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Design of two-channel causal-stable IIR filter banks by spectral factorization

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Abstract: A new method was proposed for designing two-channel filter banks (FBs) with causal-stable IIR filters. By using IIR filters with a cosine-rolloff transition band, the flatness condition required for two-channel NPR FB was automatically satisfied. Instead of designing the frequency magnitude responses of the analysis filters, the power spectra of the desired filters were designed by solving a quasi-convex problem. When the solution was found, the analysis filters desired can be obtained by spectral factorization. The polyphase components of the analysis filters were assumed to have an identical denominator to simplify the PR condition. Two-channel NPR IIR FB so obtained has a reasonably low reconstruction error and can be employed as the initial guess to constrained nonlinear optimization software for designing the PR IIR FB.

Key words: causal stable IIR filters; cosine-rolloff transition band; filter banks; perfect and nearly perfect reconstruction; spectral factorization

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基于谱分解技术的两通道因果稳定IIR滤波器组的设计

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摘要:提出一种新的两通道因果稳定IIR滤波器组的设计方法.首先基于所定义的一种余弦滚降IIR滤波器,两通道滤波器组所要求的平坦条件完全被满足;其次该方法是依赖于分析滤波器的功率谱特性,而不是该滤波器的幅度频率特性来设计滤波器组.IIR滤波器的设计问题可转化成一个准凸问题,采用凸优化算法来求解;然后对所得优化解进行谱分解,得到目标滤波器组.该滤波器组具有合理的完全重构误差,并且其多相因子的分母相同,简化了完全重构条件.采用该滤波器组作为约束非线性优化法的有效初值,进一步优化得到完全重构2通道IIR滤波器组.

关键词:因果稳定的IIR滤波器;余弦滚降;滤波器组;近似完全重构和完全重构;谱分解

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0 Introduction

Much attention has been paid to the design of two-channel maximally decimated perfect reconstruction filter banks (PR FBs) with IIR filters during the last few decades[1-10]. Compared to FIR filters, IIR filters have the potential to offer lower system delay, sharper cutoff and higher stopband attenuation than their FIR counterparts. However, the PR condition is considerably complicated for two-channel IIR FBs since all the analysis and synthesis filters are IIR filters. In particular, the system is PR if the determinant of the polyphase matrix is a minimum phase function[2]. In addition to the more complicated PR condition, it is also difficult to ensure that the IIR filters are causal-stable.

The design of two-channel PR IIR FBs is usually realized by iterative procedures involving nonlinear constrained optimization. When the number of variables and constraints increases, the optimization procedure is rather sensitive to the initial guess[5-10]. In Ref. [9], a modified model-reduction technique is used to obtain a nearly-PR (NPR) IIR FB from a PR low-delay (LD) FIR FB and the NPR IIR FB is used as an initial guess for some nonlinear constrained optimization to design the PR IIR FB. The drawback of this method is that much time is needed for designing a PR LD FIR FB which also involves nonlinear constrained optimization. By making use of a class of cosine-rolloff (CR) filters defined in Ref. [11], the flatness condition required by FBs is automatically satisfied and the design problem of two-channel IIR FBs is reduced to the design of FIR filers since the poles of IIR filters are located in advance therein[10]. The design problem can be formulated as a convex minimax optimization problem which is solved by second order cone programming (SOCP). However, the poles of the proposed IIR FBs should be predefined artificially to start the design procedure.

In this paper, a new method for the design of two-channel IIR FBs is proposed, inspired when modifying the filter design method via spectral factorization originally proposed in Refs. [12-13] for FIR and IIR filters, respectively. Compared to FIR filters, the design problem of IIR filter is much more difficult to be transformed into a convex form due to the stability property as well as the low group delay. Recently, some works were devoted to designing IIR filters by convex programming[13-16]. In Ref. [13], instead of designing the frequency magnitude responses of the IIR filters, the squared magnitude response, that is, the power spectra of the desired filters are designed. The design problem is reformulated into a quasi-convex problem which can be solved by bisection. Each section involves solving a linear program. When the solution of the new design problem is found, the filter desired can be obtained by spectral factorization which is based on the fast Fourier transform (FFT). Based on pole-zero mapping, IIR filters were designed by a linear programming[14]. In Ref. [15], a semidefinite programming (SDP) relaxation technique was adopted to formulate the design problem of IIR filters in a convex form and a regularized feasibility problem was used together to find the solution. The design problem of IIR filters with minimax phase error was reformed into a convex one by using the Levy-Sanathanan-Koerner (L-SK) strategy and a least-squares method[16]. However, IIR filters proposed in Refs. [13-16] are not suitable to be used as analysis filters due to the complexity of PR condition for the implementation of PR IIR FBs.

In this work, the method in Ref. [13] is modified for solving the design problem of two-channel IIR FBs. To simplify the PR constraints of IIR FBs, it is suggested that the denominators of the desired IIR filters be a polynomial in z^2 and hence the denominator of its polyphase components can be made identical[8-10]. Design results show that two-channel NPR IIR FBs obtained by the proposed method have a good frequency magnitude response and a reasonably low system reconstruction error. By using the IIR FBs designed as the initial guesses to constrained optimization software, high quality PR IIR FBs can be readily obtained.

1 Two-channel IIR filter banks

Fig. 1 shows the structure of a two-channel critically decimated multirate FB, where the input is filtered by two analysis filters, $H_0(z)$ and $H_1(z)$, and is decimated by a factor of two to form two subband signals[1]. To reconstruct the original signal, the subband signals are upsampled, filtered by two synthesis filters, $G_0(z)$ and $G_1(z)$, and added together. The system is called a PR system if the output $\hat{x}(n)$ is identical to the input, $x(n)$, except for some constant scaling and time delay. To cancel the aliasing term, the synthesis filters are chosen as:

$$G_0(z) = H_1(-z), G_1(z) = -H_0(-z) \quad (1)$$

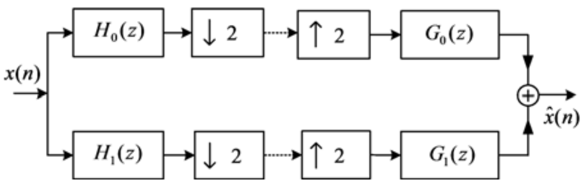

Fig. 1 Two-channel multirate filter bank

If $H_0(z)$ and $H_1(z)$ have good magnitude responses in both passband and stopband, see Fig. 2, the magnitude response of the transfer function is almost constant in the whole frequency region except the transition bands of $H_0(z)$ and $H_1(z)$. Assume the desired frequency responses of two analysis filters, $H_0(z)$ and $H_1(z)$, are defined to have an approximate CR transition band as:

$$\left.\begin{aligned} H_0(\omega) &= \begin{cases} e^{-j\omega D/2}, & \omega \in [0, \omega_{p0}] \\ \cos\left[\frac{\pi}{2\Delta\omega_0}(\omega - \omega_{p0})\right] \cdot e^{-j\omega D/2}, & \omega \in [\omega_{p0}, \omega_{s0}] \\ 0, & \omega \in [\omega_{s0}, \pi] \end{cases} \\ H_1(\omega) &= \begin{cases} 0, & \omega \in [0, \omega_{s1}] \\ \sin\left[\frac{\pi}{2\Delta\omega_1}(\omega - \omega_{s1})\right] \cdot e^{-j\omega D/2}, & \omega \in [\omega_{s1}, \omega_{p1}] \\ e^{-j\omega D/2}, & \omega \in [\omega_{p1}, \pi] \end{cases} \end{aligned}\right\} \quad (2)$$

where $D/2$, ω_{pi}, ω_{si} and $\Delta\omega_i = |\omega_{si} - \omega_{pi}|$, $i=0,1$,

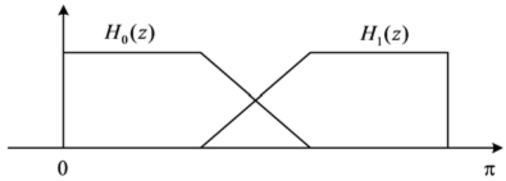

Fig. 2 $H_0(z)$ **and** $H_1(z)$

are the group delay over the passband, the passband and the stopband cutoff frequencies and the transition bandwidths of $H_0(z)$ and $H_1(z)$. $\omega_{p0} = \omega_{s1} = \omega_p$ and $\omega_{s0} = \omega_{p1} = \omega_s$ to make the obtained FB having approximately symmetrical analysis filters. Therefore, the frequency response of the system transfer function $T(z)$ is now given by

$$T(e^{j\omega}) = \frac{1}{2}[\,|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2\,]e^{-j2\omega D} = \frac{1}{2}e^{-j2\omega D} \quad (3)$$

for all $\omega \in [0, \pi]$. Here the system delay $D_{fb} = D$. It can be seen that the magnitude distortion is eliminated in the whole frequency region. Since the flatness condition can be structurally imposed, the design problem is reduced to make $H_0(z)$ and $H_1(z)$ be the filters given in Eq. (2). For IIR FBs, the discussion above still holds except that the analysis filters and synthesis filters are rational functions.

$$\text{Assume: } H_i(e^{j\omega}) = \frac{A_i(e^{j\omega})}{B(e^{j\omega})} = \frac{\sum_{n=0}^{L_{num}-1} a_i(n)e^{-j\omega n}}{\sum_{n=0}^{L_{den}-1} b(n)e^{-j\omega n}},$$

where $i=0,1$, L_{num} and L_{den} are the lengths of the numerator and the denominator, respectively. Note that $H_0(z)$ and $H_1(z)$ have the same denominator and the denominator coefficients are assumed to be a polynomial in z^2 to simplify the PR condition of the FBs[8-10].

2 Design method

Generally, the frequency magnitude response design method for designing $H_0(z)$ in Eq. (2) can be written as:

$$\begin{aligned} &\text{Minimize } \max |H_0(e^{j\omega})| \quad \text{for } \omega \in [\omega_s, \pi] \\ &\text{Subject to } L(\omega) < |H_0(e^{j\omega})| < U(\omega) \\ &\qquad \text{for } \omega \in [0, \omega_s] \end{aligned} \quad (4)$$

where $L(\omega)$ and $U(\omega)$ denote the allowed upper and lower magnitude bounds during the considered frequency region. One can simplify the design problem of $H_1(z)$. However, instead of designing the frequency magnitude responses of the analysis filters, the squared magnitude responses are designed in the proposed method. Let $P_i(e^{j\omega})$, $i=0,1$, be the power spectrums of $H_i(z)$, $i=0,1$:

$$P_i(e^{j\omega}) = R_i(e^{j\omega})/V(e^{j\omega}) = |A_i(e^{j\omega})|^2/|B(e^{j\omega})|^2 = |H_i(e^{j\omega})|^2,\ i=0,1 \tag{5}$$

where $R_i(e^{j\omega})$, for $i=0,1$, and $V(e^{j\omega})$ are the Fourier transform of the sequences $\{r_i(n)\}$ for $i=0,1$, and $\{v(n)\}$. Thus, these sequences are the autocorrelation coefficient of the analysis filters numerators and denominators $\{a_i(n)\}$ for $i=0,1$, and $\{b(n)\}$:

$$\left.\begin{aligned} r_i(n) &= \sum_{k=-\infty}^{\infty} a_i(k)a_i(k+n), \\ &\qquad i=0,1,-(L_{\text{num}}-1)\leqslant n\leqslant L_{\text{num}}-1 \\ v(n) &= \sum_{k=-\infty}^{\infty} b(k)b(k+n), \\ &\qquad -(L_{\text{den}}-1)\leqslant n\leqslant L_{\text{den}}-1 \end{aligned}\right\} \tag{6}$$

The sequences $\{r_i(n)\}$ for $i=0,1$, and $\{v(n)\}$ are all symmetric around $n=0$. The design problem of $H_0(z)$ in Eq. (4) is now rewritten as a quasi-convex problem to find $\{r_0(n)\}$ and $\{v(n)\}$, for $n\geqslant 0$, via Ref. [13].

$$\begin{aligned} &\text{Minimize } \max|P_0(e^{j\omega})| \quad \text{for } \omega\in[\omega_{s1},\pi] \\ &\text{Subject to } L^2(\omega) < P_0(e^{j\omega}) < U^2(\omega) \\ &\qquad\qquad \text{for } \omega\in[0,\omega_{s1}] \end{aligned} \tag{7}$$

Similarly, the design of $H_1(z)$ can also be reformulated as a quasi-convex problem to find $\{r_1(n)\}$ and $\{v(n)\}$ instead of $\{a_1(n)\}$ and $\{b(n)\}$. Here, the denominators of $P_i(e^{j\omega})$ for $i=0,1$ should be the same with each other to simplify the PR condition of FBs.

When a solution of $\{r_i(n)\}$ for $i=0,1$ and $\{v(n)\}$ is found, $\{a_i(n)\}$ for $i=0,1$ and $\{b(n)\}$ can be obtained by spectral factorization. Note that if the denominator of $P_0(e^{j\omega})$, that is $\{v(n)\}$, is constrained to be a polynomial in z^2, the denominator of $H_0(z)$, that is $\{b(n)\}$, is a polynomial in z^2 instantly.

Based on the defined CR filters, the flatness condition required for FBs is satisfied automatically. Thus the design problem of FBs is reduced to the design of CR filters. The filter design problem can be reformulated into a quasi-convex problem by finding the autocorrelation coefficients of filters' coefficients to satisfy the magnitude constraints on the power spectrum of filters. This quasi-convex problem can be solved by bisection and each iteration of the bisection is a linear program.

Since NPR FBs so obtained are of very high quality, they can serve as initial guesses to some constrained nonlinear optimizer to solve for the PR FB with the same filter parameters. The PR condition of IIR FB is given by Refs. [8-10]:

$$A_{00}(z)A_{11}(z) - A_{10}(z)A_{01}(z) = \beta\cdot z^{-s}\hat{B}^2(z) \tag{8}$$

where $\{A_{i0}(z), A_{i1}(z)\}$, $i=0,1$, are the type-I polyphase components of the numerators of $A_i(z)$, $i=0,1$, respectively and $\hat{B}(z)$ is the type-I polyphase components of $B(z)$. β is a non-zero constant and s is an integer. A possible objective function is given as:

$$\Phi(\boldsymbol{x}) = \int_0^{\omega_p} |1-|H_0(e^{j\omega})||^2 d\omega + \int_{\omega_s}^{\pi} |H_0(e^{j\omega})|^2 d\omega + \int_{\omega_s}^{\pi} |1-|H_1(e^{j\omega})||^2 d\omega + \int_0^{\omega_p} |H_1(e^{j\omega})|^2 d\omega \tag{9}$$

where ω_p and ω_s are the passband and the stopband cutoff frequencies of $H_0(z)$. The variables vector $\boldsymbol{x}$ contains the part of $A_{ij}(z)$, $i,j=0,1$, as well as the part of $\hat{B}(z)$. The constrained optimization can be stated as follows:

$$\begin{aligned} &\min \Phi(\boldsymbol{x}) \\ &\text{subject to: } \{\max(|t_i|) < 1\} \text{ and the PR condition} \end{aligned} \tag{10}$$

where t_i's are the roots of $B(z)$ which should remain in the unit circle for the stability of the IIR filters. The PR condition in Eq. (8) suggests that the length of the denominator in the polyphase components should not be longer than that of the numerator in order to balance the powers of z on both sides of Eq. (10).

3 Design procedure and examples

In Ref. [13], a general lowpass IIR filter is designed by the spectral factorization method which is modified and used to solve the design of the proposed FB with CR filters in this work. To keep the denominators of $P_i(e^{j\omega})$ for $i=0,1$ the same as each other, $\{b(n)\}$ is determined in the design of $H_0(z)$ and used in the design of $H_1(z)$. It is well known that there is a tradeoff among the transition bandwidth, the ratio of passband and stopband ripples of analysis filters and the reconstruction error of the FB. The stopband attenuation of the designed analysis filters is adjusted iteratively to minimize the lower reconstruction error. A design procedure is presented as follows.

Design procedure:

Given the length of the numerator and the denominator, L_{num} and L_{den}, of the desired analysis filters, the passband and the stopband cutoff frequencies, ω_p and ω_s. Initialize the desired stopband attenuation ripple error δ_s and the step sizes to update δ_s: δ_{istep} and δ_{dstep}. The flag to stop the iteration is initialized as t=0.

1. Solve the CR filter $H_0(z)$ in (2) with the given (ω_p, ω_s) and δ_s:
   1.1 find the solution of $P_0(z)$ in (7), i. e. the solution of $\{r_1(n)\}$ and $\{v(n)\}$.
   1.2 obtain $\{a_1(n)\}$ and $\{b(n)\}$ by spectral factorization.
2 obtain the CR filter $H_1(z)$ in (2) by using $\{b(n)\}$:
   2.1 find the solution of $P_1(z)$, i. e. the solution of $\{r_2(n)\}$
   2.2 obtain $\{a_1(n)\}$ by spectral factorization.
3 Calculate the reconstruction error E_{PP} which is the maximum peak-to-peak ripple of the transfer function T(z).
   If $E_{PP} < E_{PP\_min}$
   {Record the FB and set $E_{pp\_min} = E_{PP}$;
   increase δ_s to $\delta_s * \delta_{istep}$;
   t=0;}
   Else
   {decrease δ_s to $\delta_s * \delta_{dstep}$;
   t=t+1; {if t>5, stop;}
   Go back to step 1.

If E_{PP} does not decreased in t (such as $t=5$ in this work) for successive times, the local solution is assumed to have been found. To keep the procedure from becoming an endless loop, the step sizes to update δ_s are set to $\delta_{istep} * \delta_{dstep} \neq 1$. After the NPR IIR FB is obtained, they will be used as the initial guesses for some nonlinear constrained software to obtain the PR IIR FBs. Note that the group delay of filters is not considered during the design of NPR IIR FBs. However, we can control the whole system delay D during the design of PR FBs. To illustrate the efficiency of the proposed method, some examples are given in this part.

Example 1 Design of two-channel IIR FBs with $L_{num}=24$

The numerator lengths L_{num} of all the FBs in this example are 24 while the denominator lengths L_{den} are different from each other as illustrated in Tab. 1. The passband and stopband cutoff frequencies of $H_0(z)$ are set to be $(\omega_p, \omega_s) = (0.4\pi, 0.6\pi)$. The desired stopband attenuation ripple error δ_s is initialized to be 1e−5. The step sizes are set as: $\delta_{istep}=5$ and $\delta_{dstep}=0.25$. For obtaining a good NPR FB, the proposed method costs 7 or 8 iterations which are denoted as kt in Tab. 1. A_{s_NPR} (dB) is the highest stopband attenuation of the analysis filters for NPR FBs and E_{PP_NPR} is the maximum peak-to-peak ripple of the transfer functions $T(z)$.

Tab. 1 Two-channel IIR FBs with $L_{num}=24$

FBs	L_{den}	kt	A_{s_NPR} /dB	E_{PP_NPR}	A_{s_PR} /dB	E_{PP_PR}	D	Iters
FB-1	4	8	−40.70	7.59e−2	−29.94	6.99e−15	9	467
FB-2	6	7	−41.69	9.59e−2	−32.80	1.37e−15	13	408
FB-3	8	8	−41.35	3.55e−2	−33.78	1.66e−15	17	266

By using these NPR FBs as initial guesses for Fmincon in Matlab, PR IIR FBs are obtained. The highest stopband attenuation and the maximum peak-to-peak ripple of the transfer functions of the PR IIR FBs, A_{s_PR}(dB) and E_{PP_PR}, are also listed in Tab. 1. D indicates system delays which are different from each other for the designed PR IIR

FBs. All NPR IIR FBs in Tab. 1 have almost the same highest stopband attenuation of the analysis filters because the desired stopband attenuation ripple errors δ_s are initialized to be the same to start the design procedure. Iters denotes the number of iterations for $F_{\rm mincon}$ to converge to a PR FB. It can be seen that FB-2 has a higher stopband attenuation than FB-1 by 2. 86 dB and FB-3 has a higher stopband attenuation than FB-2 by 0. 98 dB due to a longer denominator used for FB-2 and FB-3, respectively. The magnitude responses of the NPR and PR analysis FB-1 and FB-3 are plotted in Figs. 3(a) and 3(b), respectively.

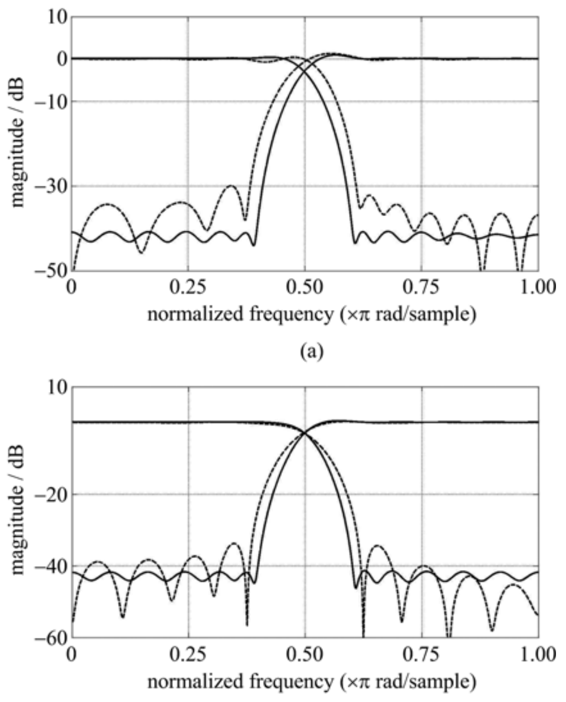

Fig. 3 The magnitude responses of analysis FB of (a) NPR IIR FB-1 (in solid-line) and PR IIR FB-1 (in dash-line); (b) NPR IIR FB-3 (in solid-line) and PR IIR FB-3 (in dash-line)

For comparison purposes, PR IIR FBs with the same specifications of FBs in Tab. 1 are designed using the model-reduction method proposed in Ref. [9]. Low delay (LD) PR FIR FBs with a longer filter length are designed first and then model reduced to NPR IIR FBs which are used as initial guesses for some constrained nonlinear optimizer to obtain PR IIR FBs. The performances of PR FIR and IIR FBs are listed in Tab. 2. L is the length of LD PR FIR FBs and ($L_{\rm num}$, $L_{\rm den}$) denote the length of the numerator and the denominator of PR IIR FBs.

Tab. 2 Two-channel FBs designed with the model-reduction method

FBs	L($L_{\rm num}$, $L_{\rm den}$)	A_{s_PR}/dB	E_{PP_PR}	D	Iters
PR FIR FB-1	34	−31. 10	6. 88e−15	9	643
PR IIR FB-1	(24,4)	−28. 72	2. 66e−15	9	2419
PR FIR FB-2	34	−31. 16	3. 10e−15	13	392
PR IIR FB-2	(24,6)	−30. 30	1. 88e−15	13	483
PR FIR FB-3	34	−39. 24	1. 33e−15	17	406
PR IIR FB-3	(24,8)	−31. 12	7. 66e−15	17	226

It is clear that the proposed method leads to two-channel PR IIR FBs with a higher stopband attenuation (−29. 94 dB vs. −28. 72 dB for FB-1, −32. 80 dB vs. −30. 30 dB for FB-2 and −33. 78 dB vs. −31. 12 dB for FB-3). On the other hand, a lot of time is needed for the model reduction method to obtain a LD PR FIR FB with a longer filter length.

Example 2 Design of two-channel IIR FBs with $L_{\rm num}=16$

Some two-channel NPR and PR IIR FBs with $L_{\rm num}=16$ are designed with the proposed method as listed in Tab. 2. The passband and stopband cutoff frequencies of $H_0(z)$ are set to be $(\omega_p, \omega_s)=(0.41\pi, 0.59\pi)$. The desired stopband attenuation ripple error is initialized to be 1e−3. The step sizes are set as: $\delta_{istep}=5$ and $\delta_{dstep}=0.25$. D equals 7 for all FBs.

For comparison, two-channel IIR FB-6 listed in Tab. 3 was designed with the same filter parameters with the method reported in Ref. [10]. As can be seen from Tab. 3, the proposed NPR and PR IIR FB-5 has a better stopband attenuation than the NPR and PR IIR FB-6 by 10. 51 dB and 1. 3 dB, respectively. Fig. 4 illustrates the magnitude response of the proposed PR FB-5.

Tab. 3 Two-channel IIR FBs with $L_{\rm num}=16$

FB	$L_{\rm den}$	A_{s_NPR}/dB	E_{PP_NPR}	A_{s_PR}/dB	E_{PP_PR}	D
FB-4	4	−22. 35	3. 15e−2	−26. 92	9. 43e−15	7
FB-5	6	−22. 71	2. 06e−2	−26. 69	7. 27e−15	7
FB-6[10]	6	−12. 20	8. 61e−2	−25. 39	3. 83e−15	7

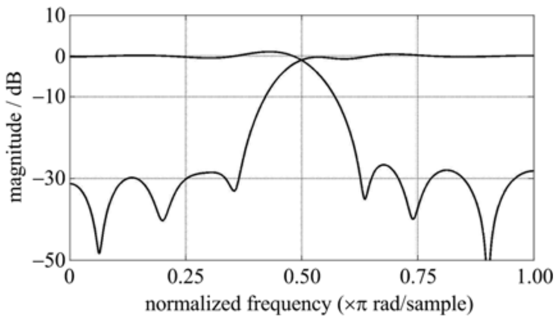

Fig. 4 The magnitude response of analysis PR IIR FB-5

4 Conclusion

The IIR lowpass and the highpass filters of two-channel IIR FBs are assumed to be with a CR transition band. The design problem of IIR FB is reduced to the design of the CR filters since the flatness condition required by FBs is automatically satisfied. Instead of designing the frequency magnitude responses of the analysis filters, the power spectra of the desired filters are designed by solving a quasi-convex problem. When the solution is found, the analysis filters desired can be obtained by spectral factorization. To simplify the PR condition, the polyphase components of the analysis filters have an identical denominator. After the NPR IIR FBs has been obtained, they are further optimized to obtain high quality PR IIR FBs.

References

[1] Vaidyanathan P P. Multirate Systems and Filter Banks [M]. NJ, USA: Prentice Hall, 1993.

[2] Vaidyanathan P P. Multirate digital filters, filter banks, polyphase networks and applications: A tutorial [J]. Proceedings of IEEE, 1990, 78(1):56-93.

[3] Basu S, Chiang C H, Choi H N. Wavelets and perfect reconstruction subband coding with causal stable IIR filters [J]. IEEE Transactions on Circuits and Systems-II, 1995, 42(1): 24-38.

[4] Phoong S M, Kim C W, Vaidyanathan P P, etc al. A new class of two-channel biorthogonal filter banks and wavelet bases [J]. IEEE Transactions on Signal Processing, 1995, 43(3): 649-665.

[5] Tsui K M, Chan S C. Multi-plet two-channel perfect reconstruction FIR and causal stable IIR filter banks [J]. IEEE Transactions on Circuits and Systems-I, 2006, 53(12): 2 804-2 817.

[6] Bregovicć R, Saramäki T. Design of causal stable perfect reconstruction two-channel IIR filter banks [C]// Proceedings of the 2nd International Symposium on Image and Signal Processing and Analysis. Pula, Yugoslavia: IEEE Press, 2001: 545-550.

[7] Okuda M, Fukuoka T, Ikehara M, et al. Design of causal IIR filter banks satisfying perfect reconstruction [C]// Proceedings of the International Symposium on Circuits and Systems. Hongkong, China: IEEE Press, 1997, 4: 2 433-2 436.

[8] Mao J S, Chan S C, Ho K L. Theory and design of causal stable IIR PR cosine-modulated filter banks [C] // Proceedings of the IInternational Symposium on Circuits and Systems. Orlando, USA: IEEE Press, 1999, 3: 427-430.

[9] Yin S S, Chan S C. Design and factorization of two-channel perfect reconstruction filter banks with causal-stable IIR filters [J]. Journal of Signal Processing Systems, 2010, 68(3): 353-366.

[10] Yin S S. New method for designing of two-channel causal-stable IIR filter banks[C] // Proceedings of the 10th International Conference on Signal Processing. Beijing, China: IEEE Press, 2010, 1: 22-26.

[11] Yin S S, Chan S C, Tsui K M. On the design of nearly-PR and PR FIR cosine modulated filter banks having approximate cosine-rolloff transition band[J]. IEEE Transactions on Circuits and Systems-II, 2008, 55(6): 571-575.

[12] Wu S P, Boyd S, Vandenberghe L. FIR filter design via semidefinite programming and spectral factorization [C]// Proceeding of the 35th IEEE Conference on Decision and Control. Boston, USA: Birkhäuser, 1996, 1:271-276.

[13] Rabiner L R, Graham N Y, Helms H D. Maximize stopband attenuation of a lowpass IIR filter[EB/OL]. http://cvxr. com/cvx/examples/filter_design/iir_mag_design_lowpass_max_atten. m.

[14] Quelhas M F, Petraglia A, Petraglia M R. Linear programming for the design of IIR filters [C]// Proceedings of the 3rd Latin American Symposium on Circuits and Systems. Playa del Carmen, Mexico: IEEE Press, 2012: 1-4.

[15] Jiang A M, Kwan H K. Minimax design of IIR digital filters using SDP relaxation technique [J]. IEEE Transactions on Circuits and Systems-I, 2010, 57(2): 378-390.

[16] Lai X P, Lin Z P. Minimax phase error design of IIR digital filters with prescribed magnitude and phase responses [J]. IEEE Transactions on Signal Processing, 2012, 60(2): 980-986.