light-by-light scattering. The statistical accuracy obtained with the analysis scheme described above is unprecedented for $Q^2 \leq 1.5 \text{ GeV}^2$; for larger momentum transfers it is still compatible with the CLEO^[16] result. The systematics are still under study, where the largest contribution comes from the method of background subtraction. The publication of the result is expected in the near future.

An improved understanding of the contribution of the π^0 TFF to α_μ^{HLbL} is, however, only sufficient on the level of the current experimental precision of α_μ . In view of the announced new direct measurements and

their planned four-fold improvement in accuracy, it is necessary to include and to improve the understanding of the contributions of η and η' .

A dedicated measurement of the respective TFF has been started, based on the decay modes $\eta \rightarrow \pi^0 \pi^+ \pi^-$ and $\eta' \rightarrow \pi^0 \pi^+ \pi^-$, which result in the same final state, taking into account the subsequent decays of π^0 and η into two photons. The analysis strategy is analogous to the analysis of the π^0 production described above, except for the conditions to suppress radiative Bhabha scattering, which is not of concern in this channel. Hadronic background contributions involve decays of ω and φ mesons into three pions. In the analysis of the η mesons, there is also background from incompletely reconstructed decays of η' mesons, produced in the two-photon collisions. By kinematically fitting the decay systems to the masses of η and η' , respectively, background is suppressed completely.

Based on this selection, the TFF can be extracted from data taken at $\sqrt{s} = 3.77 \text{ GeV}$, for momentum transfers between $0.3 \leq Q^2 \text{ (GeV}^2\text{)} \leq 3.5$. The statistical accuracy is compatible with the published results of the CELLO and CLEO experiments^[15-16]. However, it should be noted that in contrast to the previous measurements, here, only one decay channel has been evaluated. Adding more decay channels in the analysis, as well as analyzing the remaining data sets will improve the BES III result significantly.

3 Transition form factors of $\pi\pi$

Recently, the BES III collaboration started a measurements of $\pi^+ \pi^-$ pair production in two-photon collisions. The measurement is not only motivated by its relevance for α_μ^{HLbL} , especially for the new dispersive calculations. It is also of interest, due to the possibility of extracting parameters of resonances in the two-pion final state and the possibility to study pion rescattering effects at low invariant masses. Previous measurements used untagged measurements, i. e. both scattered leptons escaped detection, and were restricted to invariant masses larger than approximately $500 \text{ MeV}/c^2$. Only a few data points at lower masses

have been published by the MARK II experiment^[24-26].

The analysis of the $\pi^+ \pi^-$ final state at BES III follows the single-tag strategy, successfully used in the analysis of single pseudoscalar meson production. The main background contributions stem from the two-photon production of muon pairs, and the radiative Bhabha scattering process, in which the photon couples to a ρ -meson, decaying into two pions.

The QED background of muon production is well understood from the $\gamma\gamma$ physics studies at LEP^[27]. Monte Carlo generators are used to produce training samples for the application of an artificial neural network, which was already successfully applied in the measurement of hadronic cross sections, to separate pions from muons^[28]. Muon background surviving the condition found with the neural net is subtracted using Monte Carlo distributions.

Background involving the ρ -meson is subtracted by fitting the clearly visible peak in the $\pi^+ \pi^-$ invariant mass with the Kühn-Santamaria parameterization^[29]. The remaining events allow to study the production of $\pi^+ \pi^-$ in bins of the momentum transfer Q^2 , the pion invariant mass, and the pion helicity angle $\cos\theta^*$. This is the first measurement in the invariant mass region of $2m_\pi \leq M_{\pi\pi} (\text{GeV}) \leq 2.0$ and the momentum transfer region of $0.2 \leq Q^2 (\text{GeV}^2) \leq 2.0$ with a full coverage of the helicity angle.

4 Double-tagged measurements

In case of common detector setups at $e^+ e^-$ colliders, double-tagged measurements refer to the analysis of events, where both leptons have been scattered into the detector volume, i. e. the scattering angles with respect to the beam axis are in general larger than approximately 20° . Thus, the momentum transfer of each lepton is large. Due to vanishing cross sections for events with the corresponding kinematics, currently, experimental information on TFFs depending on two virtualities $F(Q_1^2, Q_2^2)$ is not available.

First exploratory studies for a double-tagged measurement of the π^0 TFF have been started at BES III. The aim is to exploit the large data sets, originally collected for charm physics and charmonium-like

spectroscopy, which correspond to almost 8 fb^{-1} . Monte Carlo studies show that a direct measurement of $F(Q_1^2, Q_2^2)$ is possible for $0.5 \leq Q_1^2, Q_2^2 (\text{GeV}^2) \leq 1.5$, which, depending on the reconstruction efficiency, can already have a significant impact on α_μ^{HLBL} .

The available statistics at BES III should at least be sufficient for a comparison between two hadronic models, which was recently suggested by Nyffeler. The momentum transfer dependence of the TFF in VMD and in the LMD + V model^[13] differs by a damping factor of Q^2 . The difference could be as large as 25% for $F(1 \text{ GeV}^2, 1 \text{ GeV}^2)$, and, thus, be resolved by a measurement.

Another aspect of double-tagged measurements is related to the production mechanism in two-photon collisions. The general term for the cross section can be separated into individual terms depending on the transverse or longitudinal polarization of the individual photons, the relative parallel or perpendicular polarization of the two photons and the helicity of the two-photon system^[30]. Another parameter is the dihedral angle of the leptons in the rest frame of the two photons. It can be used to separate multi-meson and tensor contributions to the TFF. Its determination requires knowledge of the four-momentum of both scattered leptons. In single-tag measurements there is, however, a large uncertainty on the azimuthal angle of the untagged lepton. Double-tagged measurements, in turn, suffer from a vanishing cross section.

A way out is the installation of special tagging detectors, which cover small angles. In this way, double-tagged measurements are no longer restricted to events with two large virtualities of the exchanged photons. A first tagging detector has been installed at BES III. It is a sampling calorimeter, made from lead and scintillating fibers, which covers polar angles from 1 to 10 m · rad. Its benefits will be tested in the upcoming data taking periods. The tagging detector can also be used to measure photons emitted by initial state radiation. Motivated by the additional use, there are plans to replace the current detector design with crystal calorimeters, which will be installed in both hemispheres of the BES III setup.

5 Conclusion

The $\gamma\gamma$ physics program at BES III is motivated by the need of new, high precision data on transition form factors of pseudoscalar mesons as input to the calculations of the contribution of hadronic light-by-light scattering to the anomalous magnetic moment of the muon. The measurement of the π^0 TFF results in unprecedented statistical accuracy for momentum transfers in the region of $0.3 \leq Q^2 (\text{GeV}^2) \leq 1.5$ and compatible precision with previous measurements for larger Q^2 . The results for η and η' are currently limited in statistics by restricting the analysis to only a single decay mode.

The analysis of $\pi^+ \pi^-$ in two-photon collisions will provide the first single-tag measurement at low invariant masses and small momentum transfers, with the full coverage of the pion helicity angle. The analysis is being extended to the neutral channels $\pi^0 \pi^0$, $\pi^0 \eta$ and $\eta\eta$.

In addition to the single-tag studies, first double-tagged investigations have been started. It is the first attempt to obtain a direct and model independent parameterization of the TFF of π^0 .

The great potential of the BES III experiment, to contribute valuable information on the field of $\gamma\gamma$ physics is currently being extended by the installation of tagging detectors. These add new prospects to the physics program, allowing to measure the scattered leptons with small momentum transfer, which has, so far, escaped detection.

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